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Control of nonholonomic wheeled mobile robots via i-PID controller

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***Abstract*— An intelligent PID controller (i-PID controller) is applied to control the nonholonomic mobile robot with measurement disturbance. Because of the particularity of the nonholonomic systems, this paper propose to use a switching parameter α in the i-PID controller. We show in simulations that the proposed method is able to control the nonholonomic mobile robots with measurement disturbance, and it can also stabilize the robot at a static point.**

I. INTRODUCTION

The problem of wheeled mobile robot control has been widely studied and attracted the interest of many researches because of its wide application in industries and theoretical challenges [1], [2]. Generally the robot control problem can be divided into two main problems: the trajectory tracking problem and stabilization problem. The control problem of trajectory tracking can also be categorized into two types: linear control and nonlinear control. [3] proposed a linear controller which is robust to the perturbation in robot velocity control. Separated feedback loops control for robot position and velocity was used in [4]. The kinematic model of the robot was linearized in [5], and in which a proportional linear control was applied. The famous PID controller was applied in [6], in which a simple linearized mobile robot model is used.

The linear control indeed has great advantages because of its simplicity in linear control theory, while however when comparing with nonlinear control its robustness are very limited. In linear control the initial states are often required to stay close to the reference to ensure the stability, instead nonlinear control is able to guarantee the stability without this kind of problems. Moveover, it is known that the feedback stabilization at a given posture cannot be obtained by smooth time-invariant control [7], this implies that the problem is truly nonlinear, and linear control is ineffective here. For nonlinear nonholonomic robot systems, there are usually open loop controls where the inputs are calculated from the reference trajectory [8], flatness based control [9] is a kind of open-loop control, whose robustness can be strengthened [10], which is widely applied in optimal control problems. However, it is well known that the open-loop control is not robust to modeling errors so that it cannot guarantee the mobile robot to move along the desired trajectory. Nonlinear feedback control for mobile robots is used in [11] to solve the trajectory tracking problem, and the dynamic feedback linearization is also used in [12]. [13] proposed a nonlinear control law based on partial state feedback linearization and Lyapunov's direct method, but the disturbance and uncertainty were not considered in the control design.

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Recently, an intelligent PID controller (i-PID controller) introduced in [14] exhibits the robustness to the unmodeled dynamics and disturbance in the system [15], and it has been widely studied and applied to many electrical and mechanical processes [16], [17]. This paper aims at applying the so-called i-PID controller to the nonholonomic robots in order to control the robot with measurement disturbance. However, due to the particularity of the nonholonomic system, this controller can not be simply applied, for this a switching parameter is selected and a robust controller is proposed to control the robot with measurement disturbance.

The paper is structured as follows. Section II presents the problem statement. Section III explains the determination of the controller. Simulation results are detailed in Section IV.

II. PROBLEM STATEMENT

This paper considers the unicycle-type mobile robot whose kinematic model under the nonholonomic constraint of pure rolling and no slipping can be described as follows:

$$\begin{cases} \dot{x} &= \nu\cos\theta \\ \dot{y} &= \nu\sin\theta \\ \dot{\theta} &= \omega \end{cases} \tag{1}$$

where ν and ω are linear and angular velocity respectively, x and y represent the location of the robot, θ is the orientation of the robot with respect to x-axis(see Fig. 1).

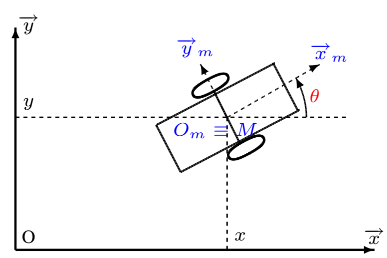

Fig. 1. Unicycle-type mobile robot

It can be shown that x and y are flat outputs for the studied system [9]. Indeed, θ, υ and ω can all be expressed by x, y and their first and second-order derivatives as follows:

$$\begin{cases} \theta &= \arctan\frac{\dot{y}}{\dot{x}} \\ \upsilon &= \sqrt{\dot{x}^2+\dot{y}^2} \\ \omega &= \frac{\ddot{y}\dot{x}-\ddot{x}\dot{y}}{\dot{x}^2+\dot{y}^2} \end{cases} \tag{2}$$

