

Humanoid push recovery control in case of multiple non-coplanar contacts

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Abstract—This paper presents a method for humanoid push recovery in the general context of multiple non-coplanar contacts. The method consists of a controller that minimizes the kinetic energy of a perturbed whole body humanoid system, while controlling the support change to achieve the stabilization (or push recovery) of the system. The controller uses a simple model based approach to determine the necessity of support change and in this case to approximate a new contact position that allows to stabilize the system. The controller is tested on a simulated humanoid robot and it succeeds in stabilizing the robot in coplanar and non-coplanar environments.

I. INTRODUCTION

This study has been motivated by safety issues at perturbed workplaces. [5] gives the statistics of accidents that occur in leveled environments of such workplaces. In order to assess safety, tests are usually performed on passive dummies or human subjects, which are respectively non representative and too costly. Within this context, our objective is to develop new controllers for Virtual Humans (VH) in order to make them react to environmental perturbations in a sufficient realistic way. This paper presents a push recovery method for a whole body humanoid system, that we test on a simulated VH with 45 degrees of freedom (dof) to control its push recovery in coplanar and non-coplanar environments.

A. Related Work

The addressed *push recovery* problem is widely treated in literature. It is mainly based on various stability criteria and on the study of simple and whole body humanoid models. The most usual stability criteria are the Zero Moment Point (ZMP) [22] and the Foot Rotation Indicator (FRI) [6]. They present main limitations as the ZMP deals with coplanar contacts and the FRI requires one single foot contact. An interesting extension of the ZMP has been proposed in [13], [8] for multiple non-coplanar contacts, but the Generalized ZMP on a virtual plane assumes sufficient friction, similarly to ZMP. [7] proposes the Rate of change of Angular Momentum (RAM) about the Center Of Mass (COM) as a general criterion for rotational stability; the derived Zero RAM point that measures rotational instability is limited to bipeds with coplanar feet or with one foot support.

A lot of works on push recovery consider the simple Linear Inverted Pendulum Model (LIPM). Introducing the notion of

Capture Points, *N-Step capture points* and *Capture Regions* [21], the step to make for push recovery is calculated for LIPM in [19], [24], [26]. In [25], the authors propose a push recovery approach based on predictive control with LIPM and that is very similar to [4] with additional objectives for the COM. Some works went beyond the LIPM by varying the COM height as in [20], or by adding an inertia about the COM in [19], [24], [26] to recover the stability through a change of angular momentum. Still, all these methods are restricted to coplanar feet contacts. Non coplanar stepping is treated in [27]; a planar rimless wheel model with two massless spokes is used to compute the Generalized Foot Placement Estimator which is the step position, on level or non level ground, that assures push recovery. This method based on a 2D model is for bipeds that have coplanar contacts in initial state and sufficient friction is assumed.

Some works consider whole body humanoid models to treat push recovery. In [1], the rate of change of angular and linear momentums of a biped are controlled to compensate the perturbation by maintaining the COM and Center Of Pressure (COP) above the feet. The control does not deal with contact change when it can not recover from perturbation. In [14] and [10], [11], [9], a desired and heuristically chosen COM position and a desired null COM velocity are predefined to perform a momentum control.

In [14], the desired COM state and a desired null centroidal AM determine, through a feedback law, the desired rate of change of centroidal momenta. Then an optimization based controller makes the actual momenta converge to the desired values while respecting the model dynamics and constraints. In [10], [11], [9], the desired COM state determines the desired change in centroidal linear momentum through a feedback and feedforward law. Then the desired contact forces are calculated using an inverse pendulum model. Finally, a passivity based force control is performed under quasi-static assumptions to achieve the desired forces and though push recovery. This control is valid for a biped with quasi-coplanar feet contacts. When the controller fails to stabilize the model, symmetric foot steps are performed.

B. Scope and contribution

Our main contribution consists in proposing a push recovery control that dynamically stabilizes a whole body humanoid system, in the general case of multiple non coplanar contacts. In our previous work, we developed two algorithms based on a simple humanoid model, that determine the necessity of support change for stabilization and that compute new contact positions (not restricted to feet contacts) that assure

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push recovery in the general contact case. In this paper, we apply the developed algorithms on a whole body humanoid model to control its push recovery. We present:

- A complete approach for the push recovery of the simple model in case of multiple non-coplanar contacts. In our previous work, we assumed an instantaneous support change. In this paper, the proposed approach considers all the push recovery phases and time durations; it takes into account the time duration of support change during which it controls the model and considers its dynamics for a proper computation of new stabilizing contact positions.
- A push recovery method for a whole body humanoid model. It consists of a whole body controller that minimizes the kinetic energy of the perturbed model during all the push recovery stages. The controller uses the results of the simple model approach as approximations to control the support change of the whole body model. The controller does not predefine desired and heuristic COM and posture references; the controlled system automatically reaches a final stable state, when possible. This control is valid in the case of multiple non-coplanar contacts.

