2013 IEEE/RSJ International Conference on
Intelligent Robots and Systems (IROS)
November 3-7, 2013. Tokyo, Japan

A 3T2R Parallel and Partially Decoupled Kinematic Architecture

Matteo Malosio[1,2], Simone Pio Negri[1], Nicola Pedrocchi[1], Federico Vicentini[1] and Lorenzo Molinari Tosatti[1]

Abstract— **This paper presents a parallel and partially decoupled mechanism characterized by three translational and two rotational degrees of freedom. A set of parallel kinematic chains actuates five degrees of freedom of the mobile platform and constrains one of its rotations. Its kinematics combines advantages typical of parallel architectures, as high dynamics, with positive aspects of partially decoupled ones, in terms of mechanical design, control and motion planning, through a relatively simple direct kinematic formulation. The presented architecture constitutes the mechanical heart of a robotic prototype designed to actively support the patient's head in open-skull awake surgery.**

I. INTRODUCTION

Parallel kinematics architectures are typically suitable in applications where dynamics, stiffness and positioning precision play a significant role. On the other side, workspace limitations and tricky aspects related to the synthesis and optimization of the mechanism are topics constantly objects of study and research. Moreover parallel manipulators are typically characterized by a complete, and in some cases a partial, joint coupling. This aspect, sometimes negligible, can be important when it is convenient to select the degrees of freedom (*dof*, hereafter) to be controlled. In this regard, the concept of *group decoupling* has been recently introduced to define and categorize partially decoupled parallel manipulators, by which different motion groups of the degrees of freedom are controlled by different actuators following a certain order [1]. Finally, joint decoupling typically leads to simpler direct kinematics formulations. This aspect led to develop some methods to solve the direct kinematics of parallel structures, some of them efficient but usually quite complex as [2], exploiting also unconventional approaches as Neural Network Solutions [3] and Support Vector Machines [4].

A number of feasible parallel kinematic architectures have been conceived so far to face the problem of dof decoupling.

The CBM-Motus robot is based on a 2dof fully decoupled parallel Cartesian kinematic architecture [5]. Similarly, the Hemisphere is a 2dof fully decoupled parallel spherical mechanism [6].

Examples of 3dof translational parallel and fully decoupled architectures are the Tripteron [7] and the Pantopteron [8]. Both the solutions are subjected to bending moments, which can considerably affect the overall positioning accuracy, unless relatively cumbersome links are employed. Extensions of these architectures with one rotational dof are the Quadrupteron [9] and the Pantopteron-4 [10], both featuring 4dof partially decoupled architectures. Examples of partially decoupled parallel mechanisms with more rotational than translational dof are the ones developed by Kim et al. [11], specifically 1T2R and 1T3R.

In some proposed solutions joint decoupling is obtained giving up the characteristic of actuation parallelism. Jang et al. proposed a 6dof partially decoupled architecture [12]. In an interesting architecture conceived by Jin et al. the end effector of the manipulator can produce 3dof spherical, 3dof translational, 3dof hybrid or complete 6dof spatial motion, depending on the types of the actuation (rotary or linear) chosen for the actuators, and the manipulator architecture completely decouples translations and rotations of the end effector [13]. However, in both these solutions, each parallel limb is characterized by two actuated dof serially arranged.

In the mechanism proposed by Moreno et al. [14] translations and rotations are decoupled exploiting the spherical parallel mechanism conceived by Gosselin [15] as rotational wrist of the mobile platform. However, flexional and torsional deformations affecting links of the spherical mechanism can reduce the overall mobile platform accuracy.

Parallel partially decoupled architectures with 5dof are rarer; an interesting architecture is the one conceived by Altuzarra et al. which achieves a 5dof partially decoupled kinematics using multiple platforms [16]. Other interesting

