2013 IEEE/RSJ International Conference on
Intelligent Robots and Systems (IROS)
November 3-7, 2013. Tokyo, Japan

Design of Nonlinear $\mathcal{H}_\infty$ Optimal Impedance Controllers

Min Jun Kim and Wan Kyun Chung

Abstract— **In this paper, nonlinear $\mathcal{H}_\infty$ optimal design of impedance controllers is proposed based on the nonlinear robust internal-loop compensator (NRIC) framework. Simply adding PD-type auxiliary input to the original control law, the robust performance and the robust stability are achieved. Nonlinear $\mathcal{H}_\infty$ optimality is guaranteed by solving Hamilton-Jacobi-Isaacs (HJI) equation and the disturbance input-to-state stability (ISS) is guaranteed by finding ISS-Lyapunov function. Moreover, it is shown that the proposed method preserves the passivity of the impedance controllers. The proposed method can be applied to various types of impedance controllers in a unified way. Through simulations and experimental studies, the proposed method is verified.**

I. INTRODUCTION

Impedance control has been extensively and intensively studied since it was firstly proposed in [1]. Especially in recent years, impedance control methods are gathering more and more interests with growing interest of force control and/or safety robotics [2], [3]. The goal of the impedance control is to realize a desired impedance behavior of the end-effector. Conceptually, as long as the perfect robot model is available, the impedance control is able to render the end-effector dynamics into the desired impedance behavior exactly. When implementing an impedance controller to a real robot, however, the performance is sometimes not satisfactory.

The main causes of this problem are disturbances such as modeling uncertainty, joint friction/damping from various sources, etc. One typical source is high reduction ratio transmission which causes high friction. Moreover, the highly reduced transmission makes an electric motor far from being an ideal torque source since it amplifies all the negative effects in the motor-side such as back-emf effects, frictions, etc. Equipment of the gear transmission, however, is usually unavoidable due to the limited performance of electric motors, especially for the light-weight-oriented robots. Therefore, it can be said that the ideal performance of the impedance controllers is no longer guaranteed when applied to real robots due to the disturbances from various sources of which identification/compensation is difficult in practice.

To deal with robustness problems, several methods have been proposed to implement robust impedance controllers by many. For example, adaptive law is incorporated in [4], [5]. However, adaptive methods are sensitive to unmodeled dynamics. In [6], sliding mode control scheme is applied. This method, however, requires information about modeling-parameter error bounds which are unknown in practice. The error bounds are treated as control gains in the implementation and make it difficult. One interesting recent result is [7] which is a method based on the internal model control. However, extension to a general redundant robots seems not clear. One of the state-of-the-art is [8] which uses the joint torque measurement feedback in order to reshape motor-side inertia and reduce adverse effects of the motor-side dynamics. This approach, however, requires joint torque sensors which many of current robot systems usually are not equipped with.

As a solution to the robustness issues, this paper proposes robust impedance controllers based on the nonlinear $\mathcal{H}_\infty$ optimal control technique. The proposed method can be applied to redundant robots and does not necessarily require joint torque measurements. Furthermore, aforementioned disturbances can be treated in a systematic way; $\mathcal{H}_\infty$ control approach is attractive since it successfully brings the disturbance into the formulation. For this reason, $\mathcal{H}_\infty$ optimal control methods are widely adopted in the linear control systems. However, it is well-known that solving nonlinear $\mathcal{H}_\infty$ control problems is very difficult since we have to deal with a multi-variable partial differential equation which is known as Hamilton-Jacobi-Isaacs (HJI) equation [9], [10].

Fortunately, recently developed nonlinear robust internal-loop compensator (NRIC) framework provides a somewhat general method to deliver the nonlinear $\mathcal{H}_\infty$ optimality to a controller [11]. Thus, using the NRIC framework, it is possible to incorporate nonlinear $\mathcal{H}_\infty$ optimality to impedance controllers. The key idea is to find an auxiliary input that attenuates the difference between real and nominal plant in the sense of nonlinear $\mathcal{H}_\infty$ optimality. The resulting auxiliary input is given in a simple PD-type form. Simply adding the auxiliary input to the existing impedance control inputs, the robust performance and robust stability can be achieved. Moreover, it is shown that the proposed method preserves the passivity of the impedance controllers by finding a proper storage function. It should also be mentioned that the proposed method is not restricted to a specific type of impedance controller. In other words, it can be applied to various types of impedance controllers in a unified way.

This paper is organized as follows. In the section 2, classical impedance controllers are revisited. Section 3 introduces the design of nonlinear $\mathcal{H}_\infty$ optimal impedance controllers based on the NRIC framework. Optimality, stability, and passivity analysis are provided. Section 4 verifies the proposed method through simulation and experimental studies. Finally, section 5 concludes the paper.

