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Position Regulation of Flexible-Joint Robots with Input/Output Constant Delays

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Abstract— **In this paper, the problem of set-point control for flexible-joint robotic manipulators with input/output time delays is investigated. By utilizing scattering transformation with an input-output passive controller, it is demonstrated that the flexible-joint robotic control system can be stabilized when there are time delays in the communication channels. Although stabilization is achieved, the flexible-joint robot cannot be regulated to the desired configuration when utilizing the scattering variables. Hence, a new control framework without scattering transformation is subsequently studied in this paper to guarantee both stability and position regulation provided that the control gain is appropriately selected based on a bound on the time delays. The proposed control algorithms are validated via numerical examples on a two-link flexible-joint robotic manipulator.**

I. INTRODUCTION

Due to the advantages of increased flexibility and modularity [1], [2], [3], [4], [5], control of robotic systems over a communication network has attracted the attention of several researchers. In contrast to wired interconnections, by transmitting input and output signals of robots and controllers through communication networks, manipulators can implement various tasks without being constrained by wired connections. However, possible time delays in the communication networks should be taken into account in order to ensure stability and performance of the closed-loop control system [6]. Therefore, several control algorithms have been developed for set-point control of rigid-joint robot with input/output delays [2], [5], [7].

The idea of scattering transformation, which was originally developed to stabilize a bilateral teleoperation system under constant communication delays [8], [9], has been widely applied for the studies of networked systems [3], [5], [10]. By invoking the fundamental passivity theorem [11], scattering transformation has been exploited for set-point control for rigid-joint robot by utilizing the passivity of the controller and robotic manipulator. The stability of controlling nonlinear robotic systems with non-collocated controller has been studied in [2] for constant delays. However, the aforementioned robotic control system was developed for rigid robots.

Due to practical considerations, the presence of joint flexibility in a robotic manipulator is inevitable. If the controller is designed without taking into account the joint flexibility, instability can occur due to the non-collocated nature of the control action [12]. Even though the problem of controlling flexible-joint robots has been studied for decades [13], [14], [15], [16], [17], the signals were assumed to exchanged via a perfect communication between the robotic manipulator and the controller. This assumption limits the application of flexible-joint robot where the controller is not collocated with the robot.

The objective of this paper is to develop control algorithms for flexible-joint robotic manipulator to ensure both stability and position regulation in the presence of input/output delays. We first demonstrate that with the use of scattering transformation for a position regulation controller [18], the closed-loop control system with input/output constant delays is stable. However, due to the gravity compensation term in the controller and joint flexibility in the robotic manipulator, the robot is unable to achieve position regulation. Hence, an alternative controller without utilizing the scattering transformation is studied under the assumption of internal damping. It is demonstrated that the flexible-joint robot is stable and can be regulated to the desired configuration. The proposed control architecture does not require utilization of the scattering transformation, and can guarantee stability by adjusting the control gains.

The rest of this paper is organized as follows. A brief background on flexible-joint robotic manipulator and the set-point controller is presented in Section II. Subsequently, the main results of the paper are detailed in Section III. Numerical studies of the proposed control algorithms are accomplished in Section IV. Finally, the results are summarized and the future work is discussed in Section V.

II. PRELIMINARIES

The flexible-joint robotic manipulator considered in this paper is modeled as an Euler-Lagrange system [15], [19], and the equations of motion in the absence of friction and disturbances are given as

$$\begin{cases} M(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + g(q_1) + K(q_1-q_2) = 0 & (1) \\ J_m\ddot{q}_2 + K(q_2-q_1) = -\tau_m + \tau_e = \tau_t & (2) \end{cases}$$

where $q_1,\ q_2 \in R^n$ are the vector of joint angles and the vector of motor shaft angles, $\tau_m \in R^n$ is motor torque acting on the system, $\tau_e \in R^n$ is the external torque acting on the system, $M(q_1) \in R^{n\times n}$ is the positive definite inertia matrix, $C(q_1,\dot{q}_1)\dot{q}_1 \in R^n$ is the vector of Coriolis/Centrifugal forces where $C(q_1,\dot{q}_1) \in R^{n\times n}$, $J_m \in R^{n\times n}$ is the actuators' inertia matrix, $K \in R^{n\times n}$ represents the positive diagonal matrix

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of the joint stiffness, and $g(q_1)=\frac{\partial U_g(q_1)}{\partial q_1}$, where $U_g(q_1)$ is the gradient of the potential energy due to gravity. For the gravitational potential energy, there exists a global minimum such that $U_{g\min}:=\min_{q_1} U_g(q_1)$. The above equations exhibit several fundamental properties due to their Lagrangian dynamic structure [14], [19].

Property 2.1: Under an appropriate definition of the matrix $C(q_1,\dot{q}_1)$, the matrix $\dot{M}(q_1)-2C(q_1,\dot{q}_1)$ is skew symmetric.

Property 2.2: The matrix $M(q_1)$ is symmetric positive definite, and considering revolute joints in the robot, there exist positive constants λ_m and λ_M such that $\lambda_m I_n \le M(q_1) \le \lambda_M I_n$, where $I_n \in R^{n\times n}$ is an identity matrix.