Suppose that we can only measure the position (x, y) of the robot, which implies that the relative degree of those measurements is equal to 1, since the first part of system (1) is of the following form:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

It is clearly that the second control input ω is not involved in the above equation, since the decoupling matrix is singular. Due to this fact, one classic solution is to add an integrator to the first input in order to overcome the singularity of the decoupling matrix [18]. For this, let us consider the following extended system:

$$\begin{cases} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \omega \\ \dot{v} &= \xi \end{cases}$$

with $u = [\xi, \omega]$ being the new input. One can check that, with the extended system, the relative degree for both output is equal to 2. Then one obtains:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = G(x, y, \dot{x}, \dot{y})u \tag{3}$$

where

$$\begin{aligned} G &= \begin{bmatrix} \cos\theta & -\nu\sin\theta \\ \sin\theta & \nu\cos\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos(\arctan\frac{\dot{y}}{\dot{x}}) & -\sqrt{\dot{x}^2+\dot{y}^2}\sin(\arctan\frac{\dot{y}}{\dot{x}}) \\ \sin(\arctan\frac{\dot{y}}{\dot{x}}) & \sqrt{\dot{x}^2+\dot{y}^2}\cos(\arctan\frac{\dot{y}}{\dot{x}}) \end{bmatrix} \end{aligned} \tag{4}$$

which is invertible if $v = \sqrt{\dot{x}^2+\dot{y}^2} \neq 0$.

If there is no disturbance in the measurement, a classic PID controller, which needs the exact value of G^{-1}, can be used to achieve non-vanishing Cartesian trajectories tracking (the linear velocity of the robot is assumed to be always non-zero), since G is singular when $v = 0$. It has been shown ([19]) that this method cannot be used to stabilize the robot to a static point due to the same reason.

In addition, noises and disturbance are inevitable in real situations, thus the exact computation of G^{-1} in (4) cannot be obtained. Consider the output under disturbance as $Y = [x, y]^T + D$, where $D = [d_1, d_2]^T$ is the disturbance in the measurement, Thus the estimated values of θ, v and ω are disturbed:

$$\begin{cases} \theta_d &= \arctan\frac{\dot{y}+\dot{d}_2}{\dot{x}+\dot{d}_1} \\ v_d &= \sqrt{(\dot{x}+\dot{d}_1)^2+(\dot{y}+\dot{d}_2)^2} \end{cases} \tag{5}$$

thus we have

$$\ddot{Y} = G(Y, \dot{Y})u + \ddot{D} \tag{6}$$

with

$$G(Y, \dot{Y}) = \begin{bmatrix} \cos(\theta_d) & -v_d\sin(\theta_d) \\ \sin(\theta_d) & v_d\cos(\theta_d) \end{bmatrix} \tag{7}$$

where θ_d and v_d are defined in (5).

It is clear that the system (6) can not be controlled with the classical PID controller, since $G(Y, \dot{Y})$ defined in (7) can not be accurately estimated due to the unknown disturbance. Moreover, $G(Y, \dot{Y})$ becomes singular when $v_d = 0$. In order to overcome the two drawbacks when applying the simple PID controller, this paper uses the recently proposed i-PID controller to control the robot with measurement disturbance.

III. DETERMINATION OF THE CONTROLLER

Since the controller proposed in this paper is based on the i-PID controller, let us firstly present the basic idea of this controller, and then detail how to apply this controller into the control of the unicycle model with measurement disturbance.

A. i-PID controller

Generally speaking, the method of i-PID controller locally approximates the system model by a simple local model with unknown term, and the unknown term can be estimated by the measurements of the input and output of the system, then a so-called i-PID controller can be deduced to realize the control goal.

In this paper the system model (6) is approximated by the following local model over a small time interval $T = [t_k, t_{k+1}]$ with $k \in Z^+$:

$$\ddot{Y}(t) = F(t) + \alpha(Y, \dot{Y})u(t) \tag{8}$$

where u and Y are known input and output signals with disturbance, $\alpha(Y, \dot{Y})$ is a non singular 2×2 dimensional matrix which should be well chosen in order to achieve the control goal. $F \in \mathbb{R}^2$ represents all unknown terms including the disturbances, which can be estimated by using the information of Y, u and α.