The authors are with the Robotics Laboratory, School of Mechanical Engineering, Pohang University of Science and Technology (POSTECH), Pohang, 790-784, Gyung-buk, Korea {mjkim0229, wkchung}@postech.ac.kr

II. Design of Impedance Controllers

This section introduces design of classical impedance controllers. Starting from (kinematically) non-redundant case, two solutions for redundancy problem are introduced. The details of derivation are omitted since the contents in this section are already well-established results.

A. Non-redundant Case

Robot kinematics and dynamics are described by:

$$\dot{p} = J(q)\dot{q} \tag{1}$$

$$\tau + \tau_{ext} = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) \tag{2}$$

with known notations. Moreover, the transpose of the Jacobian matrix, $J(q)^T$, defines relation between joint torque and force at the end-effector:

$$\tau = J(q)^T f \tag{3}$$

The purpose of impedance control is to render the end-effector behavior into a desired impedance behavior. To this end, it is sometimes convenient to describe the robot dynamics in the operational space instead of joint space. Substituting (1) into (2), the operational space dynamics is obtained as:

$$f + f_{ext} = \Lambda(q)\ddot{p} + \Gamma(q,\dot{q})\dot{p} + \zeta(q) \tag{4}$$

where, Λ, Γ, and ζ represent operational space Inertia, Coriolis/centrifugal force, gravity matrices/vector, respectively. Now, it is clear that the control input $f_c = -(D\dot{p} + Ke_p) + \Gamma\dot{p} + \zeta$ renders (4) into the desired impedance behavior

$$f_{ext} = \Lambda\ddot{p} + D\dot{p} + Ke_p \tag{5}$$

where, $e_p = p - p_D$ and subscript D stands for desired. Using the relation (3), the resulting impedance control law is $\tau_c = J(q)^T f_c$. This control law, however, is valid only for the non-redundant robots. Therefore, the following sections describe methods that deal with redundancy issue.

B. Redundant Case: Null-space Projection Method

One of the most common approach to deal with redundancy is projecting damping to the null-space [12]. In the redundant case, Jacobian matrix $J(q)$ is no longer square (i.e., non-invertible) and therefore weighted pseudo-inverse $J_W^\dagger = W^{-1}J^T(JW^{-1}J^T)^{-1}$ is usually adopted instead of J^{-1}. Furthermore, (3) is extended as

$$\tau = J(q)^T f + (I - J(q)^T J(q)_W^{\dagger T})\xi \tag{6}$$

where ξ is an arbitrary vector. Note that, letting $W(q) = M(q)$ gives the dynamically consistent generalized inverse [13].

Under the redundant situation, the control law $\tau_c = J(q)^T f_c$ is still a stabilizing controller, but the asymptotic stability is not guaranteed anymore. By properly selecting ξ, the asymptotic stability can be guaranteed. One specific example of null-space control vector is $\xi_c = D\dot{q}$, where D is a positive definite matrix [13].

Consequently, the impedance control law for redundant robot is

$$\tau_c = J(q)^T f_c + (I - J(q)^T J(q)_W^{\dagger T})\xi_c \tag{7}$$

C. Redundant Case: Null-space Parameterization Method

Sometimes, it is useful to express null-space explicitly rather than simply projecting joint damping to the null-space [14]–[16]. Minimal parameterization of the null-space can be done by incorporating the null-space basis matrix $Z(q)$ [17], [18]. The forward kinematics (1) can be extended as[1]

$$\begin{pmatrix} \dot{p} \\ \dot{n} \end{pmatrix} = \begin{bmatrix} J(q) \\ Z(q)W(q) \end{bmatrix} \dot{q} \tag{8}$$

Using (8), operational space dynamics can be expressed as

$$\begin{aligned} \begin{pmatrix} f + f_{ext} \\ \eta + \eta_{ext} \end{pmatrix} &= \begin{bmatrix} \Lambda_{W,pp} & \Lambda_{W,pn} \\ \Lambda_{W,np} & \Lambda_{W,nn} \end{bmatrix} \begin{pmatrix} \ddot{p} \\ \ddot{n} \end{pmatrix} \\ &+ \begin{bmatrix} \Gamma_{W,pp} & \Gamma_{W,pn} \\ \Gamma_{W,np} & \Gamma_{W,nn} \end{bmatrix} \begin{pmatrix} \dot{p} \\ \dot{n} \end{pmatrix} + \begin{pmatrix} \zeta_{W,p} \\ \zeta_{W,n} \end{pmatrix} \end{aligned} \tag{9}$$

Again, by letting $W(q) = M(q)$, we arrive at the dynamically decoupled property, i.e., $\Lambda_{W,pn} = \Lambda_{W,np} = 0$, which is consistent to the null-space projection method [19].

Various control strategies can be implemented using (9). This paper, for example, considers null-space suppression control by letting the null-space control input as $\eta_c = \Lambda_{W,nn}(\ddot{n} + k_{nv}\dot{e}_n + k_{np}e_n) + \Gamma_{W,np}\dot{p} + \Gamma_{W,nn}\dot{n} + \zeta_{W,n}$.