Property 2.3: For $q,\ \dot{q},\ \xi \in R^n$, there exists a positive constant k_c such that the matrix of Coriolis/Centrifugal torques is bounded by $\|C(q,\dot{q})\xi\| \le k_c\|\dot{q}\|\|\xi\|$.

Property 2.4: A positive constant β exists such that $\|\partial g(q_1)/\partial q_1\| \le \beta,\ q_1 \in R^n$. The above inequality implies that $\|g(q_1)-g(\bar{q}_1)\| \le \beta\|q_1-\bar{q}_1\|,\ \forall q_1,\bar{q}_1 \in R^n$.

In this paper, the notations $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalue of the enclosed matrix. The norm of vector $x \in R^n$ is defined as $\|x\| = (\sum_{i=1}^n x_i^2)^{1/2}$, and the norm of matrix $A \in R^{n\times n}$ is defined as $\|A\| = \left(\lambda_{\max}(A^TA)\right)^{1/2}$, which implies that if A is symmetric positive definite, we have $\|A\| = \lambda_{\max}(A)$. In addition, for the sake of simplicity the notation $\xi^{[i]} = d^i\xi/dt^i$ is used as required.

The problem of position regulation for flexible-joint robots has been studied in [15], [20] in the absence of time delays. The control law for (1) and (2) was proposed in [20] that

$$\tau_m = k_p(q_2-q_{2d})+k_d\dot{q}_2-g(q_{1d}), \tag{3}$$

where k_p and k_d are positive constant gains, $q_{1d} \in R^n$ is the desired configuration of the links, and $q_{2d} = q_{1d} + K^{-1}g(q_{1d}) \in R^n$ is given as the desired position of the motor. By defining the positive matrix $\bar{K} \in R^{2n\times 2n}$ as

$$\bar{K} = \begin{bmatrix} K & -K \\ -K & K+k_pI_n \end{bmatrix},$$

it has been demonstrated that using the controller (3) under perfect communication channels and provided the control gain satisfies $\lambda_{min}(\bar{K}) > \beta$ [20], the equilibrium configuration $(q_1,q_2,\dot{q}_1,\dot{q}_2) = (q_{1d},q_{2d},0,0)$ is globally asymptotically stable.

However, if the control of flexible-joint robots is subjected to input/output delays, then the closed-loop control system is unstable (as shown in Section IV). Therefore, the objective of this paper is to study the closed-loop control system in the presence of communication delays. The control architecture for such a system with input/output delays is illustrated in Fig. 1, where T_1 and T_2 are time delays in the communication network. In order to deal with the stability issue, two control algorithms and architectures are presented in the subsequent section.

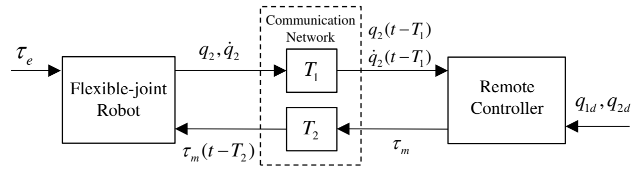


Fig. 1. Control of flexible-joint robot with input/output delays.

III. STABILITY AND POSITION REGULATION

It is first demonstrated in this section that the flexible-joint robot with the use of scattering transformation can stabilize an otherwise unstable system under unknown constant input/output delays. However, position regulation of flexible-joint robot cannot be guaranteed by using the scattering variables. Therefore, another controller is proposed subsequently to ensure stability and ensure regulation performance without utilizing the scattering transformation.

A. Control Architecture with Scattering Transformation

The idea of scattering or wave-variable transformation was proposed in [8], [9], [21] to stabilize teleoperation system with communication delays. Encoded from the input and output signals of the robotic system, wave-variables are transmitted through delayed network instead to passify the communication block with constant delays. Therefore, by invoking the fundamental passivity theorem [11], the closed-loop teleoperation system interconnecting with passive robotic systems, human operator, and remote environment is stable. Since the flexible-joint robotic manipulator is passive with $(\tau_t,\ \dot{q}_2)$ as the input-output pair [14], the use of scattering representation is considered in this paper to stabilize the position control of flexible-joint robots with input/output delays.

The control architecture utilizing scattering representation is shown in Fig. 2, where $T_1,\ T_2 \ge 0$ are both constant and bounded delays. By considering the scattering variables as [8], [9]

$$\begin{array}{ll} v_r = \frac{1}{\sqrt{2b}}(\tau_m+b\dot{q}_2) & ,\quad z_r = \frac{1}{\sqrt{2b}}(\tau_m-b\dot{q}_2) \\ v_c = \frac{1}{\sqrt{2b}}(y_c+bu_c) & ,\quad z_c = \frac{1}{\sqrt{2b}}(y_c-bu_c) \end{array} \tag{4}$$

where the wave impedance b is a positive constant, the controller (3) can be modified and given as

$$\begin{cases} \dot{x}_c = u_c \\ y_c = k_p(x_c-q_{2d})+k_du_c-g(q_{1d}) \end{cases} \tag{5}$$

where the control gains $k_p,\ k_d$ and the desired configurations $q_{1d},\ q_{2d}$ are the same as in (3), x_c is the state of the controller, u_c is the input of the controller that is obtained from decoding the scattering variables v_c, and y_c is the output of the controller that is fed back to the flexible-joint robot.