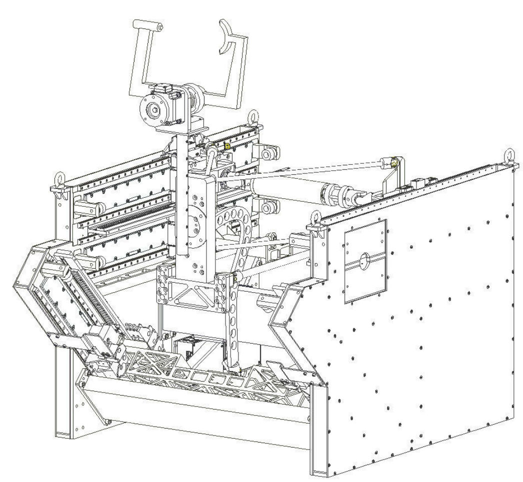
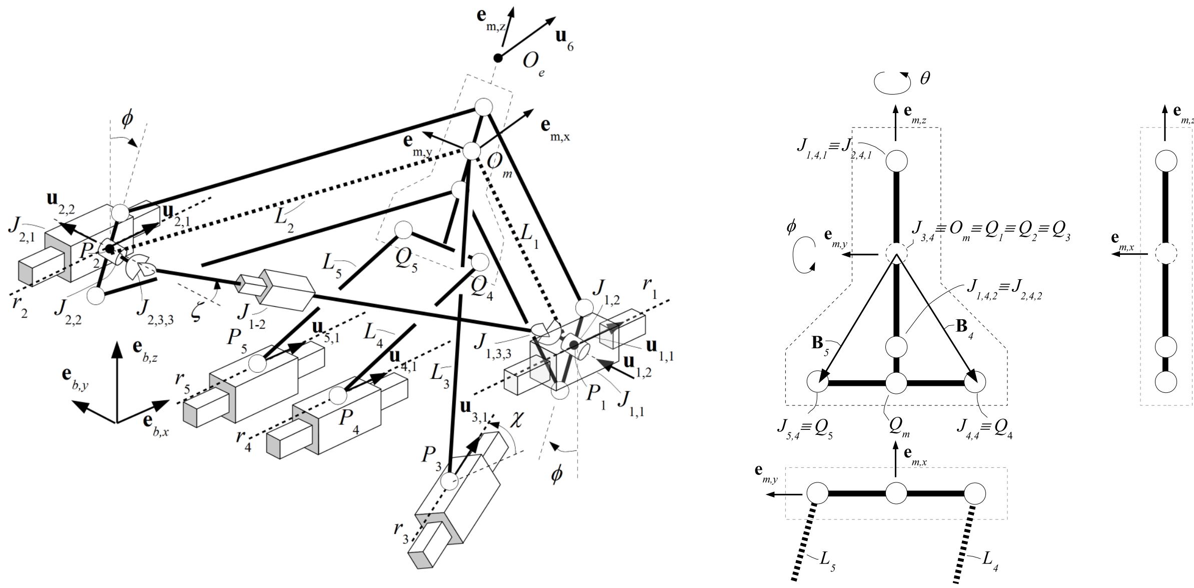


Fig. 1. The presented architecture constitutes the mechanical heart of the Active Headframe, a robotic prototype designed to actively support the patient's head in open-skull awake surgery.

This work was partially supported by the European Commission with the Collaborative Project no. FP7-ICT-2009-6270460, ACTIVE: Active Constraints Technologies for Ill-defined or Volatile Environments.

[1]Matteo Malosio et al. are with the Institute of Industrial Technologies and Automation of the Italian National Research Council, via Bassini 15, 20133 Milan, Italy. matteo.malosio at itia.cnr.it

[2]Matteo Malosio is with the University of Brescia, Piazza del Mercato 15, 25121 Brescia, Italy.



(a) Schematic view of the mechanism. $\mathbf{u}_{i,j}$ denotes the axis of joint $J_{i,j}$ (Fig. 3). P_i and Q_i are the endpoints on the base and on the mobile platform of link L_i.

(b) Mobile platform

Fig. 2. Vectorial representation of the 5dof partially decoupled 3T2R parallel kinematic architecture.

solutions are [17] and [18]. The kinematic architecture presented in this paper has been applied in the prototype named Active Headframe, a robotic head support, conceived, designed and optimized within the EU FP7 ACTIVE Project to actively control the position and orientation of the patient's head in open-skull surgical operations [19]. It is a 5dof architecture with only one element, identified as double universal prismatic joint, loaded not axially (specifically torsionally), whose position is not an impediment during the normal functioning of the machine and which can be opportunely dimensioned to guarantee the required stiffness. All the other links are loaded axially, allowing the parallel structure to be designed with slim and light elements.

The paper is organized as follows: in sec. II an overall description of the mechanism is given; in sec. III the main mechanism subsystems are detailed; in sec. IV inverse and direct kinematics are presented, underlying aspects related to dof partial decoupling; conclusions are drawn in sec. V.

II. OVERALL DESCRIPTION

The presented mechanism is a 5dof parallel and partially decoupled kinematic architecture. A schematic representation of the mechanism is given in Fig. 2 and the scheme of joints connections is shown in Fig. 3. It is characterized by four limbs: three simple - c_3, c_4, c_5 - and one complex - the group (c_1, c_{12}, c_2) - defining a complex limb as a kinematic chain combining at least one closed loop [20]. It can be classified as a wrist-partitioned parallel manipulator in which three of the five actuated joints are used to control the position of a point on the moving platform and the other two are used to control its orientation [21].

Referring to Fig. 2 and Fig. 3, $\{b\}$ and $\{m\}$ denote the base and the mobile platform frames of the parallel architecture, respectively, denoting by:

- $\{f\}$ a generic coordinate frame;
- $\mathbf{O}_f = [O_{f,x}, O_{f,y}, O_{f,z}]^T$ the origin of $\{f\}$;
- $\{\mathbf{e}_{f,x}, \mathbf{e}_{f,y}, \mathbf{e}_{f,z}\}$ axes unit vectors of $\{f\}$.

For completeness, $\{e\}$ denotes the end effector frame of the Active Headframe prototype [19]. Hereinafter this last frame will be neglected and only the peculiarities of the parallel structure will be described and investigated.

The parallel architecture is actuated by five linear actuators configured as follows: a) $J_{1,1}$ and $J_{4,1}$ are arranged symmetric to $J_{2,1}$ and $J_{5,1}$ with respect to the plane $(\mathbf{e}_{b,x}, \mathbf{e}_{b,z})$; b) $J_{3,1}$ is coplanar to plane $(\mathbf{e}_{b,x}, \mathbf{e}_{b,z})$.

III. MECHANISM SUBSYSTEMS

The main subsystems of the mechanism are presented in this section pointing out how the partial decoupling is achieved. Referring to Fig. 4, they are denoted by fbl (four-bar linkage), $dupj$ (double universal prismatic joint) and mp (mobile platform).