Consequently, the resulting control input is

$$\tau_c = J(q)^T f_c + W(q)Z(q)^T \eta_c \tag{10}$$

D. Miscellaneous Notes

The impedance control laws (7), (10) both guarantee the globally asymptotically stability of the equilibrium point. Therefore, by the converse Lyapunov theorem, a Lyapunov function that satisfies the following conditions can be found (Theorem 23 and definitions (viii), (x) in [20]).

$$V(x,t) > 0 \tag{11}$$

$$V(x,t) \to \infty \quad as \quad \|x\| \to \infty \tag{12}$$

$$\dot{V}(x,t) \leq -\gamma(\|x\|) \tag{13}$$

It should also be mentioned that the impedance control laws can be easily modified to the passive version. Since the skew-symmetricity of $\dot{M} - 2C(q,\dot{q})$ plays a key role in the passivity analysis, if we do not cancel out the Coriolis/centrifugal forces in the control law, it automatically becomes a passive impedance controller.

The performance and the stability, however, are sometimes not satisfactory when implemented to real robots due to the various kinds of disturbances. The source of the disturbance can be anything, e.g., modeling uncertainty, joint friction, joint damping, and so on. Thus, the following sections describe a method to overcome the robustness problem using nonlinear $\mathcal{H}_\infty$ optimal control technique.

[1] Using $Z(q)W(q)$, not $Z(q)$, in the forward kinematics (8) is to establish kinematically decoupled relation. Namely, $(ZW)J_W^\dagger\dot{p} \equiv 0$ and $JZ^\#\dot{n} \equiv 0$, where $Z^\# = Z^T(ZWZ^T)^{-1}$ is a pseudo-inverse of $Z(q)W(q)$.

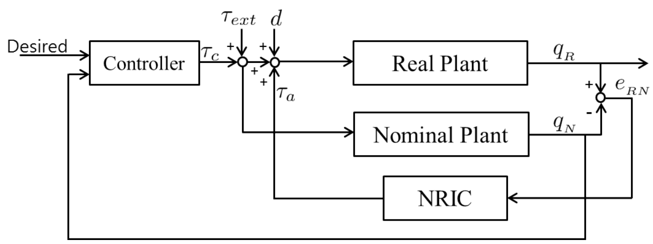


Fig. 1. The structure of NRIC.

III. Incorporating Nonlinear $\mathcal{H}_\infty$ Optimality

In this section, the nonlinear $\mathcal{H}_\infty$ optimality will be brought into the impedance controllers.

Before going further, several notations used in this paper must be defined. In the Fig. 1, subscript D denotes desired signal, subscript R denotes real (measured) signal and subscript N denotes nominal signal which is the expected signal from the nominal plant under the control input τ_c. Thus, for example, q_D is a desired displacement vector, q_R is a displacement vector of the real plant and q_N is that of a nominal plant. Furthermore, error-related quantities are defined as

$$e_{RN} = q_R - q_N \tag{14}$$

$$e_{DN} = q_D - q_N \tag{15}$$

$$e_{DR} = q_D - q_R \tag{16}$$

Also, $x_{(\cdot)}$ is a state related to $(\cdot)$. For example, x_{RN} is a state related to e_{RN}, i.e., $x_{RN} = \left[\begin{array}{cc} e_{RN}^T & \dot{e}_{RN}^T \end{array}\right]^T$.

A. Nonlinear Robust Internal-loop Compensator Framework

In order to incorporate nonlinear $\mathcal{H}_\infty$ optimality, so called nonlinear robust internal-loop compensator (NRIC) framework is applied whose structure is shown in the Fig. 1. As can be seen in the Fig. 1, a nominal plant is augmented to a real one. Therefore, the number of states and equations is increased. From the figure, the closed loop dynamics of real plant and nominal plant is given as

$$\tau_{ext} + \tau_c + \tau_a + d = M_R\ddot{q}_R + C_R\dot{q}_R + g_R \tag{17}$$

$$\tau_{ext} + \tau_c = \hat{M}_N\ddot{q}_N + \hat{C}_N\dot{q}_N + \hat{g}_N \tag{18}$$

where, M, C, g are known notations and $\hat{(\cdot)}$ represents estimated (nominal) parameter of $(\cdot)$. Moreover, M_R represents an inertia matrix of which variable is q_R, i.e., $M_R \triangleq M(q_R)$. Likewise, M_N, $\hat{M}_R$, C_R, etc can be defined in a similar way. Note that any kinds of disturbances could be contained in d.

In the framework, instead of nominal plant equation (17), the difference between real and nominal plant is used. Subtracting (18) from (17), we obtain the difference system as follows

$$\tau_a + w = M_R\dot{s} + C_R s \tag{19}$$

where, $s \triangleq \dot{e}_{RN} + K_P e_{RN}$ and K_P is a symmetric positive definite matrix. w is treated as an extended disturbance and can be expressed as follows after some algebraic manipulation.

$$\begin{aligned} w = &M_R\big((M_R^{-1} - \hat{M}_N^{-1})\tau_c + K_P\dot{e}_{RN}\big) \\ &+ C_R\big((I - C_R^{-1}M_R\hat{M}_N^{-1}\hat{C}_R)\dot{e}_{DN} + K_P e_{RN}\big) \\ &+ (M_R\hat{M}_N^{-1}\hat{g}_N - g_R) + (M_R\hat{M}_N^{-1}\hat{C}_N - C_R)\dot{q}_D + d \end{aligned} \tag{20}$$

The extended disturbance w contains modeling uncertainty as well as disturbance d.