If there exists no time delays in the communication channels ($T_1 \equiv T_2 \equiv 0$) such that $v_c = v_r$ and $z_r = z_c$, we have $u_c = \dot{q}_s$ and $\tau_m = y_c$ from the scattering transformation (4). Therefore, the controller (5) is identical to (3) if $x_c(0) = q_2(0)$. However, in the presence of constant input-output

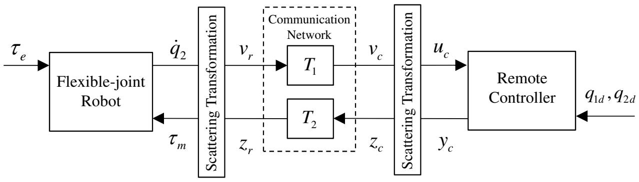


Fig. 2. Control of flexible-joint robot under input/output delays with the utilization of scattering transformation.

delays, the transmission equations between the robot and the controller become

$$v_c(t) = v_r(t - T_1) \quad , \quad z_r(t) = z_c(t - T_2). \tag{6}$$

It is worth noting that the constant time delays are unknown to the controller when utilizing the scattering transformation.

Before stating the result using the scattering transformation, defining an auxiliary function $H(x_c)$ given by

$$H(x_c) = \frac{k_p}{2}(x_c - q_{2d})^T(x_c - q_{2d}) - x_c^T g(q_{1d}). \tag{7}$$

The extremum points of the auxiliary function occur at $\nabla H(x_c) = 0$, which gives $x_c = q_{2d} + k_p^{-1} g(q_{1d}) := \bar{x}_c$. Since $\nabla^2 H(x_c) = k_p > 0$ and is independent of x_c, the function $H(x_c)$ has a global minimum at $x_c = \bar{x}_c$.

The following proposition demonstrates that the closed-loop system can be stabilized with the use of scattering transformation in the presence of constant input/output communication delays.

Proposition 3.1: Consider the closed-loop system described by (1), (2), (4), (5), and (6) with $\tau_e \equiv 0$. The signals of the system are bounded independent of the constant communication delays, and $\lim_{t\to\infty} \dot{q}_1(t) = \lim_{t\to\infty} \dot{q}_2(t) = 0$, $\lim_{t\to\infty} \ddot{q}_1(t) = \lim_{t\to\infty} \ddot{q}_2(t) = 0$.

Proof: Consider a positive-definite storage functional for the closed-loop system as

$$\begin{aligned} S = {} & \frac{1}{2}\dot{q}_1^T M(q_1)\dot{q}_1 + \frac{1}{2}\dot{q}_2^T J_m \dot{q}_2 + \frac{1}{2}(q_1 - q_2)^T K(q_1 - q_2) \\ & + U_g(q_1) - U_{g\min} + H(x_c) - H(\bar{x}_c) + \frac{1}{2}\int_{t-T_1}^{t} \|v_r(\sigma)\|^2 d\sigma \\ & + \frac{1}{2}\int_{t-T_2}^{t} \|z_c(\sigma)\|^2 d\sigma \geq 0, \end{aligned}$$

where $U_{g\min}$ is the minimum potential energy of $U_g(q_1)$ for the flexible-joint manipulator, and $H(x_c)$ is the auxiliary function defined in (7). Taking the derivative of the storage functional and using the transmission equations (6) yields

$$\begin{aligned} \dot{S} = {} & \dot{q}_1^T(-C\dot{q}_1 - g(q_1) - K(q_1 - q_2)) + \frac{1}{2}\dot{q}_1^T \dot{M}\dot{q}_1 \\ & + \dot{q}_2^T\big(-\tau_m - K(q_2 - q_1)\big) + (\dot{q}_1 - \dot{q}_2)^T K(q_1 - q_2) \\ & + \dot{q}_1^T g(q_1) + k_p \dot{x}_c^T(x_c - q_{2d}) - \dot{x}_c^T g(q_{1d}) + \frac{1}{2}(\|v_r\|^2 - \|z_r\|^2 \\ & + \|z_c\|^2 - \|v_c\|^2). \end{aligned}$$

By utilizing Property 2.1, the scattering variables (4), and the output of controller (5), $\dot{S}$ becomes

$$\begin{aligned} \dot{S} = {} & -\dot{q}_1^T g(q_1) - \dot{q}_1^T K(q_1 - q_2) - \dot{q}_2^T \tau_m - \dot{q}_2^T K(q_2 - q_1) \\ & + (\dot{q}_1 - \dot{q}_2)^T K(q_1 - q_2) + \dot{q}_1^T g(q_1) + u_c^T\big(y_c - k_d u_c \\ & + g(q_{1d})\big) - u_c^T g(q_{1d}) + \tau_m^T \dot{q}_2 - u_c^T y_c \\ = {} & -k_d u_c^T u_c \leq 0. \end{aligned} \tag{8}$$

