Link Position Control of a Compliant Actuator with Unknown Transmission Friction Torque

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Abstract—This paper proposes a control strategy for a compliant actuator, the CompAct™ actuator, which is equipped with semi active friction dampers in its transmission system. Both the transmission flexibility and the nonlinearity of the friction based damping torque makes the control of this actuator not a trivial task. This paper studies model of the presented actuator and the control problem of accurate link position tracking based on sliding mode approach that considers the friction torque as an uncertainty. Stability analysis and simulations highlight the effectiveness of the proposed controller in compensating for the deflections and unknown friction torque of the actuator. The performance of the controller is also validated by experiment results that demonstrate the tracking performance of the CompAct™ actuator achieved by the presented control strategy.

I. INTRODUCTION

The classical design template of industrial robots traditionally involves the use of highly geared stiff position based actuation units that grant these robots with high precision and repeatability. However, as robots are entering living and work environments, new demands are placed on the available robotic systems related to safety, efficiency and ability to interact with the environment and humans.

To address the issue and develop robots that can intrinsically demonstrate adequate motion tolerance to interaction constraints, actuators with embodied physical elasticity have been developed. Considering the electric motor technologies, an early development towards the realization of actuator units with inherent compliance is the Series Elastic Actuator (SEA) [1-3] which employs a fixed compliant element between a high impedance actuator and the load. To address the limitation of the fixed compliance within the SEA, actuation units with the ability to modulate compliance have also been developed [4-10].

Nevertheless, the introduced elasticity makes precise position/velocity control much more challenging. Compared with the performance of the traditional rigid robots, the precise tracking of the passively compliant robots is limited due to the reduced bandwidth of the actuation system and the effect of natural dynamics which become significant introducing oscillations in the robot motion. In addition, as collaborative robots are entering factory environments to be used for industrial applications [11] the coexistence of safety [8] and performance (accuracy, repeatability) becomes more important than in any other service or assistance application.

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Figure 1. Prototype of the CompAct™ actuator

To satisfy both requirements, semi active damping transmission systems have been recently developed. They either perform as clutches [12, 13], or combine with compliant transmission systems, are used to make the arm rigid on demand to achieve higher precision and controllability, permitting at the same time the exploitation of the benefits of compliance during interaction, [14, 15]. Such actuation concept was discussed and implemented in our previous work, [14], shown in Fig.1. In this system a set of four piezoelectric actuators are employed to actuate a semi active friction damper placed in parallel to the elastic transmission. In our previous works the amount of torque generated by these friction dampers was regulated to emulate viscous damping effects on demand [14, 16].

Although the dedicated design can improve the performance of the robot, to achieve high accuracy and repeatability appropriate control algorithms are also necessary. For the actuation unit in Fig. 1 both the joint elasticity and the introduced (nonlinear) friction torque necessitate more challenging control algorithms to achieve precise position tracking. The compliance of the system does not facilitate this as conventional PID controller at the link side (after the elasticity) cannot be used to precisely control link trajectories as the controlled plant will result unstable when link trajectories are fed back, [17]. A possible solution to this is the use of linear-quadratic regulator control, [18] however this method is based on a linear model and does not match well with the strong nonlinearity introduced by the friction generated by the transmission clutches of the systems explained above. In contrast to this, a sliding mode control-based strategy can permit the precise control of the link trajectory without incurring in instability issues and can easily take into account for nonlinear terms such as dry friction. Sliding mode approach is an effective control method [19] for robust control of nonlinear systems which include uncertainties. Furthermore, many studies evaluate the effectiveness of the sliding mode control (SMC) of flexible manipulators, [20] and [21] present the sliding mode
control of flexible arms for position control; [22] and [23] use the adaptive sliding approach for the flexible joint with mismatched uncertainties.