Although no guarantees on push recovery can be made for the whole body humanoid model using the simple model approach, we show in this paper that it works in practice; a VH with 45 dof, perturbed in different directions, recovers its stability in coplanar and non-coplanar environments, when we control it using the proposed method.

In section II, we present the push recovery approach based on a simple humanoid model; we first review some algorithms we developed in previous papers; we then present the push recovery approach that involves the reviewed algorithms. In section III, we develop the push recovery method for a whole body humanoid model; we first describe a whole body controller; we then present the push recovery method that combines the controller with the simple model based approach developed in section II, to achieve stabilization.

II. PUSH RECOVERY APPROACH BASED ON A SIMPLE HUMANOID MODEL

The approach is based on a simple humanoid model and it uses two algorithms that we proposed in previous papers. The first algorithm is a fall indicator that determines the necessity of support change to stabilize the perturbed model. The second is a support change algorithm that computes the new stabilizing contact configuration, in case of support change. We first review the simple model. We then describe the push recovery approach and finally we review the two algorithms used by the approach.

A. Review of the simple model and related assumptions

The simple model is detailed in [16] and is illustrated in Fig.1. It has completely actuated massless limbs. It consists of a point mass m at the COM and of n non-coplanar contact surfaces. We assume a null inertia of the body about the COM. The control variables of the system are the

ground reaction wrenches with $\mathbf{W}_i = \begin{pmatrix} \mathbf{f}_i & \boldsymbol{\tau}_i \end{pmatrix}^t$ being the wrench at the i^{th} contact and $\mathbf{f}_i \in \mathbb{R}^3$ and $\boldsymbol{\tau}_i \in \mathbb{R}^3$ being respectively the contact force and torque.

The dynamics of the model are described by Newton-Euler equations and the model is subject to several constraints:

- The COP of each contact belongs to the contact surface.
- To avoid slipping contacts, forces \mathbf{f}_i belong to friction cones defined by $\|\mathbf{f}_i - (\mathbf{f}_i \cdot \mathbf{n}_i) \mathbf{n}_i\| \leq \mu_i \mathbf{f}_i \cdot \mathbf{n}_i$ with μ_i and \mathbf{n}_i being respectively the friction coefficient and the normal vector to the i^{th} contact surface. We discretize the cone into several facets to linearize the constraint
- The normal force of the i^{th} contact is limited to a maximum admissible normal force: $\mathbf{n}_i^t \mathbf{f}_i \leq f_{n_{i_{max}}}$
- To avoid rotational slipping of contacts, the friction torque at the COP of the i^{th} contact is limited proportionally to the contact normal friction force and friction coefficient: $\|\boldsymbol{\tau}_{i_{COP}}\| \leq \alpha \mu_i (\mathbf{n}_i^t \mathbf{f}_i)$

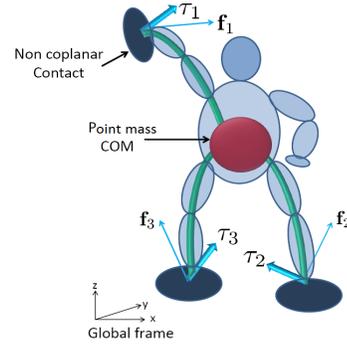


Fig. 1: Simple Model

Discretization of the model

We discretize the model dynamics using a simple Taylor series expansion. Let $(\mathbf{X}, \dot{\mathbf{X}})$ be the COM state and T the time sampling period. We have for $X(t \in]t_k, t_{k+1}[)$: $\mathbf{X}_{k^-} = \mathbf{X}_{k^+}$, $\dot{\mathbf{X}}_{k^-} = \dot{\mathbf{X}}_{k^+}$, $\ddot{\mathbf{X}}_{k^+} \neq \ddot{\mathbf{X}}_{k^-}$, and we assume $\frac{d^3 \mathbf{X}}{dt^3}(t \in]t_k, t_{k+1}[) = 0$.

The dynamics and constraints of the model are linearly expressed in terms of the wrench vector \mathbf{W}_k with $\mathbf{W}_k = (\mathbf{W}_{1k}^t \dots \mathbf{W}_{ik}^t \dots \mathbf{W}_{nk}^t)^t$ where \mathbf{W}_{ik} is the wrench at the i^{th} contact for $t \in]t_{k-1}, t_k]$.

Perturbation of the model

We consider that the simple model, initially at rest, is subject to a horizontal instantaneous perturbation (force impulse) that implies an initial non static COM state $(\mathbf{X}_0, \dot{\mathbf{X}}_0)$, with $\dot{\mathbf{X}}_0$ negligible about the vertical gravity axis. The COM trajectory is assumed to keep the same horizontal direction \mathbf{d} impulsed by the perturbation and determined by choosing \mathbf{d} collinear to $\dot{\mathbf{X}}_0$. We propose to stabilize the perturbed model (or make it recover from perturbation) by minimizing its kinetic energy until it stops in a static state $(\dot{\mathbf{X}} = \mathbf{0}, \ddot{\mathbf{X}} = \mathbf{0})$.