Therefore, the storage functional S is bounded which implies that signals $\dot{q}_1$, $\dot{q}_2$, $q_1 - q_2$, $x_c \in \mathscr{L}_\infty$, and from (8) $u_c \in \mathscr{L}_2$. Using the scattering variables (4) and the transmission equations (6), the relationships between signals are

$$y_c(t) + b u_c(t) = \tau_m(t - T_1) + b\dot{q}_2(t - T_1), \tag{9}$$

$$y_c(t - T_2) - b u_c(t - T_2) = \tau_m(t) - b\dot{q}_2(t). \tag{10}$$

Substituting the control output $y_c(t)$, the above equations become

$$\begin{aligned} & k_p(x_c(t) - q_{2d}) + k_d u_c(t) - g(q_{1d}) + b u_c(t) \\ & \qquad = \tau_m(t - T_1) + b\dot{q}_2(t - T_1), \end{aligned} \tag{11}$$

$$\begin{aligned} & k_p(x_c(t - T_2) - q_{2d}) + k_d u_c(t - T_2) - g(q_{1d}) - b u_c(t - T_2) \\ & \qquad = \tau_m(t) - b\dot{q}_2(t). \end{aligned} \tag{12}$$

By letting $k_d = b$ to avoid wave reflections [9], (11) and (12) can be rewritten as

$$2b u_c(t) + k_p(x_c(t) - q_{2d}) - g(q_{1d}) = \tau_m(t - T_1) + b\dot{q}_2(t - T_1) \tag{13}$$

$$k_p(x_c(t - T_2) - q_{2d}) - g(q_{1d}) = \tau_m(t) - b\dot{q}_2(t). \tag{14}$$

Since x_c, $\dot{q}_2$ are bounded and q_{2d}, $g(q_{1d})$ are constant, from (14) we get that τ_m is bounded. Utilizing the result in (13) yields that $u_c \in \mathscr{L}_\infty$, and further using the result in the dynamic model (2) implies that $\ddot{q}_2 \in \mathscr{L}_\infty$. Differentiating (14) yields that $\dot{\tau}_m$ is bounded, and this result additionally implies that $\dot{u}_c$ is bounded by differentiating (13).

As a square integrable signal with a bounded derivative approaches the origin [19], $u_c \in \mathscr{L}_2$ and $\dot{u}_c \in \mathscr{L}_\infty$ implies that $\lim_{t\to\infty} u_c(t) = 0$. Delaying the transmission equation (14) by T_1 and subtracting from (13) yields

$$2b u_c(t) + k_p(x_c(t) - x_c(t - T_1 - T_2)) = 2b\dot{q}_2(t - T_1). \tag{15}$$

Taking the limit of the above equation for $t \to \infty$ with $\lim_{t\to\infty} u_c(t) = 0$, we get $\lim_{t\to\infty} k_p\big(x_c(t) - x_c(t - T_1 - T_2)\big) = \lim_{t\to\infty} 2b\dot{q}_2(t - T_1)$. As $\dot{x}_c = u_c$, the previous equation can be rewritten as $\lim_{t\to\infty} k_p \int_{t-T_1-T_2}^{t} u_c(\tau)d\tau = \lim_{t\to\infty} 2b\dot{q}_2(t - T_1)$. Thus, we obtain that $\lim_{t\to\infty} \dot{q}_2(t) = 0$ as $\lim_{t\to\infty} u_c(t) = 0$. Since $\dot{q}_1, \dot{q}_2, \dot{\tau}_m \in \mathscr{L}_\infty$, differentiating (2) yields that $q_2^{[3]} \in \mathscr{L}_\infty$. As $\lim_{t\to\infty} \dot{q}_2(t) = 0$ and $\ddot{q}_2$ is uniformly continuous, by invoking Barbalat's Lemma [22] we have $\lim_{t\to\infty} \ddot{q}_2(t) = 0$.

By differentiating (14) twice, we get $k_p \ddot{x}_c(t - T_2) = \ddot{\tau}_m(t) - b q_2^{[3]}(t)$. Since $\dot{u}_c$, $q_2^{[3]} \in \mathscr{L}_\infty$ and $\ddot{x}_c = \dot{u}_c$, we have $\ddot{\tau}_m \in \mathscr{L}_\infty$. As $\dot{q}_1, q_1 - q_2 \in \mathscr{L}_\infty$, from observing (1) with Property 2.2 and Property 2.3, we get $\ddot{q}_1 \in \mathscr{L}_\infty$. Then, differentiating (2) twice with $\ddot{q}_1, \ddot{q}_2, \ddot{\tau}_m \in \mathscr{L}_\infty$ yields $q_2^{[4]} \in \mathscr{L}_\infty$. Since $\lim_{t\to\infty} \ddot{q}_2(t) = 0$ and $q_2^{[3]}$ is uniformly continuous, by invoking again Barbalat's Lemma, we have $\lim_{t\to\infty} q_2^{[3]}(t) = 0$. Taking the time derivative of (14) gives $\dot{\tau}_m = k_p \dot{x}_c(t - T_2) + b\ddot{q}_2(t) = k_p u_c(t - T_2) + b\ddot{q}_2(t)$. Substituting $\dot{\tau}_m$ to the derivative of (2), we have $J_m q_2^{[3]} + K(\dot{q}_2 - \dot{q}_1) = -\dot{\tau}_m = -k_p u_c(t - T_2) - b\ddot{q}_2$. From the above equation, since $\dot{q}_2, \ddot{q}_2, q_2^{[3]}$, and u_c