Given the generic vector $\mathbf{V} = [V_x, V_y, V_z]^T$, let us conventionally denote by:

- $v = \|\mathbf{V}\|$, norm of $\mathbf{V}$;
- $\mathbf{v} = \mathbf{V}/\|\mathbf{V}\|$, unit vector associated to $\mathbf{V}$;
- V the geometric entity identified by $\mathbf{V}$, as a point P identified by the position vector $\mathbf{P}$ or a link L associated to the vector $\mathbf{L}$.

A. Four-bar linkage

Let us isolate one of the two fbl of Fig. 4 as represented in Fig. 5. Its components are two pairs of links (D_1-D_2, G_1-G_2), connected by four spherical joints ($S_{p,1}$, $S_{p,2}$, $S_{m,1}$, $S_{m,2}$). The middle point of D_1, denoted by O_p, is constrained to the ground by the revolute joint J_p aligned

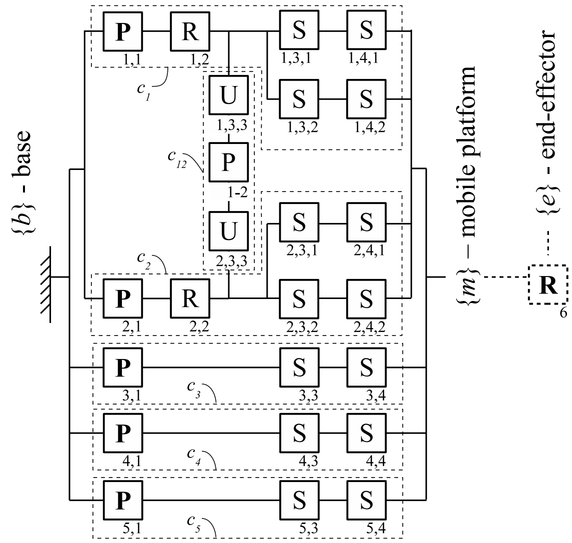


Fig. 3. Joint connection graph: horizontal branches depict the $i = 1, \ldots, 5$ kinematic chains c_i, composed by joints $J_{i,j,k}$ of type prismatic (P), rotational (R), universal (U) or spherical (S). Elements belonging to the same i-th chain are identified by indexes j,k. A homokinetic double universal prismatic joint (U-P-U, denoted by c_{12}) connects c_1 and c_2 constraining one rotational dof of the mobile platform. The actuated joints are identified by bold characters. J_6 is a rotational joint placed in series with the parallel structure, present in the Active Headframe prototype.

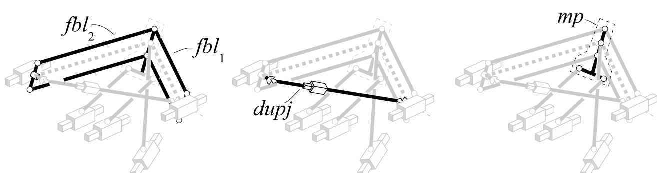


Fig. 4. Mechanism subsystems are denoted by fbl (four-bar linkage), $dupj$ (double universal prismatic joint) and mp (mobile platform).

to $\mathbf{e}_{p,y}$. Its angular displacement is denoted by the angle $\phi = \arccos(\mathbf{d}_1 \cdot \mathbf{e}_{p,z})$. Link D_2 is connected to D_1 by links G_1 and G_2. The position of O_m with respect to O_p can be identified by angles α and β. Conveniently, let us consider $\{m\}$ centered in O_m, defined by $\mathbf{e}_{m,z} \parallel \mathbf{d}_2$ and $\mathbf{e}_{m,x} \perp \mathbf{e}_{p,y}$.

With this configuration, it is possible to identify the correspondence between fbl and components of c_1 and c_2. Assuming $i = 1, 2$, O_p corresponds to P_i, J_p to $J_{i,2}$, G to L_i, and D_2 to mp.

Let us now consider the configuration in which links are coplanar (plane ξ). Underlying that in the planar configuration $\mathbf{g}_1 \parallel \mathbf{g}_2$ and $\mathbf{d}_1 \parallel \mathbf{d}_2$, and assuming $\mathbf{d}$=$\mathbf{d}_1$=$\mathbf{d}_2$ and $\mathbf{g}$=$\mathbf{g}_1$=$\mathbf{g}_2$, the unit vector normal to ξ is $\mathbf{e}_\xi = \mathbf{g} \times \mathbf{d}$.