For the above locally approximated continuous model over time interval T, one can estimate F by discretizing it. Precisely, denote T_s the sampling period, so at each sampling time $k = t/T_s$, one has

$$\ddot{Y}_k = F_k + \alpha(Y, \dot{Y})u_k$$

then it yields $F_k = \ddot{Y}_k - \alpha(Y, \dot{Y})u_k$, where Y_k and u_k are measurable signals at time k, and $\ddot{Y}_k$ is the 2^{nd} order differentiation of the output Y at sampling time k. If it is assumed that T_s is small enough such that $F_{k-1} \to F_k$, then the so-called i-PID controller can be designed as follows:

$$u_k = \alpha^{-1}(Y, \dot{Y})(-F_{k-1} + e_k) \tag{9}$$

where $e_k = \ddot{Y}_{ref,k} - K_2(\dot{Y}_k - \dot{Y}_{ref,k}) - K_1(Y_k - Y_{ref,k})$ with Y_{ref} being the references of the output to be tracked, and K_1 and K_2 being the freely chosen coefficients such that the polynomial $s^2 + K_2 s + K_1$ is Hurwitz.

As one can see in the controller (9), there are two parameters to be determined, $\alpha(Y, \dot{Y})$ and F_k, which will be discussed in the following.

B. Discussion on $\alpha(Y, \dot{Y})$

The determination of $\alpha(Y, \dot{Y})$ is the most important issue when applying such a controller. A good parameter $\alpha(Y, \dot{Y})$ should well approximate $G(Y, \dot{Y})$ defined in (7), and be always invertible, and change as fewer times as possible as time goes on, and it is best that $\alpha(Y, \dot{Y})$ is time-invariant.

In [20] and [21], similar controllers are presented, which use an unknown term to represent unknown parameters and disturbance in the system. However, in [20] the similar $G(Y,\dot{Y})$ in the system is a time-invariant scalar. In [21] the determination of $\alpha(Y,\dot{Y})$ is discussed, but the similar parameter $G(Y,\dot{Y})$ in the system is assumed to be always invertible and time-invariant. Thus in their controller, $\alpha(Y,\dot{Y})$ can be set as a fixed number or a fixed invertible matrix.

We aim to find out an invertible time-invariant α to well approximate $G(Y,\dot{Y})$ in the controller. However, let us take a look at $G(Y,\dot{Y})$ in our system (6). Firstly it is a matrix whose entries vary as time goes on, and the sign of all entries in $G(Y,\dot{Y})$ is changing, which makes it impossible to use a time-invariant α to approximate $G(Y,\dot{Y})$.

In order to approximate $G(Y,\dot{Y})$ with $\alpha(Y,\dot{Y})$, $\alpha(Y,\dot{Y})$ needs to vary with $G(Y,\dot{Y})$. One can of course set that

$$\alpha(Y,\dot{Y}) = \begin{bmatrix} \cos\hat{\theta} & -\hat{v}\sin\hat{\theta} \\ \sin\hat{\theta} & \hat{v}\cos\hat{\theta} \end{bmatrix}$$

where $\hat{\theta} = \arctan\frac{\hat{\dot{y}}}{\hat{\dot{x}}}$ is the estimation of θ with noises, $\hat{\dot{x}}$ and $\hat{\dot{y}}$ are the estimation of $\dot{x}$ and $\dot{y}$ with noises, and $\hat{v}$ is the estimation of v defined in (2). In this way, this method is in fact equivalent to the controller linked to exact linearization by feedback with the estimate of θ and v. However, one can notice that $\alpha(Y,\dot{Y})$ will be singular when $\hat{v} = 0$. In order to make $\alpha(Y,\dot{Y})$ being invertible and well approximate $G(Y,\dot{Y})$, another intuitive choice is to remove $\hat{v}$ in the above matrix and one obtains:

$$\alpha(Y,\dot{Y}) = \begin{bmatrix} \cos\hat{\theta} & -\sin\hat{\theta} \\ \sin\hat{\theta} & \cos\hat{\theta} \end{bmatrix}$$