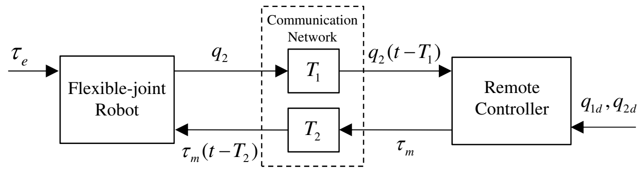


Fig. 3. Control architecture for flexible-joint robots without utilizing scattering transformation.

converge to zero as $t \to \infty$, we conclude that $\lim_{t\to\infty} \dot{q}_1(t) = 0$. Differentiating (1) with $\ddot{q}_1, \dot{q}_1, \dot{q}_2 \in \mathscr{L}_\infty$ implies that $q_1^{[3]} \in \mathscr{L}_\infty$ (note that the derivative of $M(q_1)$ and $C(q_1, \dot{q}_1)$ are bounded because $\ddot{q}_1, \dot{q}_1, \dot{q}_2 \in \mathscr{L}_\infty$). Since $\ddot{q}_1$ is uniformly continuous and $\lim_{t\to\infty} \dot{q}_1(t) = 0$, from Barbalat's Lemma it can be obtained that $\lim_{t\to\infty} \ddot{q}_1(t) = 0$. ■

The above result demonstrates that the flexible-joint robot with constant input/output communication delays can be stabilized with the use of scattering transformation. Additionally, the convergence of $\dot{q}_1, \dot{q}_2, \ddot{q}_1, \ddot{q}_2$ to the origin is guaranteed. Even though Proposition 3.1 provides a simple method to stabilize the proposed system, the robot is not guaranteed to be regulated to the desired configuration. The inability to achieve position regulation will be validated via simulations in Section IV.

B. Control Architecture without Scattering Transformation

Since the control architecture with the use of scattering transformation is unable to guarantee position regulation for flexible-joint robots with input/output delays, an alternative control algorithm without using the scattering transformation is developed in this section. We consider the same robotic model (1) and (2) but assume that there exists innate dissipation in the model of motor shaft. Thus, the dynamic model for flexible-joint robot becomes

$$\begin{cases} M(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + g(q_1) + K(q_1 - q_2) = 0 & (16) \\ J_m\ddot{q}_2 + b_m\dot{q}_2 + K(q_2 - q_1) = -\tau_m + \tau_e = \tau_t & (17) \end{cases}$$

where b_m denotes the internal damping in the robotic system and is assumed to be a constant.

The control architecture is shown in Fig. 3, where only the angle of motor shaft q_2 is transmitted to the controller. In the absence of scattering transformation, the controller output is

$$y_c = k_p(q_2(t - T_1) - q_{2d}) - g(q_{1d}). \tag{18}$$

The controller output y_c is transmitted to the flexible-joint robot directly via the communication channel with time delays T_2. Thus, the control input applied to the robot is

$$\tau_m = k_p(q_2(t - T_1 - T_2) - q_{2d}) - g(q_{1d}). \tag{19}$$

Since the desired configuration q_{2d} and gravitational compensation $g(q_{1d})$ are constant, there is no influence of delays on these signals.

The next lemma is exploited in this paper to prove the stability and position regulation of the proposed control system shown in Fig. 3.

Lemma 3.2: [23] Given signals $x, y \in R^n, \forall T$ such that $0 < T < \infty$ and $\alpha > 0$, the following inequality holds

$$-\int_0^t x^T(\sigma)\int_{-T}^0 y(\sigma+\theta)d\theta d\sigma \le \frac{\alpha}{2}\|x\|_2^2 + \frac{T^2}{2\alpha}\|y\|_2^2$$

where $\|\cdot\|_2$ denotes the $\mathscr{L}_2$ norm of the enclosed signal.

By denoting $\bar{T} > T_1 + T_2$ the upper bound of the round-trip delays, the next result follows.

Proposition 3.3: Consider the closed-loop system described by (16), (17), and (18) for $\tau_e \equiv 0$. If $b_m > k_p\bar{T}$, then the signals of the system are bounded, and $\lim_{t\to\infty}\dot{q}_1(t) = \lim_{t\to\infty}\dot{q}_2(t) = 0$, $\lim_{t\to\infty}\ddot{q}_1(t) = \lim_{t\to\infty}\ddot{q}_2(t) = 0$. Additionally, if the control gain k_p satisfies $\lambda_{\min}(\bar{K}) > \beta$, $\lim_{t\to\infty} q_1(t) = q_{1d}$ and $\lim_{t\to\infty} q_2(t) = q_{2d}$.