Let us now analyze how the angular velocity of D_2, denoted by $\boldsymbol{\omega}_m$, is transmitted to J_p. Without loosing in generality we will consider $\boldsymbol{\omega}_m \perp \mathbf{d}_2$; in fact, any rotation of D_2 around its axis $\mathbf{e}_{m,z}$ will not be transmitted through the mechanism because of the presence of spherical joints $S_{m,1}$ and $S_{m,2}$, which makes D_2 permanently singular and free to rotate around $\mathbf{e}_{m,z}$. The direction of $\boldsymbol{\omega}_m$ with respect to $\{m\}$ can be defined by the angle γ. Referring to Fig. 6 $\boldsymbol{\omega}_m$ can be decomposed in two components: $\boldsymbol{\omega}_{m,\parallel}$ and $\boldsymbol{\omega}_{m,\perp}$,

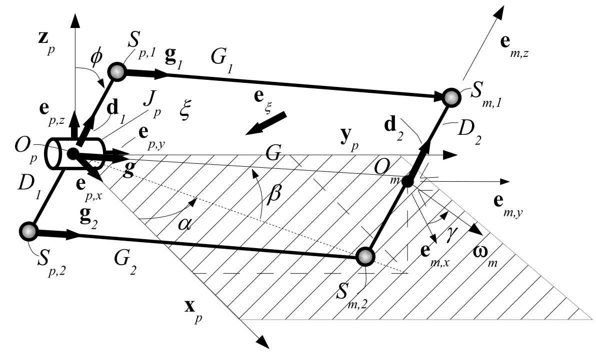


Fig. 5. The four-bar linkage

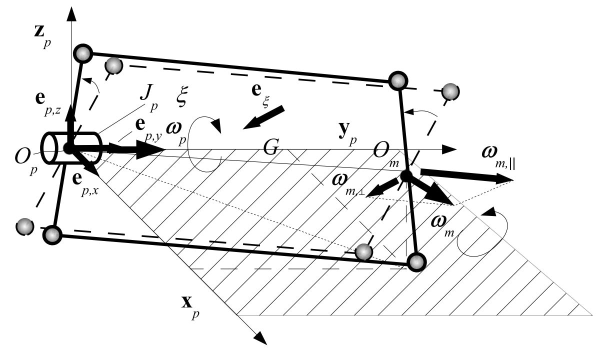


Fig. 6. Angular velocity transmission in the four-bar linkage.

parallel and perpendicular to ξ, respectively.

Considering infinitesimal rotations to keep valid the hypothesis of coplanarity, $\boldsymbol{\omega}_{m,\parallel}$ will not be transmitted through fbl to D_1 because of the four spherical joints which make D_2 singular and free to rotate around $\mathbf{g}$. Conversely $\boldsymbol{\omega}_{m,\perp}$ is transmitted to D_1 through the mechanism, exploiting a displacement of G_1 and G_2 along their axes. The same reasoning applies to $\boldsymbol{\omega}_p$, the angular velocity of joint J_p. Combining these considerations with the assumption of infinitesimal rotations in the neighborhood of the planar configuration, it is possible to infer the transmission of $\boldsymbol{\omega}_m$ to J_p by

$$\boldsymbol{\omega}_{m,\perp} = \boldsymbol{\omega}_m \cdot \mathbf{e}_\xi = \boldsymbol{\omega}_p \cdot \mathbf{e}_\xi = \omega_p \mathbf{e}_{p,y} \cdot \mathbf{e}_\xi,$$

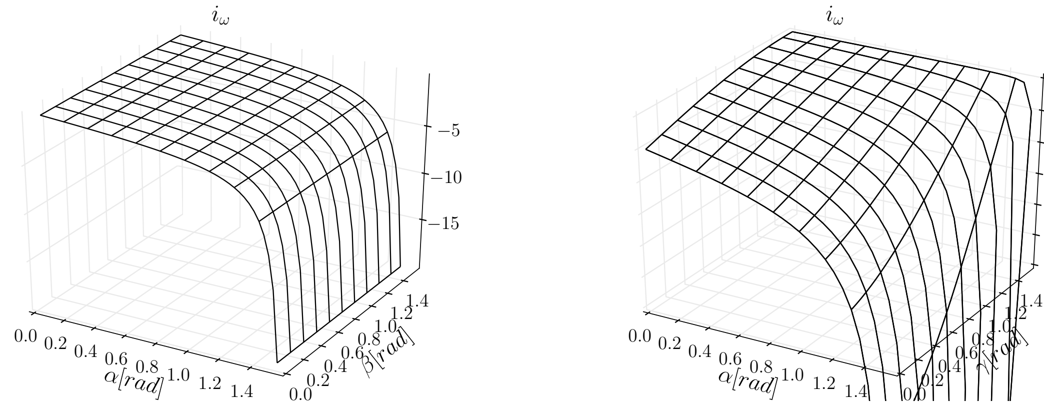
since rotation components of D_2 and D_1 around axes parallel to $\mathbf{e}_\xi$ must be equal. The scalar angular velocity ω_p of J_p is therefore

$$\omega_p = \frac{\boldsymbol{\omega}_m \cdot \mathbf{e}_\xi}{\mathbf{e}_{p,y} \cdot \mathbf{e}_\xi}.$$

Concluding, the angular velocity transmission ratio from D_2 to D_1, defined as $i_\omega = \omega_p / \|\boldsymbol{\omega}_m\|$, is represented in Fig. 7 as function of angles α, β and γ.

The obtained results can be summarized as:

- β does not influence i_ω, assuming $\boldsymbol{\omega}_m \parallel \mathbf{e}_{m,x}$ (Fig. 7(a));
- if $\boldsymbol{\omega}_m \parallel \mathbf{g}$ (*i.e.*$\gamma = \alpha$) $\Rightarrow i_\omega = 0$ fbl singularity (Fig. 7(b));
- if $\boldsymbol{\omega}_m \parallel \mathbf{e}_{p,y}$ (*i.e.*$\gamma = \pi/2$) $\Rightarrow i_\omega = 1 \quad \forall \alpha$ (Fig. 7(b)).