B. Push recovery approach for the simple model

The model, initially at rest, is perturbed. We define:

- t_I : time at the onset of the perturbation
- t_B : time at the end of the perturbation
- t_{BR} : time at which a subject triggers a support change reflex, if necessary
- *Reflex time* ($t_{BR} - t_I$): time duration, after the onset of perturbation, needed by a human to trigger a support change. On the basis of human biomechanical data in [23], we determine a *Reflex time* of 100 ms

Since we consider that the simple model is instantaneously perturbed, we have $t_I = t_B$.

Given the model initial contact configuration and the COM state ($\mathbf{X}_B, \dot{\mathbf{X}}_B$) upon perturbation at t_B , the approach consists of a linear step by step optimization formulated in terms of the wrench variable \mathbf{W}_k and that minimizes the model kinetic energy along the direction of perturbation \mathbf{d} collinear to $\dot{\mathbf{X}}_B$ while satisfying the model dynamics and constraints. This optimization is performed while keeping or changing the contact configuration and it pursues until the model reaches a full stop.

The approach is illustrated in Fig.2. The fall indicator, described in section II-C, is called at t_{BR} to indicate the necessity of support change to stabilize the model:

a) If no support change is necessary, the kinetic energy is minimized until the model is stabilized.

b) Otherwise, we arbitrarily choose to add one contact (contact addition) or to change the position (contact change) of a contact already established in the initial contact configuration. The limb changing/adding contact is also arbitrarily chosen. The chosen limb moves during a *Step time* towards its new contact position and it establishes the contact at time t_E ($t_E = t_{BR} + \text{Step time}$); we choose a *Step time* of 200 ms (average time obtained in experiments led on 20 human subjects in [15]); the limb new contact position is computed by the support change algorithm described in section II-D. Overall, in this case of support change, at t_{BR} the support change is triggered (if contact change, the chosen limb establishing a contact, removes its contact; in both contact change or addition, the chosen limb starts moving towards its new position) and the push recovery approach operates as described in Fig.2. If the support change algorithm does not find a feasible stabilizing contact position \mathbf{P} , successive contact changes/additions can be envisaged with the same method.

C. Review of the Fall indicator [17]

Given an initial contact configuration of the model and a COM perturbed state ($\mathbf{X}_{BR}, \dot{\mathbf{X}}_{BR}$), the fall indicator informs whether the simple model can be stabilized while maintaining the same contact configuration.

Bretl generates in [2], for a given contact configuration, a convex static stability region over which the COM of a static system must lie. A direct fall indicator consists of a linear step by step optimization formulated in terms of the variable

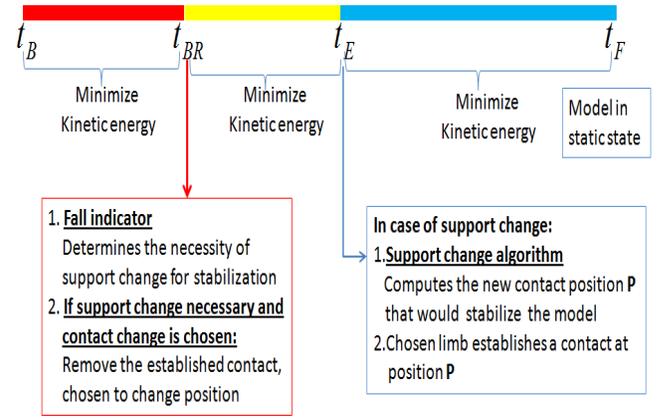


Fig. 2: Push recovery approach for the simple model

\mathbf{W}_k (details in [17]) and that minimizes the simple model (Fig.1) kinetic energy, which is a point mass kinetic energy, along the direction of perturbation \mathbf{d} collinear to $\dot{\mathbf{X}}_{BR}$ (See Fig.3) while satisfying the model dynamics and constraints; if the model reaches a static state inside the initial static region (before reaching \mathbf{X}_{bd}), then the system can remain in a static state and is therefore stabilized without support change; otherwise, the COM goes beyond \mathbf{X}_{bd} and the static region should change to include the COM and allow it to stop inside the new static region; a support change is then necessary to obtain a new different static region.

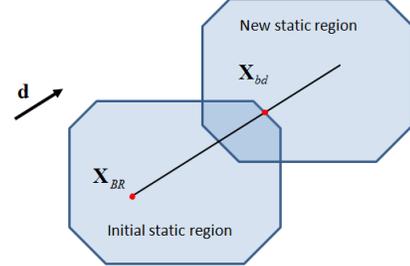


Fig. 3

A more generalized fall indicator is elaborated in [17].

D. Review of the support change algorithm

In case of a support change and given the current model contact configuration and COM state ($\mathbf{X}_E, \dot{\mathbf{X}}_E$), the support change algorithm computes the position of the contact to add to the current configuration that allows to stabilize the model during a predetermined time duration that we call *Post step time*.

The support change algorithm is a feasibility problem formulated over a predictive horizon of length (*Post step time* = $h_{Pst}T$) with h_{Pst} being a positive integer.