This paper presents a link position control scheme based on a sliding mode approach. The proposed controller is implemented on the CompAct™ actuator, Fig.1, that is equipped with a transmission system with series intrinsic elasticity in parallel to a friction based damper. The proposed control strategy deals with the unknown friction torque as a bounded uncertainty which is limited by the maximum normal clutch force. The contribution of this paper is the development of a first-order SMC to provide an accurate link position tracking of compliant actuation systems equipped with transmission clutch. By introducing a trade-off coefficient for measuring the tracking error due to the nonlinear coupling between the motor and link and designing an approximate variable for the nonlinear friction in the sense of Lyapunov, this control structure is implemented on the system.

The paper is organized as follows: in Section II the dynamic model of the compliant actuator with physical damping and the control problem statement are derived. In Section III the sliding mode control strategy is formulated including the stability analysis which outlines the performance of the method. Simulation and experiment results are analyzed in Section IV and Section V, finally in Section VI the conclusions are presented.

II. System Dynamics

A. Modeling of the Compliant Actuator

The system used for the evaluation of this control strategy is the prototype of CompAct™ Actuator presented in Fig.1. The detailed design of the above mechanism was introduced in [14]. The actuator can be modeled as an SEA provided with a clutch friction torque generated by piezoelectric actuators. A conceptual schematic is described as Fig. 2 (a). Fig. 2 (b) presents a detailed schematic of the clutch actuated by piezoelectric stacks which generate a friction torque \( \tau_c \) produced by the normal force \( F_n \).

![Figure 2. Schematics of the system: (a) the compliant variable physical damping actuator. (b) the piezoelectric actuator generating a friction torque for the damping effect and frictional surfaces on link side.](image)

Considering the system illustrated in Fig. 2, the dynamics can be written as

\[
\begin{align*}
M\ddot{q} + D_1\dot{q} + D_l(q - \dot{\theta}) + K_l(q - \theta) + \tau_c &= \tau_m, \\
B\ddot{\theta} + D_m\dot{\theta} - D_l(q - \dot{\theta}) - K_l(q - \theta) - \tau_c &= \tau_m,
\end{align*}
\]

where

\( q, \dot{q} \) : link position and link velocity

\( \theta, \dot{\theta} \) : motor position and motor velocity

\( M, B \) : link and motor inertia

\( K_l \) : transmission stiffness

\( D_1 \) : transmission damping

\( D_l, D_m \) : viscous damping of link and motor

\( \tau_m \) : torque generated by the motor

\( \tau_c, \tau_m \) : external torque and clutch friction torque

Note that the motor and link inertia are already scaled by the transmission ratio. Finally (1) explicitly takes into account the transmission as Coulomb friction torque \( \tau_c \). This friction torque is generated by the frictional forces between the two contact surfaces of the clutch. Figure 3 shows the details of the actuator prototype. Four piezoelectric stack actuators are connected in parallel to produce the normal force \( F_n \) acting directly on one of the two surfaces.

![Figure 3. (a) Frictional surfaces (b) Piezoelectric stack actuators](image)

Depending on the different relative motion between the two contact surfaces (motor and link), the clutch friction torque \( \tau_c \) is defined as [24]:

\[
\tau_c = \begin{cases} 
\mu_d R F_n \text{sign}(q - \dot{\theta}), & q \neq \dot{\theta} \\
\frac{MD_m - BD_l}{B + M} \ddot{q} - K_l(q - \theta) - \frac{M \tau_m - B \tau_c}{B + M}, & \ddot{q} = \dot{\theta}
\end{cases}
\]

where \( \mu_d \) is a dynamic friction coefficient and \( R \) is a constant parameter depending on the geometry of the contact surfaces.

B. Problem Statement

The compliance of the actuator may deteriorate static and dynamic performances of position/velocity control and this motivated the introduction of semi-active friction dampers in compliant actuators as in [14]. In theory, the generated clutch friction torque \( \tau_c \) can be calculated by solving (2). However, in practice, it is difficult to estimate the clutch friction torque because it depends on the states and the generated force by the piezoelectric actuator which has highly nonlinear dynamics [25]. Further, the dependency on the complex conditional model in (2) and the time dependency of the friction coefficient \( \mu_d \) which may vary due to wear make it more difficult to precisely estimate the clutch friction torque [16].