As a result, the equations describing the whole system are (18) and (19). The states describing the system are x_{RN} and x_{DN}.

Also, note that the state-space form of (19) can be expressed as follows.

$$\dot{x}_{RN} = Ax_{RN} + Bw + B\tau_a \tag{21}$$

where,

$$A = \left[\begin{array}{cc} 0 & I \\ -M_R^{-1}C_R K_P & -M_R^{-1}C_R - K_P \end{array}\right] \tag{22}$$

and

$$B = \left[\begin{array}{c} 0 \\ M_R^{-1} \end{array}\right] \tag{23}$$

Please note that A, B are functions of state variables which are omitted for the simplicity.

The purpose of the NRIC framework is to find an auxiliary input τ_a that attenuates the difference between real and nominal plant. To tell the result first, the auxiliary input τ_a is given as

$$\tau_a = -(K + \frac{1}{\gamma^2})(\dot{e}_{RN} + K_P e_{RN}) \tag{24}$$

In the following sections, optimality, stability, and the passivity analysis are investigated.

B. Optimality

The nonlinear $\mathcal{H}_\infty$ control problem is to find a control input that satisfies the following nonlinear $\mathcal{L}_2$-gain attenuation requirement.

$$\int_0^t (x_{RN}^T Q(x_{RN})x_{RN} + \tau_a^T R(x_{RN})\tau_a)dt \leq \gamma^2 \int_0^t w^T w dt \tag{25}$$

To solve the nonlinear $\mathcal{H}_\infty$ optimal control problem, it is sufficient to solve the Hamilton-Jacobi-Isaacs (HJI) equality: Let $\gamma > 0$ and if there exists a continuously differentiable solution $V(x_{RN}, t) \geq 0$ with $V(0,t) = 0$ satisfies

$$\begin{aligned} HJI_\gamma(x,t;V) \triangleq &V_t + V_x A x_{RN} - \frac{1}{2}V_x\Big[BR^{-1}B^T \\ &- \frac{1}{\gamma^2}BB^T\Big]V_x^T + \frac{1}{2}x_{RN}^T Q x_{RN} = 0 \end{aligned} \tag{26}$$

where V_t, V_x represent partial differentiation of V with respect to t and x_{RN}, respectively. Then, the control input

$$\tau_a = -R^{-1}B^T V_x^T \tag{27}$$

satisfies the nonlinear $\mathcal{L}_2$-gain attenuation requirement (25) for the $\mathcal{L}_2$-gain $\gamma > 0$.

However, dealing with the HJI equation is not easy since it is a multi-variable partial differential inequality problem. Therefore, this paper takes the inverse solution approach [10], [21].

Let $V(x_{RN}, t)$, R as

$$V(x_{RN}, t) = \frac{1}{2} x_{RN}^T P x_{RN}, \quad R = \Big(K + \frac{1}{\gamma^2} I\Big)^{-1} \tag{28}$$

where,

$$P = \begin{bmatrix} K_P M_R K_P + K_P K & K_P M_R \\ M_R K_P & M_R \end{bmatrix} \tag{29}$$

Using (28), $HJI_\gamma = 0$ is reduced to the differential Riccati equation:

$$\dot{P} + A^T P + PA - PB\Big(R^{-1} - \frac{1}{\gamma^2} I\Big) B^T P + Q = 0 \tag{30}$$

Now, Q can be obtained from (30):

$$Q = \begin{bmatrix} K_P^2 K & 0 \\ 0 & K \end{bmatrix} \tag{31}$$

As a result, with given weightings Q and R, we can conclude that $V = \frac{1}{2} x_{RN}^T P x_{RN}$ is a solution of $HJI_\gamma = 0$. Therefore, $\tau_a = -R^{-1} B^T V_x^T = -(K + \frac{1}{\gamma^2})(\dot{e}_{RN} + K_P e_{RN})$, provided in (24), is a solution of nonlinear $\mathcal{H}_\infty$ optimal control problem.

C. Stability

In this section, the extended disturbance w input-to-state stability (ISS) is investigated (see appendix if not familiar with ISS). To this end, before investigating the stability of the overall system, the stability of nominal system and the difference system are investigated separately[2].