(a) Linkage angular velocity ratio with $\gamma = 0$

(b) Linkage angular velocity ratio with $\beta = 0$

Fig. 7. Four-bar linkage transmission ratios assuming $\phi = 0$

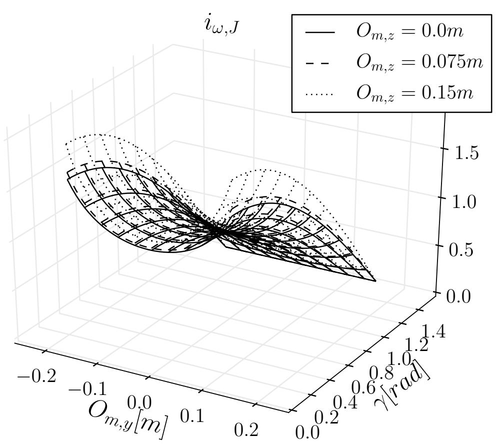


Fig. 8. Relative angular velocity between $J_{2,2}$ and $J_{2,1}$. Because of symmetry, $O_{m,z}$ is evaluated only for positive values.

B. Double universal prismatic joint

Let us now consider how two fbl are assembled in the complete mechanism. In Fig. 4 they are identified as fbl_1 and fbl_2, both connected to mp in place of D_2 on one side, and to joints $J_{1,2}$ and $J_{2,2}$, respectively, in place of J_p on the other side.

Their rotational axes $\mathbf{u}_{1,2}$ and $\mathbf{u}_{2,2}$ are parallel to $\mathbf{e}_{b,y}$ and perpendicular to the linear axes $\mathbf{u}_{1,1}$ and $\mathbf{u}_{2,1}$. Denoting by $i_{\omega,i}$ the angular velocity ratio of fbl_i, it is possible to evaluate the relative angular velocity between $J_{2,2}$ and $J_{1,2}$ as function of the mp angular velocity by

$$i_{\omega,J} = i_{\omega,2} - i_{\omega,1}.$$

Let us now consider the dimensions of the Active Headframe prototype [19]:

- distance between $\mathbf{u}_{1,1}$ and $\mathbf{u}_{2,1}$ = $0.5m$;
- length of L_1 and L_2 = $0.479m$;
- $-0.150m \leq O_{m,y} \leq 0.150m$;
- $-0.150m \leq O_{m,z} \leq 0.150m$.

In Fig. 8 $i_{\omega,J}$ is plotted against the position of O_m and of the γ angle. It can be noted that an infinitesimal rotation of mp around $\mathbf{e}_{m,y}$ ($\gamma = \pi/2$) does not cause a relative rotation between $J_{1,2}$ and $J_{2,2}$ ($i_{\omega,J} = 0$) making these joints rotate constantly in phase. Any other direction of rotation makes $J_{1,2}$ and $J_{2,2}$ rotate mutually; if $\gamma = 0$ this rotation is maximum for any specific value of $O_{m,y}$.

Exploiting this kinematic relationship, the double universal prismatic joint $dupj$, made by the universal joints $J_{1,3,3}$ and $J_{2,3,3}$ assembled in phase to guarantee homokineticity, and properly connected by the passive prismatic joint J_{1-2} to compensate the relative translation between $J_{1,1}$ and $J_{2,1}$, prevents the rotation of mp around $\mathbf{e}_{m,x}$, lowering the number of its dof from six to five. In fact, the two universal joints prevent the relative rotation between $J_{1,2}$ and $J_{2,2}$, keeping them constantly in phase and allowing mp to rotate only around $\mathbf{e}_{m,y}$. $dupj$ is the sole element loaded torsionally, making possible to have all the other elements of the parallel structure slim and light.

C. Mobile platform

The mobile platform mp is the rigid body to which all the kinematic chains are connected by a set of spherical joints (Fig. 2(b)). A proper disposition of these joints makes mp partially decouple translational to rotational dof.

The spherical joints of fbl_1 and fbl_2 connected to mp are placed symmetrically with respect to the origin O_m along $\mathbf{e}_{m,z}$. Since, as mentioned in sec. III-B, any rotation of the mobile platform around $\mathbf{e}_{m,x}$ is prevented by $dupj$, constantly guaranteeing the planarity condition mentioned in sec. III-A for fbl_1 and fbl_2, it is $\overline{P_1O_m} = L_1$ and $\overline{P_2O_m} = L_2$. Moreover $\overline{P_3O_m} = L_3$ by assembly hypothesis (Fig. 2(a)). These three constraints, given $J_{1,1}$, $J_{2,1}$ and $J_{3,1}$ coordinates, determine univocally O_m as the intersection point of three spherical surfaces centered in P_1, P_2 and P_3, excluding different assembly configurations (see section IV-B for details). Hence, the position of O_m is defined by a subset of controlled joints allowing the mechanism to be partially decoupled. The function of chains c_4 and c_5 is to constrain the two remaining rotational dof of mp (the third one is constrained by $dupj$). Besides the dof around $\mathbf{e}_{m,y}$ (section III-B), mp is free to rotate around the axis $\mathbf{e}_{m,z}$ because of the alignment of all the spherical joints connecting fbl_1 and fbl_2 to mp. To constraint these two dof, L_4 and L_5 apply moments on mp both with respect to $\mathbf{e}_{m,y}$ and $\mathbf{e}_{m,z}$, since neither Q_4 nor Q_5 lie on $\mathbf{e}_{m,y}$ and $\mathbf{e}_{m,z}$. For symmetry reasons an advantageous configuration is shown in Fig. 2(b), where Q_4 and Q_5 are configured symmetric with respect to $\mathbf{e}_{m,z}$, equidistant to $\mathbf{e}_{m,y}$ and all the spherical joints on the mobile platform are coplanar (the advantage of this configuration is clarified in sec. IV-B).