The above selected $\alpha(Y,\dot{Y})$ is suitable for the controller, since it is always invertible and it can be well approximate $G(Y,\dot{Y})$. However, since this choice of $\hat{\theta}$ is always time-varying when robot moves, which will increase the computation of the controller. In order to make the selected $\alpha(Y,\dot{Y})$ changing as fewer times as possible when robot moves, this paper proposes to choose it as follows:

$$\alpha(Y,\dot{Y}) = \begin{bmatrix} sig(\cos\hat{\theta}) & -sig(\sin\hat{\theta}) \\ sig(\sin\hat{\theta}) & sig(\cos\hat{\theta}) \end{bmatrix}$$

where $sig(\sigma)$ is the sign function which extracts the sign of the real number σ, and it is assumed that $sig(0) = 1$. For this proposed $\alpha(Y,\dot{Y})$, let us define the following switching signal $i(\hat{\theta}) : \mathbb{R} \to \mathcal{I}$ with $\mathcal{I} = \{1,2,3,4\}$:

$$i(\hat{\theta}) = \begin{cases} 1 & if\ \hat{\theta} \in (2k\pi, 2k\pi + \frac{\pi}{2}) \\ 2 & if\ \hat{\theta} \in (2k\pi + \frac{\pi}{2}, 2k\pi + \pi) \\ 3 & if\ \hat{\theta} \in (2k\pi + \pi, 2k\pi + \frac{3\pi}{2}) \\ 4 & if\ \hat{\theta} \in (2k\pi + \frac{3\pi}{2}, 2(k+1)\pi) \end{cases} \tag{10}$$

where $k \in Z$. The corresponding constant matrices can then be defined as follows:

$$\alpha_1 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \quad \alpha_2 = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$$
$$\alpha_3 = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \quad \alpha_4 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Therefore, the proposed matrix $\alpha_{i(\hat{\theta})}$ satisfies:

$$\alpha(Y,\dot{Y}) = \alpha_{i(\hat{\theta})}, \forall\hat{\theta} \in R$$

Summarily, the selected $\alpha_{i(\hat{\theta})}$ has several advantages. Firstly it has only 4 values when $\hat{\theta}$ changes in $[2k\pi, 2(k+1)\pi]$, which is able to make $\alpha(Y,\dot{Y})$ changed as fewer times as possible. Secondly it is always invertible, so that the controller can stabilize the robot at a static point with the robot velocity equals to zero.

C. Numerical differentiation

It can be seen that the choice of $\alpha(Y,\dot{Y})$ involves the estimation values $\hat{\theta}$, $\hat{\dot{x}}$ and $\hat{\dot{y}}$, then the efficient estimation of these values becomes significant. This paper uses the algebraic technique proposed by Fliess et al in [22] for the estimation. Mathematical foundation of this approach can be referred to [23], [24] and the references therein.

Generally speaking, this algebraic approach has several advantages: it provides explicit formulae, which can be directly implemented; it is of non-asymptotic nature, the desired estimation can be obtained instantaneously, which is a significant advantage for real-time applications; it does not require any assumption concerning the statistical distribution of the unstructured noise.

Consider a signal $h(t) = \sum_{k=0}^{\infty} h^{(k)}(0)\frac{t^k}{k!}$ which is assumed to be analytic around $t = 0$ and its truncated Taylor expansion $h_N(t) = \sum_{k=0}^{N} h^{(k)}(0)\frac{t^k}{k!}$, where $t > 0$. Its Laplace transform is of the form:

$$H_N(s) = \sum_{k=0}^{N} \frac{h^{(k)}(0)}{s^{k+1}} \tag{11}$$

Introducing the *algebraic derivation* $\frac{d}{ds}$, and multiply both sides of equation (11) by $\frac{d^\rho}{ds^\rho}s^N$, $\rho = 0,1,...,N$, one has a triangular system of linear equations and from which the derivatives can be obtained:

$$\frac{d^\rho s^N H_N}{ds^\rho} = \frac{d^\rho}{ds^\rho}\left(\sum_{k=0}^{N} h^{(k)}(0)s^{N-k-1}\right) \tag{12}$$

which is independent of all the unknown initial conditions, and the coefficients $h(0),...,h^{(k)}(0)$ are *linearly identifiable* [25], then the $h^{(k)}(0)$ can be obtained by taking the inverse laplace transform of (12) over a time window T.

In practice, the above algebraic technique is implemented with discrete measured data, thus it is necessary that the sampling time T_s should be small enough [24],[26].