To avoid the formulation of complex friction models and exhibit robustness towards parametric time-dependency, the clutch friction torque is considered as a bounded uncertainty with \( \tau_c = 0 \). The system in (1) can be simplified as
\[
\begin{align*}
\ddot{\theta} &= B^{-1}[ -H_2(q_\dot{q}, \theta, \dot{\theta}) + \tau_m + f ], \\
\ddot{q} &= M^{-1} \left[ -H_1(q_\dot{q}, \theta, \dot{\theta}) - f \right],
\end{align*}
\]

where
\[
\begin{align*}
H_1(q_\dot{q}, \theta, \dot{\theta}) &= D_1 \ddot{q} + D_1(\dot{q} - \dot{\theta}) + K_1(q - \theta) \\
H_2(q_\dot{q}, \theta, \dot{\theta}) &= D_2 \ddot{\theta} - D_1(\dot{q} - \dot{\theta}) - K_1(q - \theta),
\end{align*}
\]

and \( f \) denotes a substitution of \( \tau_c \) assumed as an uncertainty with the boundary \( f_L \leq f \leq f_H \). As mentioned before, since we are considering semi-active friction dampers in this work, \( f_L = 0 \) and \( f_H \) is determined by \( \mu_s R F_{\text{MAX}} \), where \( F_{\text{MAX}} \) denotes the maximum normal force and \( \mu_s \) is the static friction coefficient. Two magnetic position sensors integrated within the actuation group are used to measure the motor position \( \theta \) after the gear reduction and the flexible deflection \( \dot{\theta}_s = q - \theta \) between motor and link. The derivatives of \( q, \theta \) are calculated by means of numerical differentiation.

The target is to design a control law to generate a control input \( \tau_m \) to the system in (1) for the link position \( q \) to accurately track the desired reference trajectory \( q_d \) with the unknown \( \tau_c \), substituted by \( f \).

III. CONTROL STRATEGY

This section proposes a control law based on the sliding mode approach. A simple first-order SMC is developed to be implemented into the deflection transmission which is considered in the compliant actuator shown in Fig. 2.

Based on (3), the state-space equation is described by
\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= M^{-1} \left[ -f - H_1(x) \right], \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= B^{-1} \left[ u + f - H_2(x) \right],
\end{align*}
\]

where \( \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} = \begin{bmatrix} q & \dot{q} & \theta & \dot{\theta} \end{bmatrix} \) are the state variables while the control input is \( u = \tau_m \), whereas \( \begin{bmatrix} x_1 & x_2 \end{bmatrix} = \begin{bmatrix} q & \dot{q} \end{bmatrix} \) is the output. Let \( x_d \) represent the desired link position. The position tracking error is \( x_d - x_1 \) and its derivative \( \dot{x}_d - x_2 \).

The overall control scheme is summarized in Fig.4.

In our target system, the link position is obtained by \( q = \theta_s + \theta \) and there exists a coupling between the motor position and the link position. Thus, a new definition of error, \( e \), as indicated in Fig.4, is defined in order to simplify the controller from a second to a first order strategy. This allows not only to provide an accurate tracking but also to ensure the system’s internal stability [26].

\[
e = x_d - (\dot{\theta}_s + x_3) = x_d - (\zeta x_1 + (1 - \zeta) x_3),
\]

where the \( \dot{\theta}_s \) is the predicted deflection with a trade-off coefficient \( 0 < \zeta < 1 \). More discussion on selecting \( \zeta \) is provided in Section IV.C.