Proof: Consider a storage function for the closed-loop robotic system as

$$\begin{aligned} S = &\frac{1}{2}\dot{q}_1^T M(q_1)\dot{q}_1 + \frac{1}{2}\dot{q}_2^T J_m\dot{q}_2 + \frac{1}{2}(q_1 - q_2)^T K(q_1 - q_2) \\ &+ U_g(q_1) - U_{g\min} + H(q_2) - H(\bar{q}_2) \ge 0, \end{aligned} \tag{20}$$

where $H(q_2)$ is given as (7) by substituting x_c for q_2 with $\bar{q}_2 := q_{2d} + k_p^{-1}g(q_{1d})$. Taking the time derivative of the storage function, we get

$$\begin{aligned} \dot{S} = &\dot{q}_1^T(-C\dot{q}_1 - g(q_1) - K(q_1 - q_2)) + \frac{1}{2}\dot{q}_1^T\dot{M}\dot{q}_1 \\ &+ \dot{q}_2^T(-\tau_m - b_m\dot{q}_2 - K(q_2 - q_1)) + (\dot{q}_1 - \dot{q}_2)^T K(q_1 - q_2) \\ &+ \dot{q}_1^T g(q_1) + k_p\dot{q}_2^T(q_2 - q_{2d}) - \dot{q}_2^T g(q_{1d}). \end{aligned} \tag{21}$$

By utilizing Property 2.1 and control input (19), $\dot{S}$ becomes

$$\dot{S} \le -b_m\dot{q}_2^T\dot{q}_2 + k_p\dot{q}_2^T\int_{-\bar{T}}^0 \dot{q}_2(t+\sigma)d\sigma. \tag{22}$$

Integrating the above equation from 0 to t with the use of Lemma 3.2 yields

$$S(t) - S(0) \le -\|\dot{q}_2\|_2^2\left(b_m - \frac{k_p\alpha}{2} - \frac{k_p\bar{T}^2}{2\alpha}\right). \tag{23}$$

It is noted that the sign of the second term in (22) does not affect the calculation in (23) when utilizing Lemma 3.2. Hence, if

$$b_m - \frac{k_p\alpha}{2} - \frac{k_p\bar{T}^2}{2\alpha} > 0, \tag{24}$$

then the storage function S is a non-increasing function from observing (22). Since α is a positive constant resulting from Lemma 3.2, the inequality (24) leads to the condition that $b_m > k_p\bar{T}$. Therefore, if the control gains k_p, internal damping b_m, and the upper bound of round-trip delay $\bar{T}$ satisfy $b_m > k_p\bar{T}$, then $\dot{q}_1$, $\dot{q}_2$, q_1, $q_2 \in \mathscr{L}_\infty$ and $\dot{q}_2 \in \mathscr{L}_2$.

As $\dot{q}_2 \in \mathscr{L}_\infty$ and T_1, T_2 are bounded constants, we have $\tau_m \in \mathscr{L}_\infty$ from the joint input (19). The robot dynamics (17) with bounded τ_m results in $\ddot{q}_2 \in \mathscr{L}_\infty$. Since $\ddot{q}_2 \in \mathscr{L}_\infty$ and $\dot{q}_2 \in \mathscr{L}_2$, we have $\lim_{t\to\infty}\dot{q}_2(t) = 0$ by invoking Barbalat's Lemma [22]. By taking the time derivative of τ_m (19) with $\dot{q}_2 \in \mathscr{L}_\infty$, we obtain that $\dot{\tau}_m$ is bounded. Differentiating (17) leads to $q_2^{[3]} \in \mathscr{L}_2$. As $\int_0^t \ddot{q}_2(\sigma)d\sigma$ exists and $\ddot{q}_2$ is uniformly continuous, by invoking Barbalat's Lemma again, we conclude that $\lim_{t\to\infty}\ddot{q}_2(t) = 0$.

Taking the time derivative of the the control input (19) twice with $\ddot{q}_2 \in \mathscr{L}_\infty$ results in $\ddot{\tau}_m \in \mathscr{L}_\infty$. Thus, it is obtained by differentiating (17) twice that $q_2^{[4]} \in \mathscr{L}_\infty$. Again, since $\int_0^t q_2^{[3]}(\sigma)d\sigma$ exists and $q_2^{[3]}$ is uniformly continuous, $\lim_{t\to\infty} q_2^{[3]}(t) = 0$ by invoking Barbalat's Lemma. The time derivative of (17) gives

$$J_m q_2^{[3]} + b_m \ddot{q}_2 + K(\dot{q}_1 - \dot{q}_2) = -\dot{\tau}_m = k_p \dot{q}_2(t - T_1 - T_2). \quad (25)$$

Since $\lim_{t\to\infty} \dot{q}_2(t) = \lim_{t\to\infty} \ddot{q}_2(t) = \lim_{t\to\infty} q_2^{[3]}(t) = 0$, from the above equation, $\lim_{t\to\infty} \dot{q}_1(t) = 0$. By observing (16) with Property 2.2 and Property 2.3, we get $\ddot{q}_1 \in \mathscr{L}_\infty$. Therefore, differentiating (16) with $\dot{q}_1,\ \dot{q}_2,\ \ddot{q}_1 \in \mathscr{L}_\infty$ yields $q_1^{[3]} \in \mathscr{L}_\infty$. As $\ddot{q}_1$ is uniformly continuous and $\dot{q}_1$ has finite limit, we get $\lim_{t\to\infty} \ddot{q}_1(t) = 0$ by Barbalat's Lemma. Consequently, the system is stable with $\lim_{t\to\infty} \dot{q}_1(t) = \lim_{t\to\infty} \dot{q}_2(t) = 0$ and $\lim_{t\to\infty} \ddot{q}_1(t) = \lim_{t\to\infty} \ddot{q}_2(t) = 0$.