We define the vector of wrenches over the h_{Pst} time steps as $\mathbf{W}_h = (\mathbf{W}_1^t \dots \mathbf{W}_k^t \dots \mathbf{W}_{h_{Pst}}^t)^t$ where $\mathbf{W}_k = (\mathbf{W}_{1k}^t \dots \mathbf{W}_{ik}^t \dots \mathbf{W}_{nk}^t)^t$ and \mathbf{W}_{ik} is the reaction wrench at i^{th} contact for $t \in]t_{k-1}, t_k]$. We also define $\dot{\mathbf{X}}_F$ and $\ddot{\mathbf{X}}_F$ as the velocity and acceleration of the COM, after a *Post step time*

starting from $(\mathbf{X}_E, \dot{\mathbf{X}}_E)$.

The variables of the problem are \mathbf{W}_h and the position \mathbf{P} of the contact to add:

- We formulate the Newton equation of the model over the window h_{Pst} and then we write $\dot{\mathbf{X}}_F$ and $\ddot{\mathbf{X}}_F$ in terms of \mathbf{W}_h (details in [16])
- We formulate the model dynamics and constraints, at each time step $h = (1 \dots h_{Pst})$, in terms of \mathbf{W}_h and \mathbf{P}

The support change algorithm consists in finding \mathbf{W}_h and one reachable position \mathbf{P} that assure to the model a final static state $\dot{\mathbf{X}}_F = \mathbf{0}$ and $\ddot{\mathbf{X}}_F = \mathbf{0}$ and that satisfy the dynamics and constraints of the model at each time step $h = (1 \dots h_{Pst})$. This feasibility problem is solved by using a non linear optimization method (interior-point).

III. PUSH RECOVERY METHOD FOR A WHOLE BODY HUMANOID MODEL

We apply the simple model based push recovery approach developed in section II-B on a whole body humanoid model to control its push recovery. The control is valid in case of multiple non-coplanar contacts. The whole body model is a poly-articulated branching structure where each branch is a rigid body and each joint is one or more perfect pivot joints. We chose the hip body as the root base. The dynamics of the whole body model are written as a classical set of Euler-Lagrange equations:

$$\tilde{M}(\mathbf{T} - \mathbf{G}) + N\mathbf{T} = L\boldsymbol{\tau} + J'\mathbf{F} \quad (1)$$

where:

- $\mathbf{T} = \begin{pmatrix} \mathbf{V}_{root} \\ \dot{\mathbf{q}} \end{pmatrix}$ is the velocity vector in generalized coordinates. $\mathbf{V}_{root} = \begin{pmatrix} \boldsymbol{\omega}_{root} \\ \mathbf{v}_{root} \end{pmatrix}$ is the twist of the root base and $\boldsymbol{\omega}_{root}$ and \mathbf{v}_{root} are the angular and linear velocities of the root base. \mathbf{q} is the vector of joint angles.
- $\mathbf{F} = \mathbf{F}_{pert} + \mathbf{F}_{contact}$ with \mathbf{F}_{pert} being the vector of external perturbation wrenches and $\mathbf{F}_{contact}$ the vector of contact wrenches.
- $\boldsymbol{\tau}$ is the vector of joint torque, L expresses $\boldsymbol{\tau}$ in generalized coordinates, N is related to Coriolis and centrifugal effects, J is the basic jacobian that transforms the joint velocities to cartesian velocities (twists), \tilde{M} is the symmetric inertia matrix and G is gravitational acceleration with J , \tilde{M} and G expressed in generalized coordinates.

The whole body push recovery method interfaces the simple model based approach presented in section II-B with a whole body controller. The former makes the support change decision and computes the position at which the model establishes a contact to regain its stability (as in Fig.2). These results are given as inputs to the whole body controller; this controller minimizes the whole body model kinetic energy while controlling the support change according to the inputted results.

In the following, we first describe the whole body controller then we elaborate the push recovery method.

A. Whole body controller

The controller consists of step by step minimizing the kinetic energy of the whole body model, while satisfying the model dynamics and constraints. The total kinetic energy E_K of the model can be divided into the COM kinetic energy E_{KCOM} and the "postural" kinetic energy E_{KINT} also called internal kinetic energy [12]:

$$E_K = E_{KCOM} + E_{KINT} \quad (2)$$

The whole body controller minimizes E_{KCOM} and E_{KINT} with different weights. Actually, if we assimilate the whole body model to a flywheel centered at the COM, E_{KINT} and E_{KCOM} turn out to be respectively the rotational and the COM translational kinetic energies; the minimization of E_{KCOM} results in a minimization of the norm of the centroidal linear momentum and the minimization of E_{KINT} results in a minimization of the norm of the centroidal angular momentum (See appendix VI-A for a simple illustration of E_{KCOM} , E_{KINT} and the centroidal momentum). We have (See appendix VI-B for details):

$$E_{KCOM} = \frac{1}{2} \mathbf{T}' M_v \mathbf{T} \quad \text{with:} \quad M_v = \frac{1}{m} \tilde{M}'(4:6,:) \tilde{M}(4:6,:) \quad (3)$$

where m is the mass of the whole body model

$$E_{KINT} = \frac{1}{2} \mathbf{T}' M_\omega \mathbf{T} \quad \text{with:} \quad M_\omega = \tilde{M} - M_v \quad (4)$$