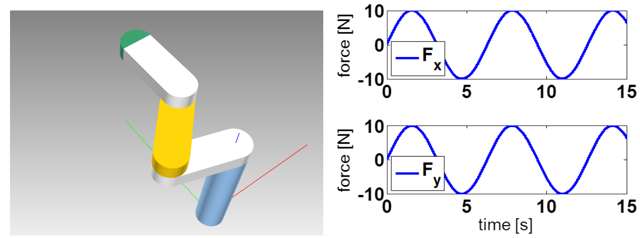
Firstly, for the nominal system (18), recall that there exists a Lyapunov function $V_{DN}(x_{DN}, t)$ that satisfies (11)-(13) since the control input is made using nominal states. Secondly, if we consider only x_{RN} as a variable in (19), $V = \frac{1}{2} x_{RN}^T P x_{RN}$ is an ISS-Lyapunov function of the difference system (19): V is positive definite and radially unbounded since V can be rewritten as

$$\begin{aligned} V &= \frac{1}{2} x_{RN}^T P x_{RN} \\ &= \frac{1}{2} s^T M_R s + \frac{1}{2} e_{RN}^T (K_P K) e_{RN} \end{aligned} \tag{32}$$

Furthermore, the derivative along the trajectory is

$$\begin{aligned} \dot{V} =& V_t + V_x A x_{RN} + V_x B \tau_a + V_x B w \\ =& \frac{1}{2} x_{RN}^T (\dot{P} + A^T P + PA) x_{RN} \\ & - x_{RN}^T PBR^{-1} B^T P x_{RN} + x_{RN}^T P w \\ =& -\frac{1}{2} x_{RN}^T (Q + PBKB^T P) x_{RN} + \gamma^2 \|w\|^2 \end{aligned} \tag{33}$$

[2]By means of "separately", only x_{RN} is considered as a variable of (19) even though both x_{RN} and x_{DN} show up in the (19). Then, we can consider (18) and (19) as separate systems that have no relation to each other. Note that only x_{DN} shows up in the (18), which means that only x_{DN} is a variable of (18) from the beginning.



(a) (b)

Fig. 2. (a) the 3-DOF planar arm used in the simulation. (b) forces applied at the end-effector.

where, $\tau_a = -R^{-1} B^T V_x^T$, $HJI_\gamma = 0$ (30), and Young's inequality $x_{RN}^T PBw \le \frac{1}{\gamma^2} \|x_{RN}^T PB\|^2 + \gamma^2 \|w\|^2$ are used. Therefore, we conclude that $V = \frac{1}{2} x_{RN}^T P x_{RN}$ is an ISS-Lyapunov function of (19).

However, when we consider both x_{RN}, x_{DN} as variables at the same time, P matrix in (29) is a function of both x_{RN} and x_{DN} since M_R contained in P can be written as $M_R = M(q_R) = M(e_{RN} - e_{DN} + q_D)$. Now, letting $V_{RN}(x_{RN}, x_{DN}, t) = x_{RN}^T P(x_{RN}, x_{DN}, t) x_{RN}$, the following properties can be derived.

$$V_{RN}(x_{RN}, x_{DN}, t) > 0 \tag{34}$$

$$V_{RN}(x_{RN}, x_{DN}, t) \to \infty \quad as \quad \|x_{RN}\| \to \infty \tag{35}$$

but,

$$V_{RN}(x_{RN}, x_{DN}, t) \nrightarrow \infty \quad as \quad \|x_{DN}\| \to \infty \tag{36}$$

$$\dot{V}_{RN}(x_{RN}, x_{DN}, t) \le -\gamma_{RN}(\|x_{RN}\|) + \gamma_w(\|w\|) \tag{37}$$

Finally, to establish the extended disturbance w ISS for the overall system, define an ISS-Lyapunov function as $V_o(x_{RN}, x_{DN}, t) = V_{RN}(x_{RN}, x_{DN}, t) + V_{DN}(x_{DN}, t)$. Then, combining (11)-(13) and (34)-(37), it is clear that the overall system is extended disturbance w ISS.

D. Passivity

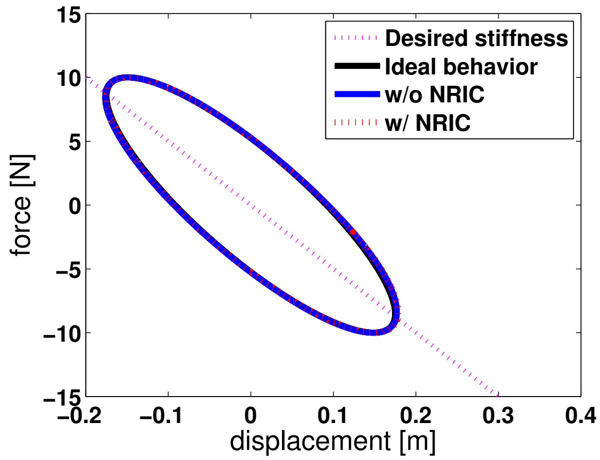
This section provides the passivity property of the overall system. First, we assume that the impedance controller is designed as a passive one. In other words, there exists a storage function S_{DN} that satisfies $\dot{S}_{DN} \le \tau_{ext}^T \dot{q}_N$. Now, let us define a storage function for the overall system as