The sliding manifold [19] is defined as
\[
\sigma = C e + \dot{e},
\]

where \( C \) is a strictly positive constant. Then, the problem of tracking the desired reference \( x_d \) is replaced by the first-order stabilization problem in terms of \( \sigma \). Because the sliding manifold is continuous and differentiable, its first derivative yields
\[
\dot{\sigma} = C \dot{e} + \ddot{e}
\]

\[
= C \left[ \dot{x}_d - (\zeta x_2 + (1 - \zeta) x_4) \right] + \ddot{x}_d - (\zeta \dot{x}_2 + (1 - \zeta) \dot{x}_4).
\]

The reaching law of \( \sigma \) is selected as, [27]:
\[
\dot{\sigma} = -\varepsilon \text{sign}(\sigma) - \lambda \sigma,
\]

where both \( \varepsilon \) and \( \lambda \) are preset positive constants. By substituting (5) into (8) and combining them with (9), the control law can be designed as follows:
\[
u = P - f \left( 1 - \frac{\zeta BM^{-1}}{1 - \zeta} \right),
\]

where
\[
P = B \frac{1}{1 - \zeta} \left[ C \left[ \dot{x}_d - (\zeta x_2 + (1 - \zeta) x_4) \right] + \ddot{x}_d + \varepsilon \text{sign}(\sigma) + \lambda \sigma \right] + H_2(x) \frac{\zeta BM^{-1}}{1 - \zeta} H_1(x).
\]

However, with a practical consideration due to the uncertainty of \( f \), it is required to introduce an approximated variable \( \hat{f} \) not only to simply realize the control law but also to guarantee the stability of the closed-loop system. The approximated control law is as follows:
\[
\hat{u} = P - \hat{f} \left( 1 - \frac{\zeta BM^{-1}}{1 - \zeta} \right).
\]

In order to find a suitable \( \hat{f} \), a closed-loop stability is analyzed in the sense of Lyapunov. When a Lyapunov function is taken as \( V = 0.5 \sigma^2 \), the derivative of \( V \) is given.
Applying the control input (11) to the plant (5), the following closed-loop dynamics can be obtained from (8):

$$\dot{\sigma} = -\epsilon \sigma \left( 1 - \zeta \right) - \lambda \sigma - Q \left( f - \hat{f} \right),$$

(12)

where $Q = \left( 1 - \zeta \right) B^{-1} - \zeta M^{-1}$. From (12), the time derivative of the Lyapunov function can be expressed as

$$\dot{V} = \sigma \left[ -\epsilon \sigma \left( 1 - \zeta \right) - \lambda \sigma - Q \left( f - \hat{f} \right) \right] = -\epsilon \left| \sigma \right| - \lambda \sigma^2 - \sigma \left( f^* - \hat{f} \right),$$

(13)

where $f^* = Qf$ and $\hat{f} = Q\hat{f}$. Here, we design $\hat{f}$ as

$$\hat{f} = \frac{f_H^* + f_L^*}{2} - \frac{f_H^* - f_L^*}{2} \text{sign} \left( \sigma \right),$$

(14)

where $f_L^* = Qf_L$ and $f_H^* = Qf_H$. Then, equation (13) can be rearranged as

$$\dot{V} = \begin{cases} -\epsilon \left| \sigma \right| - \lambda \sigma^2 - \sigma \left( f^* - f_L^* \right) & \text{if } \sigma \geq 0, \\ -\epsilon \left| \sigma \right| - \lambda \sigma^2 - \sigma \left( f^* - f_H^* \right) & \text{if } \sigma < 0. \end{cases}$$

(15)

From (15) and $f_L^* < f^* < f_H^*$, one can notice that the time derivative of $V$ is always negative. Therefore the closed-loop stability is guaranteed.

Finally, with the combination of (11) and (14), the control law is expressed by

$$\hat{u} = P - Q^{-1} \hat{f} \left( 1 - \frac{\zeta BM^{-1}}{1 - \zeta} \right).$$

(16)

IV. SIMULATION ANALYSIS

In this section simulations of the proposed control strategy are carried out on a single compliant actuator to validate its effectiveness. The parameters of the dynamic model in (1) used in the simulation are given in Table I. These are the values of the real actuator prototype obtained by parameter identification [14, 16].