The optimization problem of the controller, illustrated in Fig.4, can now be presented as follows:

1) Cost function

We minimize the following multi-objective cost function:

$$w_v E_{KCOM} + w_\omega E_{KINT} + s w_t \| \mathbf{a}_c - \mathbf{a}_c^{des} \| \quad (5)$$

- E_{KCOM} and E_{KINT} are minimized with the respective weights w_v and w_ω
- When a support change is needed for the stabilization, we arbitrarily choose to add one contact (contact addition) or to change the position of an already established contact (contact change). The model body adding/changing contact is arbitrarily chosen (e.g. body applying minimum contact force at the onset of contact change). The body tracks a desired trajectory leading after a predetermined *Step time*, to a desired final position where a new contact is established. The tracking task is assured by minimizing the third objective function ($s w_t \| \mathbf{a}_c - \mathbf{a}_c^{des} \|$) with weight w_t and during *Step time*. ($s = 1$) during *Step time* and ($s = 0$) in the remaining times.

\mathbf{a}_c and \mathbf{a}_c^{des} are respectively the controlled and desired accelerations of the chosen body. \mathbf{a}_c^{des} is determined with a PD law:

$$\mathbf{a}_c^{des} = K_p(\Delta \mathbf{P}) + K_d(\mathbf{V}^{des} - \mathbf{V}) \quad (6)$$

where K_d is the derivative PD gain, $\Delta \mathbf{P}$ is the deviation in translation and orientation between the desired and current body frames and \mathbf{V} and \mathbf{V}^{des} are the current and desired body twists.

The desired body frame and twist are given by the desired trajectory. This trajectory and the triggering of support change (or activation of tracking task) are given as input parameters to the controller (Fig.4).

2) Equality constraints:

- The model dynamics (1) are respected. Since the controller has no knowledge about the external disturbances, we consider ($\mathbf{F} = \mathbf{F}_{contact}$).
- The established contacts have a null acceleration ($J_{c_i} \dot{\mathbf{T}} + \ddot{J}_{c_i} \mathbf{T} = \mathbf{0}$) with J_{c_i} being the jacobian at the i^{th} established contact.

3) Inequality constraints:

- The joint torques τ are bounded to limit the actuation capacity.
- To avoid slipping unilateral contacts, their contact forces belong to friction cones that we discretize into several facets to linearize the constraint.
- The joint angles \mathbf{q} are bounded similarly to human articulations.

The humanoid is torque controlled. The above optimization problem is quadratic and as in [3], the cost function and the constraints are expressed in terms of the optimization variables ($\dot{\mathbf{T}}^t \ \tau^t \ \mathbf{F}_{contact}^t$). This real time control is performed using a linear quadratic programming solver.

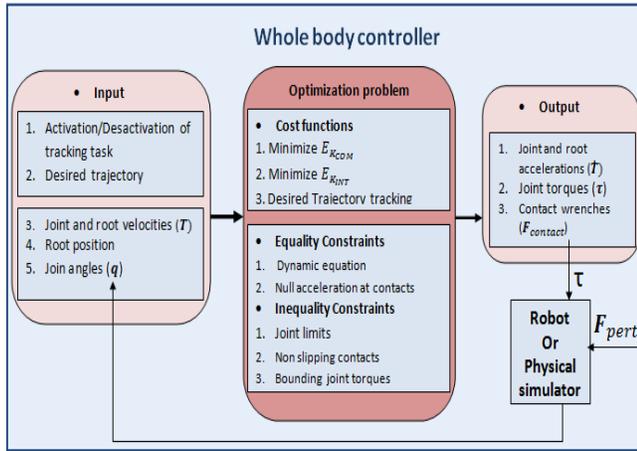


Fig. 4: Whole body controller

B. Push recovery method for the whole body model

We consider that the whole body model, initially at rest, is subject to horizontal perturbations during a short time duration. We use the times (t_I , t_B , t_{BR} , t_E , *Reflex* and *Step times*), described in section II, to characterize the push recovery; for the whole body model, the application of \mathbf{F}_{pert} begins at t_I and ends at t_B , with ($t_B \neq t_I$) and we consider short time disturbances, such that $t_{BR} \geq t_B$.