$$S_o(x_{RN}, x_{DN}, t) = \frac{1}{2} s^T M_R s + S_{DN}(x_{DN}, t) \tag{38}$$

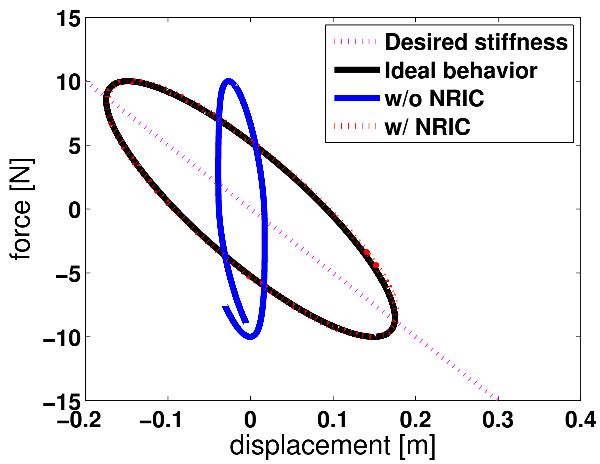
Then, the differentiation along the trajectory is

$$S_o(x_{RN}, x_{DN}, t) \le w^T s + \tau_{ext}^T \dot{q}_N = \begin{pmatrix} w \\ \tau_{ext} \end{pmatrix}^T \begin{pmatrix} s \\ \dot{q}_N \end{pmatrix} \tag{39}$$

Therefore, it can be said that the system is passive from the extended disturbance and external force input to proper output.



(a) When perfect robot model is available.



(b) When perfect robot model is not available.

Fig. 3. Simulation results. Comparison between ideal and actual impedance behaviors are shown.

IV. Simulations and Experiments

A. Simulations

In this section, the proposed method is verified through the simulation studies. The control law (10) is applied to planar 3-DOF manipulator shown in the Fig. 2 (a). It is a redundant manipulator since it moves only in the x-y plane. The control law is implemented in 1KHz control frequency using Robotics Lab which is a commercialized robot simulation software [22]. The selected target actuator model is Kollmorgen RBE01811 motor. The values of inertia, back-emf constant, frictions, etc are referenced from the catalog.

The desired impedance behavior of the end-effector is (5) with $D = diag\{30, 300\}$, and $K = diag\{50, 1000\}$ [3]. The desired position of the end-effector is fixed (i.e., constant) while the external force is applied at the end-effector in a sinusoidal form as shown in the Fig. 2 (b). The simulation is to see if the desired impedance behavior is realized or not. To this end, the comparison between the ideal impedance behavior[4] and the actual behavior is made.

The results are summarized in the Fig. 3. In the first simulation, the perfect robot model is assumed to be known and the actuators are considered as ideal torque source. Under this assumption, the desired impedance behavior is realized without help of NRIC framework (Fig. 3 (a)). The ideal impedance behavior (solid black line), the behavior of impedance controller itself (solid blue line), and that of impedance controller with NRIC (dotted red line) are almost identical. Second simulation considers motor model and some amount of joint friction/damping is added. Stick-slip friction, back-emf constant, and motor inertia are modeled and the gear ratio is set as 100:1. As shown in the Fig. 3 (b), the impedance control law itself cannot realize the desired impedance behavior. The resulting stiffness is much higher than the desired one. On the other hand, fortunately, when NRIC is applied to the impedance controller, NRIC input successfully compensates disturbances and the desired impedance behavior is recovered.

B. Experiments

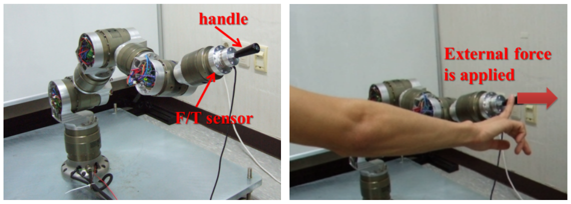
In this section, the control law (7) is implemented for the Schunk 7-DOF light weight arm 3 (LWA3) shown in the Fig. 4. The controller is implemented in a real-time OS (RTX) and runs in about 333Hz of control frequency. LWA3 has very high reduction ratio (from 300:1 to 600:1, roughly) in order to realize its light-weight specification. Two experiments are performed to verify the proposed method.

In the first experiment, similar to the simulation scenario, desired impedance behavior is defined as $D = diag\{30, 300, 300\}$, and $K = diag\{50, 1000, 1000\}$ and the desired position of LWA3 is fixed. Human operator grasps the end-effector and moves it in a sinusoidal motion (Fig. 4 (b)). Fig. 5 shows the comparison between the ideal behavior of the end-effector and the actual behavior. Ideal behavior is calculated using the measured force at the end-effector and the Fig. 5 (a) shows an example of the measured force. Fig. 5 (b) shows the result of impedance controller alone (without NRIC). High reduction ratio of LWA3 results huge disturbances and the desired behavior is not realized. On the other hand, when NRIC is applied with the impedance controller, disturbance effects are successfully compensated and the resulting actual end-effector behavior is close to the ideal one as shown in the Fig. 5 (c).