In conclusion, once defined the position of O_m by $J_{1,1}$, $J_{2,1}$ and $J_{3,1}$, the peculiar joints arrangement allows to determine the orientation of the mobile platform by joints $J_{4,1}$ and $J_{5,1}$, decoupling rotational dof from the translational ones.

IV. KINEMATICS

Inverse kinematic equations of parallel machines are typically easy to be formulated and solved, especially if compared to those of serial architectures. On the contrary direct kinematic formulations are typically tricky and quite complex. In this section the inverse and direct kinematic equations of the mechanism are presented, highlighting the advantages deriving from being partially decoupled.

Referring to Fig. 2 let us denote by:

- $\mathbf{P}_i = [P_{i,x}, P_{i,y}, P_{i,z}]^T = \mathbf{O}_b + \mathbf{A}_i$ the assembly point of link L_i on its linear axis (base endpoint), being $\mathbf{A}_i$ the offset of each endpoint w.r.t. the base origin $\mathbf{O}_b$;
- $\mathbf{Q}_i = [Q_{i,x}, Q_{i,y}, Q_{i,z}]^T = \mathbf{O}_m + \mathbf{B}_i$ the assembly point of link L_i on mp (mobile platform endpoint), being $\mathbf{B}_i$ the offset of each endpoint w.r.t. mp origin $\mathbf{O}_m$.

Moreover let us define:

- $\mathbf{Q} = [q_1, \ldots, q_5]^T$ the set of joint coordinates;
- $\mathbf{X} = [O_{m,x}, O_{m,y}, O_{m,z}]^T = \mathbf{O}_m$ the set of translational coordinates of the mobile platform w.r.t. the base origin O_b;
- $\mathbf{\Phi} = [\phi, \theta]^T$ the set of rotational coordinates of mp, denoting by ϕ and θ the rotations in sequence around $\mathbf{e}_{m,y}$ and $\mathbf{e}_{m,z}$, respectively;
- $\mathbf{S} = \left[\mathbf{X}^T, \mathbf{\Phi}^T\right]^T$ the complete set of the task-space coordinates;
- $\rho_{O,r} = \{\mathbf{R} : \|\mathbf{R} - \mathbf{O}\| = r\}$ the spherical surface centered in $\mathbf{O}$ with an r radius;
- $Rt(\mathbf{u}, \alpha)$ the 3x3 rotation matrix of α around $\mathbf{u}$;

Moreover, the rototranslation of $\{b\}$, associated to *mp*, can be expressed by the homogeneous matrix

$$ {}^b\mathbf{T}_m = \left[\begin{array}{c|c} Rt(\mathbf{e}_{m,y},\phi)Rt(\mathbf{e}_{m,z},\theta) & \begin{matrix} O_{m,x} \\ O_{m,y} \\ O_{m,z} \end{matrix} \\ \hline \mathbf{0} & 1 \end{array}\right] \tag{1} $$

A. Inverse kinematics

Given the mobile platform pose $\mathbf{S}$ and, hence, given the mobile platform endpoint Q_i for each link L_i of length l_i, the base endpoint P_i necessarily lies on the spherical surface ρ_{Q_i,l_i}. Referring to Fig. 2(a), let r_i be the line collinear to the unit vector $\mathbf{u}_{i,1}$ of the actuated prismatic joint $J_{i,1}$. As a consequence, the base endpoint of link L_i is

$$ \mathbf{P}_i = r_i \cap \rho_{Q_i,l_i} $$

which results in a quadratic equation. The equation has none, one or two real solutions, depending on the position of mobile platform, respectively outside, on the boundary or inside the reachable workspace. Two distinct real solutions of each equation represent two different assembly configurations of the kinematics. Given a reference point $\mathbf{P}_{i,0}$ on r_i for each actuated prismatic joint $J_{i,1}$, the correspondent joint coordinate is

$$ q_i = (\mathbf{P}_i - \mathbf{P}_{i,0}) \cdot \mathbf{u}_{i,1}. \tag{2} $$

B. Direct kinematics

Given the joint coordinates $\mathbf{Q}$, the base endpoint P_i derives directly from (2). Being known the distance between O_m and $P_1/P_2/P_3$, equal to $l_1/l_2/l_3$ respectively, the center of mp can be evaluated by

$$ \mathbf{O}_m = \rho_{P_1,l_1} \cap \rho_{P_2,l_2} \cap \rho_{P_3,l_3} = \mathbf{X}, $$

a typical *trilateration* problem, analytically solvable by some different techniques (*e.g.*[22]) (Fig. 9(a)). If the system of equations has no real solution L_1, L_2 and L_3 cannot be assembled. One or more distinct solutions of the system of equations denote one or more assembly configurations, respectively.