The whole body model is controlled by the whole body controller until it recovers its stability at full stop. The recovery method consists of combining the simple model based approach with the whole body controller as follows (Method illustrated in Fig.5):

- Between t_I and t_{BR} , the controller minimizes the kinetic energies (E_{KCOM} , E_{KINT}) of the whole body model.
- At t_{BR} , the whole body model and its control are suspended and offline operations are conducted using the simple model based approach (See Fig.5):
 - The simple model is initialized with the same COM state and contact configuration of the whole body model at t_{BR} . Since we consider short time horizontal perturbations, the vertical COM velocity is negligible at t_{BR} ; which is consistent with the simple model.
 - The fall indicator determines the necessity of support change for stabilization.
 - If a support change is necessary, the simple model based approach is performed between t_{BR} and t_E as in Fig.2, in the same geometric environment of the whole body model; we call \mathbf{P} the reachable contact position computed by the support change algorithm at t_E and that stabilizes the simple model during a predetermined *Post step time*.
 - In case of support change, the desired trajectory (of the chosen body adding/changing contact in the whole body model) that leads to \mathbf{P} during *Post step time*, is determined by interpolation. The triggering of support change and the desired trajectory are inputted to the whole body controller.
- The whole body model and its control are resumed at t_{BR} , after the offline operations. In case of necessary support change, it is controlled between t_{BR} and t_E as described in Fig.5. The control pursues until a full stop is reached.

We make the same choices (times, contact change or contact addition, contact to change/add) for the simple and whole body models. Various biomechanical investigations have determined that a large class of human movements conserve the total angular momentum and regulates it to zero for example during walking [18] and running. Moreover, [15] presents push recovery experiments led on 20 human subjects who are asked to spontaneously recover from a perturbation in uncluttered environments; [15] shows that the non null centroidal RAM used by the subjects had no significant effect on the stabilization. Therefore, we mostly choose ($w_\omega \gg w_v$). This limits the variation of centroidal angular momentum of the whole body model, which reduces the disparity between the whole body model and the simple model (that has null inertia) and justifies the combination between the simple model based approach and the whole body controller.

IV. SIMULATION RESULTS OF A WHOLE BODY MODEL PUSH RECOVERY

We present different simulation results of the push recovery of a VH, which is a virtual whole body humanoid model that we stabilize using the proposed push recovery method. The results are supported by a video provided with this paper. The VH in our simulations has 45 dof, is 1.46 m tall and weighs 79 kg. In order to show some assets of

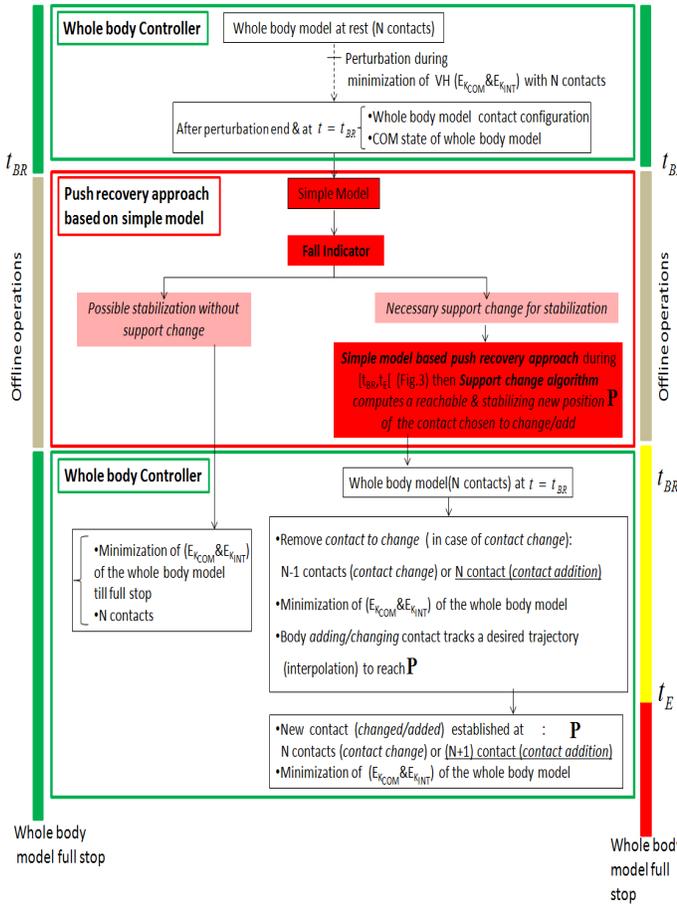


Fig. 5: Push recovery method for the whole body model

the proposed push recovery method, the VH is perturbed in different directions and is stabilized in both coplanar and non-coplanar environments. The simulations are performed using XDE¹.

A. Context

Initial posture of the VH:

Prior to perturbation, the VH is standing on a level ground, with an upright posture. Thereafter, the vectors are expressed in an inertial frame, where the x-axis and y-axis are respectively the left-right axis and dorsoventral axis of the VH in its initial posture. The VH initial contact configuration is presented in table I.

TABLE I: VH contact configuration prior to perturbation

Left ankle position (m)	$\mathbf{O}_1 = (-0.1 \ 0 \ 0)^t$
Right ankle position (m)	$\mathbf{O}_2 = (0.1 \ 0 \ 0)^t$
Normal to contact surface	$\mathbf{n}_1 = \mathbf{n}_2 = (0 \ 0 \ 1)^t$
Friction coefficient	$\mu_1 = \mu_2 = 0.8$

VH perturbation and parameters of push recovery

¹XDE: Environment of interactive physical simulation developed by CEA-LIST (see: <http://www.kalisteo.com/lisi/aucune/a-propos-de-xde>)

method:

A perturbation is modeled as a horizontal force (\mathbf{f}_{pert}) applied during a short time interval $\Delta t = 0.1$ (s). Δt is equal to the *Reflex time* and then $t_{BR} = t_B$. When a support change is necessary, we choose a *contact change* reaction. The predetermined parameters of the push recovery method are presented in table II.