<table>
<thead>
<tr>
<th>TABLE I. PARAMETERS FOR THE SYSTEM</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Moment of inertia of the rotor - B</td>
</tr>
<tr>
<td>Moment of inertia of the link - M</td>
</tr>
<tr>
<td>Viscous damping of the compliant joint - $D_t$</td>
</tr>
<tr>
<td>Viscous damping at the motor - $D_m$</td>
</tr>
<tr>
<td>Viscous damping at the link - $D_l$</td>
</tr>
<tr>
<td>Stiffness of the joint - $K_t$</td>
</tr>
<tr>
<td>Static friction coefficient - $\mu_s$</td>
</tr>
<tr>
<td>Constant factor - $R$</td>
</tr>
<tr>
<td>Maximum clutch force - $F_{nMAX}$</td>
</tr>
</tbody>
</table>

In order to evaluate the tracking performance of this controller, both sinusoidal and smoothstep [28] were selected as desired link position references.

The peak value of the torque control input in (8) is limited to 37Nm, while the deflection angle is constrained to 0.18rad. The coefficients in the controller could be selected in an appropriate range according to the SMC theory. In this paper, $C = 150$ and for reaching law, $\epsilon = 0.5, \lambda = 30$. The coefficient of tradeoff error is selected as $\zeta = 0.5$ and in the last part the analysis of the effects of $\zeta$ is studied.

A. Tracking of a sinusoidal input position signal

The semi-active friction damper used in the transmission of the analyzed actuator can introduce an amount of dry friction on demand based on the applied normal force $F_n$. Figures 5, 6 show the performance of the position tracking for sinusoidal reference of different value of normal force (variable friction torque). Initially a simulation run on the system with no additional transmission friction is analysed without friction torque to check the performance for the system working as SEA. Subsequently, the same reference was selected with $F_n = 100N$.

![Figure 5. Tracking performance and error with a 0.2Hz sinusoidal position profile with amplitude 0.5rad and $F_n=100N$](image)

The results presented in Fig. 5 show the tracking performance considering reference trajectory given by a sinusoidal position profile $x_d = 0.5 \sin(0.2 \cdot 2\pi t)$ with $F_n = 0$ and $x^T(0) = [0 \ 0 \ 0 \ 0]$. It can be noticed that after a transient lasting about 1s the oscillations of the link caused by the discontinuous transition of the velocity state from zero to about 1.5rad/s settle down. The tracking errors of position and velocity are shown in the lower part of Fig. 5, where it can also be noticed that the maximum position tracking error is 0.005rad while after the transient settles down the error becomes around 0.001rad.

The same sinusoidal position profile with $F_n = 100N$ is used to analyze how the controller works with unknown friction torque in Fig. 6. Compared with Fig. 5 the results shown in Fig. 6 exhibit smaller oscillation and bigger tracking error (the peak position tracking error is around 0.006 rad);
This is mainly due to the friction introduced by the transmission clutch. It can be seen that the proposed SMC shows high fidelity and robustness when nonlinearities due to the friction torque are introduced into the system.

B. Tracking Smoothstep position signal

To compare the performance of the proposed SMC while tracking different types of signals, a smoothstep of amplitude 0.5rad was employed as reference position profile which can avoid the generation of large control signals resulting in jerky link motions. The performances with different $F_n = 0N, F_n = 100N$ are shown in Fig.7 and Fig.8.

The curves in Fig.7 show the tracking performance of a smoothstep position for 5s with amplitude 1rad without normal clutch force. A small oscillation occurred because of the system’s compliance and no significant overshoot can be noticed.

Finally, the same simulation was repeated applying a normal force of $F_n = 100N$ to the clutch shown in Fig.8. It is evident that the proposed control provides good link position control performance and well stable behavior even when sharp position variations are required and when varying unknown levels of friction torque.