Referring to Fig. 9(b), $\mathbf{Q}_4$ and $\mathbf{Q}_5$, the connection points of links L_4 and L_5, whose ground position is defined by coordinates q_4 and q_5, must respect the condition:

$$ \mathbf{Q}_i \in c_{Q,i} = \rho_{P_i,l_i} \cap \rho_{O_m,b_i} \qquad \forall i \in \{4,5\} $$

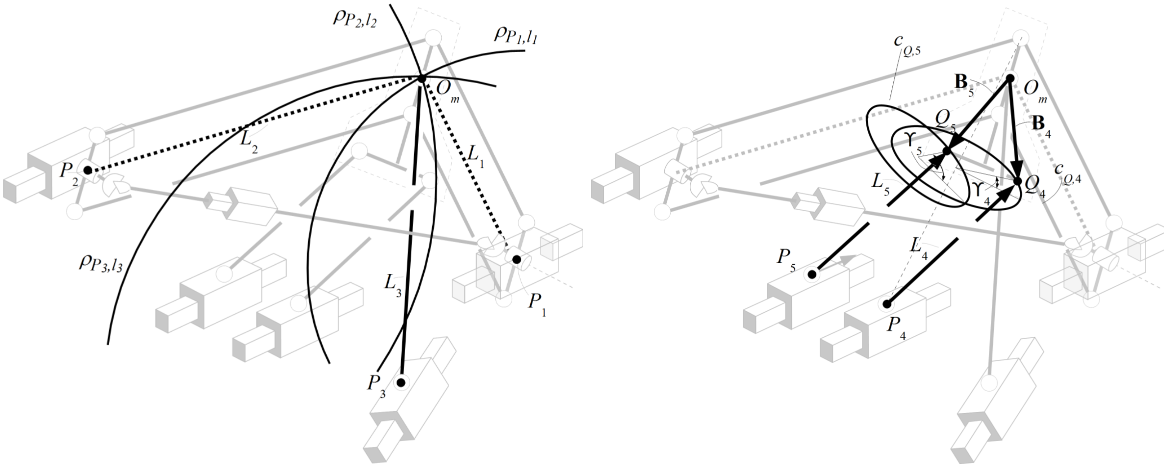
i.e. they must necessarily lie on the circumference $c_{Q,i}$, intersection of two spherical surfaces, one centered in P_i with radius $l_i = \|\mathbf{L}_i\|$ and the other centered in O_m with radius $b_i = \|\mathbf{B}_i\|$.

The position of $\mathbf{Q}_i(\Upsilon_i)$ on $c_{Q,i}$ can be expressed as function of an angular coordinate Υ_i and the actual position of $\mathbf{Q}_4$ and $\mathbf{Q}_5$ can be numerically evaluated solving the system

$$ \begin{cases} \|\mathbf{Q}_5(\Upsilon_5) - \mathbf{Q}_4(\Upsilon_4)\| = \|\mathbf{B}_5 - \mathbf{B}_4\| \\ \\ Q_{4,y} + Q_{5,y} = 2O_{m,y} \end{cases} \tag{3} $$

where: a) the first equation constraints the distance between $\mathbf{Q}_4$ and $\mathbf{Q}_5$, according to their actual distance on the mobile platform ($\mathbf{B}_4$ and $\mathbf{B}_5$ are known by design); b) the second equation constraints the y coordinate of $\mathbf{Q}_4$, $\mathbf{Q}_5$ and $\mathbf{O}_m$. In fact, explicitating (1), the position of $\mathbf{Q}_i$ in $\{b\}$ is

$$ \mathbf{Q}_i = {}^b\mathbf{T}_m \mathbf{B}_i = \begin{bmatrix} B_{i,x}c\phi c\theta - B_{i,y}s\theta c\phi + B_{i,z}s\phi + O_{m,x} \\ B_{i,x}s\theta + B_{i,y}c\theta + O_{m,y} \\ -B_{i,x}s\phi c\theta + B_{i,y}s\theta s\phi + B_{i,z}c\phi + O_{m,z} \\ 1 \end{bmatrix} $$



(a) Translation coordinates

(b) Rotation coordinates

Fig. 9. Direct kinematics.

denoting by $s\alpha$ and $c\alpha$ the sine and the cosine of α, respectively. Since $B_{4,x} = B_{5,x} = 0$ and $B_{4,y} = -B_{5,y}$ by design (sec. III-C), we obtain the second equation of (3).

Finally, being known $\mathbf{Q}_i$, $\mathbf{B}_i$ and $\mathbf{Q}$, the homogeneous matrix defining the pose of the mobile platform is

$$^{b}\mathbf{T}_m = \left[\begin{array}{c|c|c|c} \mathbf{e}_{m,x} & \mathbf{e}_{m,y} & \mathbf{e}_{m,z} & \mathbf{O}_m \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad (4)$$

where, referring to Fig.2(b),

- $\mathbf{e}_{m,y} = (\mathbf{Q}_5 - \mathbf{Q}_4)/\|\mathbf{Q}_5 - \mathbf{Q}_4\|$,
- $\mathbf{e}_{m,z} = (\mathbf{O}_m - \mathbf{Q}_m)/\|\mathbf{O}_m - \mathbf{Q}_m\|$,
- $\mathbf{e}_{m,x} = \mathbf{e}_{m,y} \times \mathbf{e}_{m,z}$.