Geometric environments:

TABLE II: Predetermined input parameters for the push recovery method

Parameters of push recovery approach based on simple model (offline operations Fig.5) <ul style="list-style-type: none"> • Step time: 0.2 (s) • Post step time: 0.55 (s) with $h_{Pst} = 4$
Parameters of the whole body controller <ul style="list-style-type: none"> • w_ω and w_v are defined for each simulation • Time period of the controller: 0.01 (s)

The VH is perturbed in two different geometric environments. The first environment Env_1 consists of a level ground. The second Env_2 consists of a level ground and a plane with a 30° slope, on which a foot step can be made (with a friction coefficient equal to 0.8, normal vector $(0 \ -0.5 \ 0.87)^t$ and a point in plane $(0 \ 0.5 \ 0)^t$). We note that a 30° slope is acceptable for our humanoid simulations; it is close to the 25° slope of the plane we used in push recovery experiments with human subjects in [15]. The geometric environments are known to the push recovery method.

B. Simulation results

In the following simulations, the VH is stabilized using the push recovery method for the whole body model (Fig.5). The whole body controller performs a real time control. For each simulation, we determine the computation time of the offline operations (Fig.5) of the simple model based approach. The offline operations are implemented in Matlab with a machine of a 2GHz core2 duo processor and 2GB RAM.

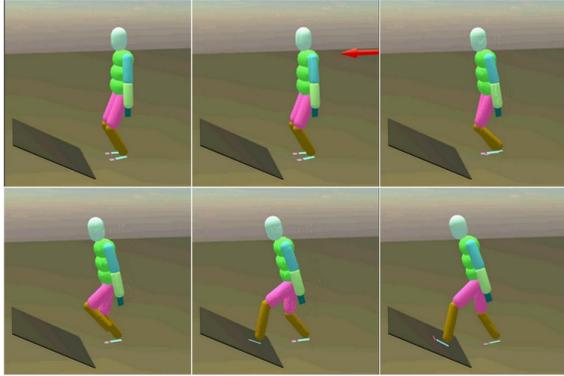
- 1) Recovery from a forward push by a non-coplanar stepping (see table III).
- 2) Recovery from a backward push by coplanar stepping (see table IV).
- 3) Recovery from a diagonal push by coplanar stepping (see table V).

The presented simulations show that the combination between the simple model based push recovery approach and the whole body controller works to stabilize the VH. The computation time of the offline operations is small, still not real time.

V. CONCLUSION

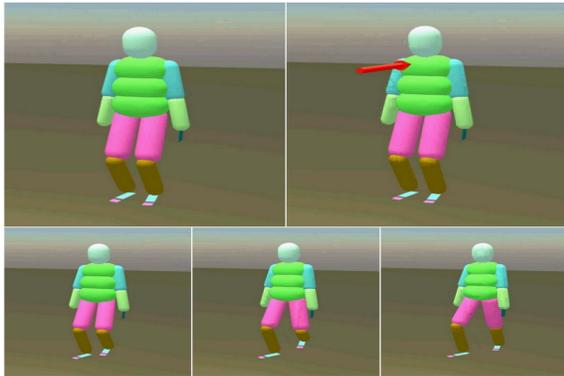
In this paper, we proposed a push recovery control for a whole body humanoid system that may have multiple non-coplanar contacts. A whole body controller minimizes the kinetic energy of the perturbed system and its uses simple model based algorithms to predict the necessity of support change and to control the support change achieving

TABLE III: Example of recovery from forward push in Env_2



Perturbation force: \mathbf{f}_{pert} (N)	(0 600 0)
Duration of \mathbf{f}_{pert} application: Δt (s)	0.1
COM position after Δt (m)	(0 0.11 0.86)
COM velocity after Δt (m/s)	(0.01 0.83 0.)
w_ω	1
w_v	1
Position of new foot contact \mathbf{P} (m)	(-0.17 0.73 0.13)
computation time of offline operations (s)	0.71

TABLE IV: Example of recovery from backward push in Env_1

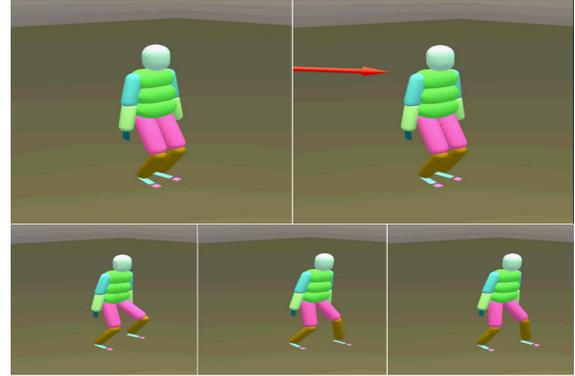


Perturbation force: \mathbf{f}_{pert} (N)	(0 -400 0)
Duration of \mathbf{f}_{pert} application: Δt (s)	0.1
COM position after Δt (m)	(0 0.01 0.86)
COM velocity after Δt (m/s)	(0.01 -0.58 0.02)
w_ω	2
w_v	1
Position of new foot contact \mathbf{P} (m)	(-0.14 -0.44 0.)
computation time of offline operations (s)	0.77

the system stabilization. We finally tested the controller to stabilize a simulated VH in coplanar and non-coplanar environments.