We next prove the position regulation of the control system. Taking the limit $t \to \infty$ for (16) and (17) yields

$$\lim_{t\to\infty} g(q_1(t)) + K \lim_{t\to\infty}(q_1(t) - q_2(t)) = 0 \quad (26)$$

$$K \lim_{t\to\infty}(q_2(t) - q_1(t)) = -k_p\big(\lim_{t\to\infty} q_2(t) - q_{2d}\big) + g(q_{1d}). \quad (27)$$

Adding the relationship $K(q_{2d} - q_{1d}) = g(q_{1d})$ to both equations (26) and (27), the above equations can be written as

$$\bar{K}\begin{bmatrix} \lim_{t\to\infty} q_1(t) - q_{1d} \\ \lim_{t\to\infty} q_2(t) - q_{2d} \end{bmatrix} = \begin{bmatrix} g(q_{1d}) - \lim_{t\to\infty} g(q_1(t)) \\ 0 \end{bmatrix} \quad (28)$$

The left term of (28) has the relationship that

$$\left\| \bar{K}\begin{bmatrix} \lim_{t\to\infty} q_1(t) - q_{1d} \\ \lim_{t\to\infty} q_2(t) - q_{2d} \end{bmatrix} \right\| \quad (29)$$
$$\geq \lambda_{\min}(\bar{K})(\|\lim_{t\to\infty} q_1(t) - q_{1d}\| + \|\lim_{t\to\infty} q_2(t) - q_{2d}\|).$$

Based on Property 2.4, the right term of (28) becomes

$$\left\| \begin{bmatrix} g(q_{1d}) - \lim_{t\to\infty} g(q_1(t)) \\ 0 \end{bmatrix} \right\| \leq \beta \|\lim_{t\to\infty} q_1(t) - q_{1d}\|$$
$$\leq \beta(\|\lim_{t\to\infty} q_1(t) - q_{1d}\| + \|\lim_{t\to\infty} q_2(t) - q_{2d}\|). \quad (30)$$

If choosing the control gain k_p such that $\lambda_{\min}(\bar{K}) > \beta$, the unique solution of (29) and (30) is $\lim_{t\to\infty} q_1(t) = q_{1d}$ and $\lim_{t\to\infty} q_2(t) = q_{2d}$ [20]. Therefore, the flexible-joint robot achieves position regulation in the presence of input/output time delays. ■

IV. SIMULATION RESULTS

A two-link flexible-joint robotic manipulator is considered in this paper to validate the proposed control algorithms. The dynamics of the two-link robot with flexible and revolute joints are given as [19] with the consideration of gravitational torques. The physical parameters are given as $m_1 = 7.848kg$, $m_2 = 4.490kg$, $I_1 = 0.176kgm^2$, $I_2 = 0.041kgm^2$, $l_1 = 0.8m$, $l_2 = 0.6m$, $l_{1c} = 0.4m$, $l_{2c} = 0.3m$, $g = 9.8$, $J_m = \mathrm{diag}\{6, 1\}$, and $K = \mathrm{diag}\{1000, 1000\}$. According to Property 2.4 and the robotic model, we have $\|\frac{\partial g(q)}{\partial q}\| < 82 = \beta$. In the following simulations, the desired configuration for the joint is given as $q_{1d} = [\pi/2,\ \pi/3]^T$rad, which leads to $q_{2d} = [1.559,\ 1.036]^T$rad.

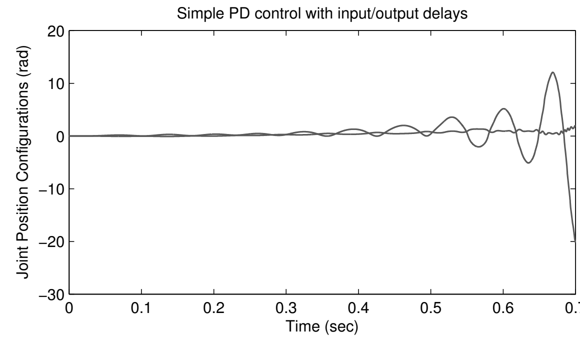


Fig. 4. Simple set-point control for flexible-joint robot becomes unstable with small input/output time delays.

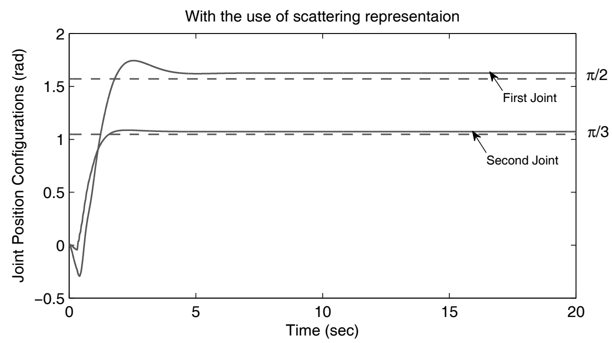
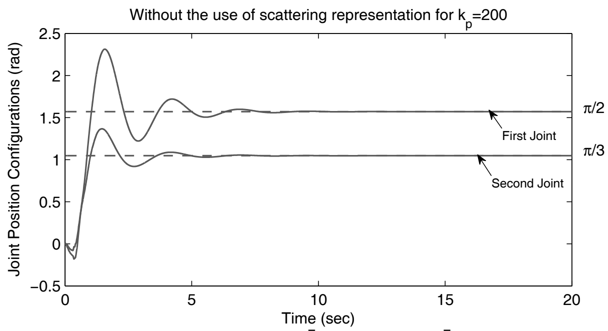