The realization of the zero stiffness is shown in the second experiment. The desired impedance behavior is defined as $D = diag\{30, 300, 300\}$ and $K = diag\{0, 1000, 1000\}$. Therefore, the end-effector of LWA3 keeps same position in the y-, z-directions due to the high stiffness but changes its neutral position along the x-direction. The measured forces and positions are shown in the Fig. 6. As can be seen in the figure, LWA3 shows very stiff behavior in the y-, z-directions and zero stiffness is realized along the x-direction.

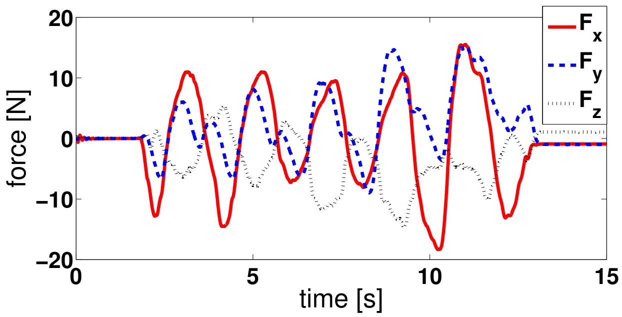
[3] In this paper, unit of stiffness is N/m, and that of damping is N·s/m.

[4] The ideal impedance behavior is calculated from (5). None of the other effects (e.g., friction) are considered in calculating the ideal impedance behavior. The only difference is that the Inertia Λ is considered as constant during the motion. Also, note that even though desired stiffness is in the linear shape, the ideal behavior has hysteresis-like shape due to the damping.

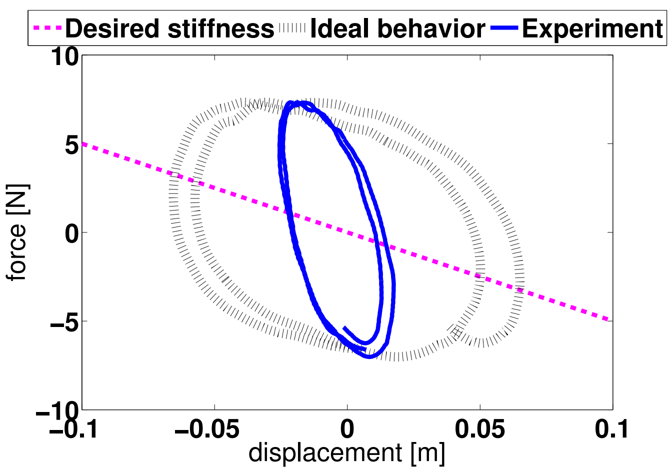


(a) (b)

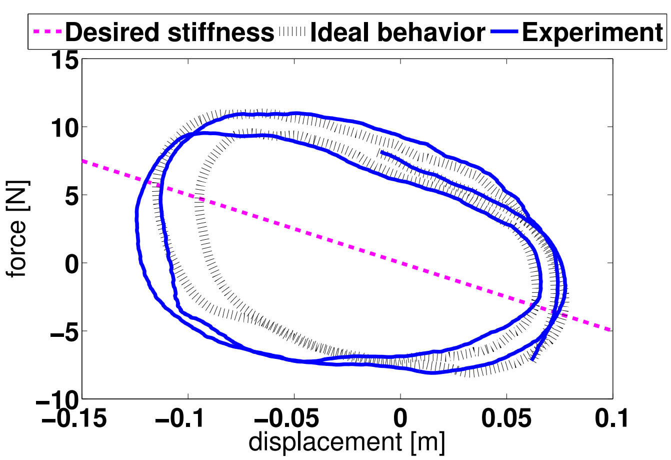
Fig. 4. 7-DOF Schunk manipulator used in the experiment.



(a) Applied force at the end-effector.

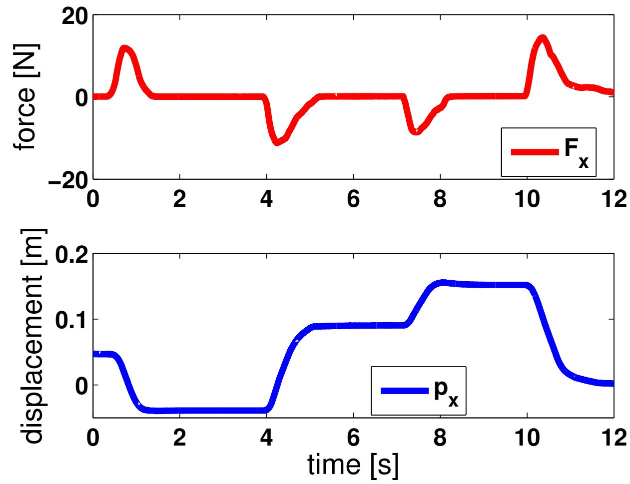


(b) Position and force in x-direction: Impedance control only.