- For a real-time context, the offline operations of our method are still time consuming. However, their low computation time promotes to use of the simple model based approach (offline operations) as a training for learning control. Which would allow a real time push recovery control.
- In our method, we heuristically choose the contact to add or change in case of support change. We will refine these

TABLE V: Example of recovery from diagonal push in Env_1



Perturbation force: \mathbf{f}_{pert} (N)	(-300 400 0)
Duration of \mathbf{f}_{pert} application: Δt (s)	0.1
COM position after Δt (m)	(-0.02 0.08 0.86)
COM velocity after Δt (m/s)	(-0.32 0.49 0.06)
w_ω	2
w_v	1
Position of new foot contact \mathbf{P} (m)	(-0.44 0.29 0.)
computation time of offline operations (s)	0.93

parameters through push recovery experiments that we led on human subjects.

- Finally, we compared the results of the simple model based approach to human push recovery through the experiments we led on human subjects [15]. We obtained a first promising feedback on the realism of the approach.

VI. APPENDIX

A. Illustration of the COM and internal kinetic energies using a Linear Inverse Pendulum (LIP) plus flywheel model

A planar LIP plus flywheel model is shown in Fig.6. The flywheel represents the inertia about the COM. $E_{K_{COM}}$ and $E_{K_{INT}}$ are then respectively the translational and rotational kinetic energies. We have:

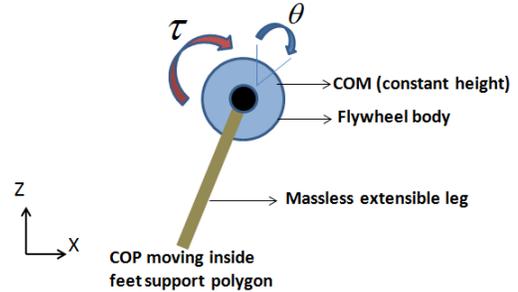


Fig. 6: Planar LIP plus flywheel model

$$\bullet E_{K_{COM}} = \frac{1}{2} m \dot{x}^2, \quad E_{K_{INT}} = \frac{1}{2} J_f \dot{\theta}^2, \quad J_f \ddot{\theta} = \tau$$

where J_f is the inertia about the COM and m is the mass of the model. x and θ are defined in Fig.6. τ is the torque generated by the flywheel; it is the RAM about the COM. We now consider the case when the COM is moving in the x positive direction and $E_{K_{COM}}$ and $E_{K_{INT}}$ are minimized respectively with the weights w_v and w_ω :

- 1) If $w_v \gg w_\omega$: a large positive τ allows greater maximum horizontal forces used to decelerate the COM. The minimization of $E_{K_{COM}}$ is though enhanced with the RAM. $E_{K_{INT}}$ is increased and should be minimized before exceeding joint limits.
- 2) If $w_\omega \gg w_v$: the priority of $E_{K_{INT}}$ minimization tends to keep the flywheel stationary while the $E_{K_{COM}}$ is less minimized through a COP control. The model behaves more like a simple LIP with zero inertia.

B. Expression of the COM and internal kinetic energies of the whole body humanoid model

- The linear momentum of the whole body model about the COM is expressed in the root body frame as:

$$\tilde{M}(4:6,:) \mathbf{T} = m \dot{\mathbf{X}}_r \quad (7)$$

with: m being the model total mass.

- The COM kinetic energy $E_{K_{COM}}$ can be written as:

$$E_{K_{COM}} = \frac{1}{2} m \dot{\mathbf{X}}_r^t \dot{\mathbf{X}}_r \quad (8)$$

$E_{K_{COM}}$ in terms of \mathbf{T} becomes:

$$E_{K_{COM}} = \frac{1}{2} \mathbf{T}^t M_v \mathbf{T} \quad \text{with:} \quad M_v = \frac{1}{m} \tilde{M}^t(4:6,:) \tilde{M}(4:6,:) \quad (9)$$

- The internal kinetic energy $E_{K_{INT}}$ can be written as:

$$E_{K_{INT}} = E_K - E_{K_{COM}} \quad \text{with:} \quad E_K = \frac{1}{2} \mathbf{T}^t \tilde{M} \mathbf{T} \quad (10)$$

$E_{K_{INT}}$ in terms of \mathbf{T} becomes:

$$E_{K_{INT}} = \frac{1}{2} \mathbf{T}^t M_\omega \mathbf{T} \quad \text{with:} \quad M_\omega = \tilde{M} - M_v \quad (11)$$

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