V. CONCLUSIONS

A 5dof parallel and partially decoupled kinematic architecture has been presented. Its peculiarity of partially decoupling translational and rotational dof is advantageous both in the design phase, making possible a better dimensioning on the basis of different rototranslational requirements, and during its use, facilitating its motion planning and control, simplifying kinematic equations. The architecture has been applied in the Active Headframe, a robotic prototype for head support in open-skull awake surgery.

ACKNOWLEDGMENTS

The authors would like to thank João Carlos Dalberto and Roberto Bozzi for the Active Headframe realization, and Francesco Paolucci for its experimental assessment.

REFERENCES

[1] Y. Jin, I.-M. Chen, and G. Yang, "Kinematic design of a family of 6-dof partially decoupled parallel manipulators," *Mechanism and Machine Theory*, vol. 44, no. 5, pp. 912 – 922, 2009.

[2] J. P. Merlet, "Solving the forward kinematics of a gough-type parallel manipulator with interval analysis," *The Intl. Journal of Robotics Research*, vol. 23, no. 3, pp. 221–235, 2004.

[3] Z. Geng and L. Haynes, "Neural network solution for the forward kinematics problem of a stewart platform," in *Robotics and Autom., 1991. Proc., 1991 IEEE Intl. Conf. on*, pp. 2650–2655 vol.3, Apr.

[4] S. Zha *et al.*, "Direct kinematics problem of parallel robot based on ls-svm," in *The Sixth WCICA 2006.*, pp. 8958–8961.

[5] L. Zollo *et al.*, "Design of a planar robotic machine for neuro-rehabilitation," in *Robotics and Automation, 2008. ICRA 2008. IEEE Intl. Conf. on*, pp. 2031–2036, May.

[6] W. Li *et al.*, "Hemisphere, a fully decoupled parallel 2-dof spherical mechanism," in *Proc. of the 7th WSEAS Intl. Conf. on Robotics, Control & Manuf. Technology*, ROCOM'07, pp. 301–306, 2007.

[7] C. Gosselin and X. Kong, "Cartesian parallel manipulators," 05 2004.

[8] S. Briot and I. Bonev, "Pantopteron: A new fully decoupled 3dof translational parallel robot for pick-and-place applications," *Journal of Mechanisms and Robotics*, vol. 1, no. 2, 2009.

[9] P.-L. Richard *et al.*, "Kinematic analysis and prototyping of a partially decoupled 4-dof 3t1r parallel manipulator," *Journal of Mechanical Design*, vol. 129, no. 6, pp. 611–616, 2006/05/26.

[10] S. Briot and I. A. Bonev, "Pantopteron-4: A new 3t1r decoupled parallel manipulator for pick-and-place applications," *Mechanism and Machine Theory*, vol. 45, no. 5, pp. 707 – 721, 2010.

[11] S. M. Kim *et al.*, "Kinematic analyses of a 1t2r and a 1t3r parallel mechanisms with closed-form position solutions," in *Control and Autom., 2009. ICCA 2009. IEEE Intl. Conf. on*, pp. 1874–1879, Dec.

[12] G. Yang *et al.*, "Kinematic design of a six-dof parallel-kinematics machine with decoupled-motion architecture," *Robotics, IEEE Transactions on*, vol. 20, no. 5, pp. 876–887, Oct.

[13] Y. Jin, I.-M. Chen, and G. Yang, "Kinematic design of a 6-dof parallel manipulator with decoupled translation and rotation," *Robotics, IEEE Transactions on*, vol. 22, no. 3, pp. 545–551, June.

[14] H. Moreno and R. Saltarn, "A novel 6 dof parallel robot with decoupled translation and rotation," in *Thirteenth World Congress in Mechanism and Machine Science*, 2011 June 19-23.

[15] C. Gosselin, E. S. Pierre, and M. Gagne, "On the development of the agile eye," *Rob.& Aut. Mag., IEEE*, vol. 3, no. 4, pp. 29–37, 1996.

[16] O. Altuzarra *et al.*, "Partially decoupled parallel manipulators based on multiple platforms.," in *12th IFToMM World Congress on the Theory of Machines and Mechanisms*, 18-21 June 2007.

[17] Q. Jin *et al.*, "Structure synthesis of a class of 5-dof parallel robot mechanisms based on single openedchain units.," in *Proc. of the ASME conf. DETC2001*, 2001.

[18] X. Kong and C. M. Gosselin, "Type synthesis of 5-dof parallel manipulators based on screw theory," *J. Robot. Syst.*, vol. 22, pp. 535–547, Oct. 2005.

[19] M. Malosio *et al.*, "The kinematic architecture of the active headframe: A new head support for awake brain surgery," in *Proc. of the Intl. Conf. of the EMBC IEEE*, pp. 1417–1421, 28 2012-Sept. 1.

[20] G. Gogu, *Structural Synthesis of Parallel Robots: Part 1: Methodology*. Dordrecht: Springer, 2008.

[21] X. Kong and C. Gosselin, "Type synthesis of six-dof wrist-partitioned parallel manipulators," *J Mech. Des.*, vol. 130, p. 062302, Jun 2008.

[22] "Trilateration." http://en.wikipedia.org/wiki/Trilateration. Accessed: 2013-07-03.