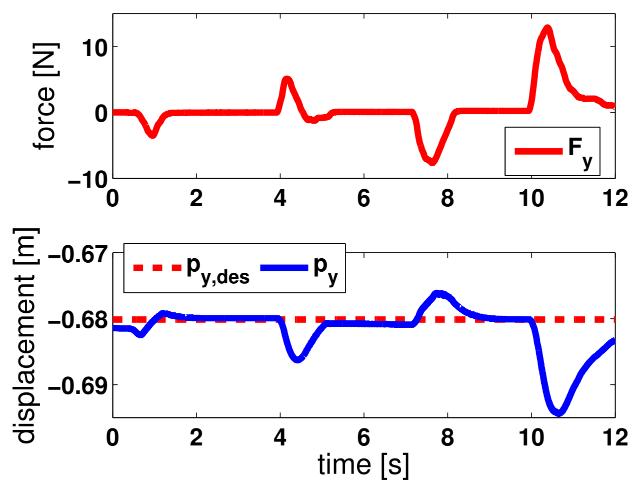


(c) Position and force in x-direction: Impedance control with NRIC.

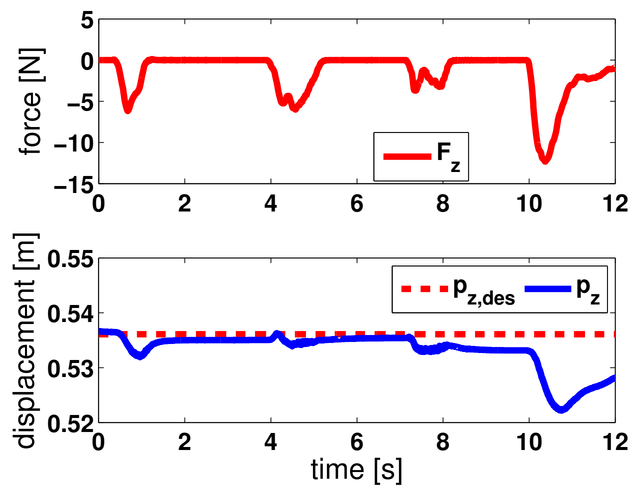
Fig. 5. Human operator moves the end-effector in a sinusoidal motion. (a) shows an example of the exerted force. In (b) and (c), the comparison between the ideal and actual impedance behavior for without-NRIC and with-NRIC cases, respectively



(a)



(b)



(c)

Fig. 6. Zero stiffness is realized in x-direction and high stiffness in y-, z-directions. Applied force and position are shown together for each joints.

V. Conclusion

In this paper, to overcome the robustness and the performance limitations of the impedance controllers, we proposed a nonlinear $\mathcal{H}_\infty$ optimal impedance control methods. Nonlinear $\mathcal{H}_\infty$ optimality is realized using the NRIC framework. Consequently, the optimality is achieved by simply adding PD-type auxiliary input to existing impedance control laws. Moreover, the extended disturbance input-to-stability (ISS) is guaranteed and the passivity of the impedance controller is preserved. The proposed method is general in some sense since it can be applied to various types of impedance controllers in a unified way. More specifically, the proposed method is applicable as long as given controller is globally (uniformly) asymptotically stable one. Through simulations and experiments, the proposed method is verified. It is shown that the desired impedance behavior is successfully realized.

Appendix

In the appendix, the definition and one central theorem for the input-to-state stability (ISS) are introduced. First, consider the system

$$\dot{x} = f(x,t) + g(x,t)w \tag{40}$$

The system (40) is said to be (disturbance w) ISS if there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ such that the solution for the (40) exists $\forall t \geq 0$ and satisfies

$$\|x(t)\| \leq \beta(\|x(0)\|, t) + \gamma(\sup_{0\leq\tau\leq t} \|w(\tau)\|) \tag{41}$$

$x(0)$ is an initial state vector and w is piecewise continuous and bounded on $[0, \infty)$.

One useful theorem for the ISS is: the system is ISS if and only if there exist a smooth positive definite radially unbounded function $V(x,t)$, a class $\mathcal{K}_\infty$ function γ_1, and a class $\mathcal{K}$ function γ_2 such that the following dissipativity inequality is satisfied:

$$\dot{V} \leq -\gamma_1(\|x\|) + \gamma_2(\|w\|) \tag{42}$$

Here, $V(x,t)$ is called as an ISS-Lyapunov function.

Acknowledgment

This work was supported in part by the Engineering Research Center (No. 2011-0030075), Global Frontier R&D Program on Human-centered Interaction for Coexistence (No.2012M3A6A3056423) and acceleration research program (No. R17-2008-021-01000-0) funded by the National Research Foundation of Korea, in part by the Industrial Source Technology Development Programs(No. 10043928 and No. 10038574) funded by the Ministry Of Trade, Industry Energ of Korea, and in part by the Unmanned Technology Research Center at KAIST funded by the Defense Acquisition Program Administration and Agency for Defense Development

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