It is worthy noting that this algebraic technique is robust with respect to noise involved into the controls and outputs, since noises are viewed here as quick fluctuations around 0. They are therefore attenuated by low-pass filters, like iterated integrals with respect to time.

D. Algebraic estimation of F

Now there is only one parameter F left in the i-PID controller to be determined. The calculation of F also uses the algebraic technique described above.

Let us consider the local approximated model:

$$\ddot{Y} = F + \alpha u \tag{13}$$

where F can be considered as constant between two sampling time. Then by taking the Laplace transformation of both sides of equation (13), one obtains

$$s^2\mathcal{Y}(s) - s\mathcal{Y}(0) - \mathcal{Y}'(0) = \frac{F}{s} + \alpha U(s) \tag{14}$$

In order to eliminate the unknown terms $\mathcal{Y}(0)$ and $\mathcal{Y}'(0)$ which are linked to unknown initial conditions, we take the 2^{nd} order derivative of both sides with respect to s:

$$s^2\mathcal{Y}''(s) + 4s\mathcal{Y}'(s) + 2\mathcal{Y}(s) = \frac{2F}{s^3} + \alpha U''(s) \tag{15}$$

By dividing both sides of equation (15) with s^3, one has:

$$\frac{\mathcal{Y}''(s)}{s} + \frac{4\mathcal{Y}(s)'}{s^2} + \frac{2\mathcal{Y}(s)}{s^3} = \frac{2F}{s^6} + \frac{\alpha U''(s)}{s^3} \tag{16}$$

Take the inverse Laplace transformation of both sides of equation (16), one obtains:

$$\begin{aligned}\int_0^T (-\tau)^2 Y d\tau + \int_0^T 4(T-\tau)(-\tau) Y d\tau + \int_0^T (T-\tau)^2 Y d\tau \\ = \frac{2FT^5}{5!} + \alpha \int_0^T \frac{(T-\tau)^2}{2!}(-\tau)^2 u d\tau\end{aligned} \tag{17}$$

where $[0, T]$ is a short time window, and the window is sliding in order to get the estimate at each time instant. Let $\tau = \delta T \in [0, T]$, where $\delta \in [0, 1]$, after simplification equation (17) becomes:

$$T^3 \int_0^1 (6\delta^2 - 6\delta + 1) Y d\delta = \frac{FT^5}{60} + \frac{T^5\alpha}{2} \int_0^1 (1-\delta)^2 \delta^2 u d\delta \tag{18}$$

Hence, at sampling step k, the numerical estimate value of F_k can be expressed as:

$$F_k = \frac{60}{T^2} \int_0^1 (6\delta^2 - 6\delta + 1) Y d\delta - 30\alpha \int_0^1 (1-\delta)^2 \delta^2 u d\delta \tag{19}$$

IV. Simulation Results

In the simulation, the parameters are set: $K_2 = 20$, $K_1 = 100$, time window $T = 3s$, sampling time $T_s = 0.01$. The reference trajectory is set as:

$$\left\{ \begin{array}{lcl} x_r & = & \sin 2t \\ y_r & = & \sin \frac{t}{2} \end{array} \right.$$

Fig. 4 to Fig. 8 show the simulation result of the designed control applied on the nonholonomic wheeled mobile robot with white Gaussian noise of $SNR = 30dB$ (signal-to-noise ratio) in the measurement (noises are shown in Fig. 2 and Fig. 3). Fig. 4 and Fig. 5 show the tracking result, $i(\hat{\theta})$ is shown in Fig. 6, and the control inputs are shown in Fig. 7 and Fig. 8. As we can see that the robot is able to track the trajectory with measurement noises, and the controller designed is effective and robust to the noises.