C. Error Analysis

The last presented simulation is to characterize the influence of the trade-off coefficient $\zeta$ on the global tracking error. The definition of global error here is based on the computation of Euclidean norm of the position tracking error vector $\|q_d - q\|_2$ which is the most commonly used method to evaluate the intuitive notion of length of the vector. The same sinusoidal reference as in Fig.5 was considered in this study. Fig.9 shows the surface of global error with respect to $\zeta \in [0.01, 0.99]$ and $F_n \in [0, 300] N$.

It can be seen that for small $F_n$ levels the choice of $\zeta$ does not relevantly affect the global tracking error, except around 0.5 there is a lowest area for error value. However, when the force $F_n$ is higher than approximately 100N, values of $\zeta$ ranging from 0.01 to 0.5 should be employed to obtain lower errors. This means that selecting 0.5 as value for the 'trade-off' coefficient reveals to be an appropriate choice for most situations.
V. EXPERIMENT RESULTS

Experiments were carried out on the CompAct™ actuator [10], shown in Fig. 1, to validate the results obtained in the simulation. The link position references used in simulations performed in section IV, are also exploited for these experiments. The configuration setup including the parameters of the system and the control gains in (16) are in accordance with those shown in Tab. I and reported in Section IV.

The results obtained for tracking a sinusoidal reference which was analyzed in Section IV. A, are illustrated in Fig. 10 and 11. These figures present time history of actual position in comparison to the reference position and motor torque for $F_n = 0N$ and $F_n = 100N$, respectively.

![Figure 10. Tracking performance and motor torque with a 0.2Hz sinusoidal position profile with amplitude 0.5rad and $F_n=0N$](image1)

![Figure 11. Tracking performance and motor torque with a 0.2Hz sinusoidal position profile with amplitude 0.5rad and $F_n=100N$](image2)

It can be seen that, according to the robustness of the proposed SMC controller, the tracking performance of the system does not depend on the unknown friction torque. However, the influence of applying normal clutch force on reducing the oscillations of the control input can be noticed.

Similarly, additional experiments were carried out to evaluate the results of the controller for tracking the different reference position. The smoothstep of amplitude 1.0rad which was studied in Section IV. B is used as the desired link position here. Fig. 12 and 13 show change in actual position in comparison with the reference position and motor torque versus time for $F_n = 0N$ and $F_n = 100N$, respectively.

![Figure 12. Tracking performance and motor torque with a smoothstep reference for 5s with amplitude 1 rad and $F_n=0N$](image3)

![Figure 13. Tracking performance and motor torque with a smoothstep reference for 5s with amplitude 1 rad and $F_n=100N$](image4)

Throughout the experiments, the proposed control shows accurate link position tracking inspite of the low joint stiffness (103 Nm/rad) and its robustness against the variation of the clutch friction torques. Therefore, proper tracking accuracy of the system, regardless of applied friction torque, is achieved by employing the proposed control approach.

VI. CONCLUSION

In this paper a sliding mode controller capable of accurately regulating the link position of a compliant actuator with unknown friction torque in its transmission was introduced. The key feature of the proposed control strategy is related to its ability of controlling compliant actuators equipped with transmission clutches, using a friction
model-free approach. This is the first contribution of this work as friction is typically hard to model and its parametric time-dependency makes estimation difficult. Therefore, the transmission clutch friction is regarded as an uncertain disturbance, which allows considering only linear terms in the model employed for the formulation of the control strategy. Another attractive aspect of the proposed method is its simple structure, i.e. that of the first-order sliding mode. Simulation results showed robust performance with respect to unknown variable friction torque level for different types of reference signal. The implementation of the trade-off coefficient made the simplification of the computation order possible and the unknown friction torque was efficiently compensated by selecting its approximation under the sufficient condition of stability. The results obtained in the numerical simulations and through experiments confirmed the tracking accuracy of the proposed control.

Future work will focus on implementing the proposed control into a 4-DOF CompAct™ arm [29].

REFERENCES