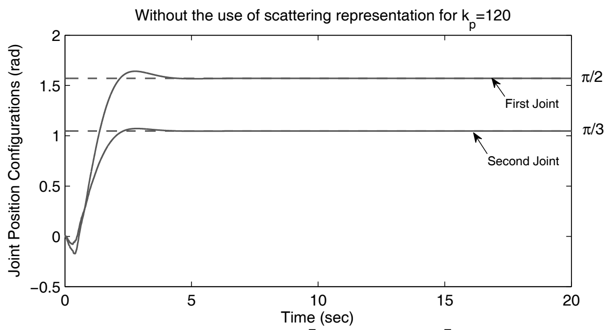
Fig. 5. The flexible-joint robot can be stabilized by employing scattering transformation but with degradation of position regulation.

In the first simulation, we demonstrate that the control system introduced in [20] becomes unstable if the robot is subjected to input/output communication delays. By selecting control gains $k_p = 200$ and $k_d = 100$ with relatively small time delays $T_1 = T_2 = 0.01$sec, the simulation result is illustrated in Fig. 4. It can be observed that the closed-loop control system for flexible-joint robot is unstable under input/output time delays. Subsequently, the control architecture shown in Fig. 2 is utilized to stabilize the control system. The control gains are given the same as in the previous case, and the wave impedance is chosen by $b = k_d = 100$. For larger input/outout delays $T_1 = 0.1$sec and $T_2 = 0.3$sec, the simulation results with the use of scattering transformation are shown in Fig. 5. Even though the control gains k_p satisfies $\lambda_{\min}(\bar{K}) = 95 > \beta = 82$, the flexible-joint robot can not be regulated to the desired configuration, which are the dashed lines in the figure.

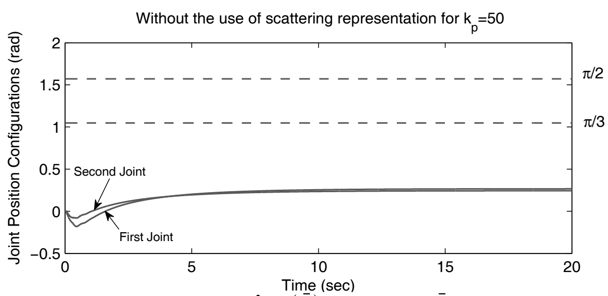
In order to ensure position regulation, the architecture without utilizing scattering transformation is validated subsequently. The innate dissipation b_m is assumed to be 100. Following the same control gains $k_p = 200$ and time delays, the simulation results is shown in Fig. 6 (a). The flexible-joint robot is stable as the control gains k_p and time delays $\bar{T}$ satisfy the condition $b_m = 100 > k_p\bar{T} = 200 \times 0.4 = 80$ that was addressed in Proposition 3.3. In addition, the control gain k_p satisfies the condition $\lambda_{\min}(\bar{K}) = 95 > \beta = 82$, hence the position regulation is guaranteed. If k_p decreases to 120, the robotic manipulator can still be regulated to the desired



(a) For $k_p = 200$, $\lambda_{min}(\bar{K}) = 95$ and $k_p\bar{T} = 80$.



(b) For $k_p = 120$, $\lambda_{min}(\bar{K}) = 58$ and $k_p\bar{T} = 48$.



(c) For $k_p = 50$, $\lambda_{min}(\bar{K}) = 24$ and $k_p\bar{T} = 20$.

Fig. 6. Simulation results of the proposed control architecture without utilizing scattering transformation.

position even if the control gain fails the sufficient condition $\lambda_{\min}(\bar{K}) > \beta$, as shown in Fig. 6 (b). However, if k_p is too small, the flexible-joint robot can not achieve position regulation as shown in Fig. 6 (c).

V. CONCLUSIONS

The problem of controlling flexible-joint robotic manipulators under input/output delays was presented in this paper. With the utilization of scattering transformation, we first demonstrated that the closed-loop control system is stable, but position drift occurs due to the joint flexibility. In order to improve regulation performance and reduce the complexity of the control system, another control architecture was developed without using the scattering transformation and with transmission of only the position signals to the controller. Provided that the control gain k_p is contingent to the innate dissipation in the robotic system $b_m > k_p\bar{T}$, the closed-loop control system is guaranteed to be stable. Additionally, if the condition $\lambda_{\min}(\bar{K}) > \beta$ is satisfied, the flexible-joint robot can be regulated to the desired configuration. The efficacy of the proposed control systems was demonstrated through numerical examples with a two-DOF flexible-joint robotic manipulator. Future work in this research topic includes not only time-varying communication delays but also trajectory tracking with non-collocated controller.

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