Fig. 9 to Fig. 13 illustrate the simulation of stabilize the robot at point $(4, 1)$, a white Gaussian noise of $SNR = 30dB$ is added to the measurement as well. Tracking result is shown in Fig. 9 and Fig. 10, control inputs are shown in Fig. 12 and Fig. 13. As we can see that the controller is able

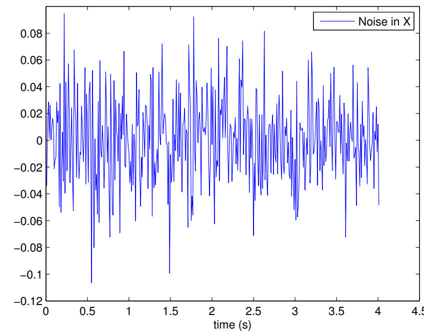

Fig. 2. Noise imposed in X $SNR = 30dB$

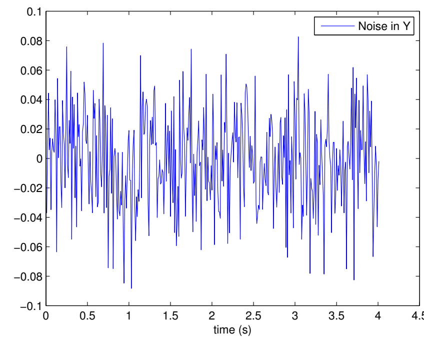

Fig. 3. Noise imposed in Y $SNR = 30dB$

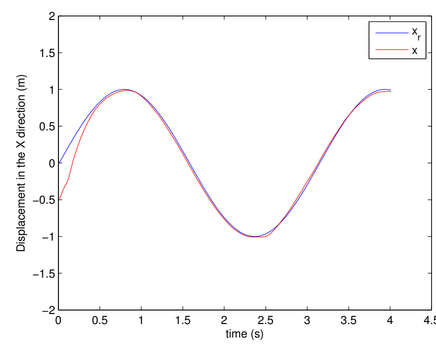

Fig. 4. Tracking of position X with white Gaussian noise $SNR = 30dB$

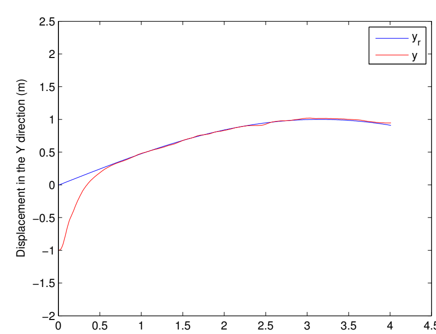

Fig. 5. Tracking of position Y with white Gaussian noise $SNR = 30dB$

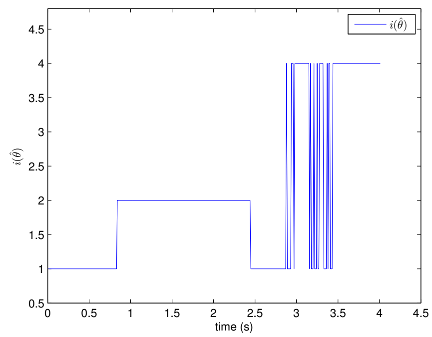

Fig. 6. $i(\hat{\theta})$ with white Gaussian noise $SNR = 30dB$

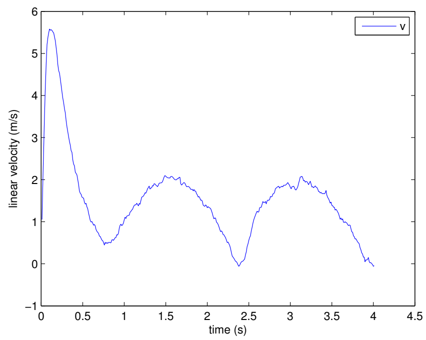

Fig. 7. Linear velocity control with white Gaussian noise $SNR = 30dB$

to stabilize the robot at a static point with the velocity equals to 0.

Two more real-time 3D simulations are made in the attached video by using ROS (Robot Operating System), one is reference tracking simulation and the other is the stabilization of the robot at a static point with the robot velocity equals to zero. Besides, a real implementation is going on.

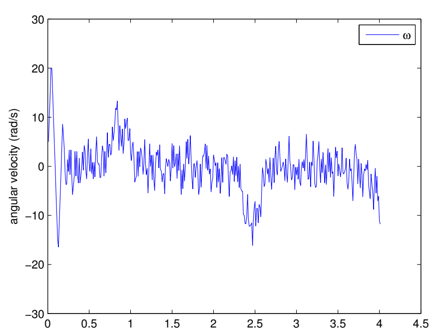

Fig. 8. Angular velocity control with white Gaussian noise $SNR = 30dB$

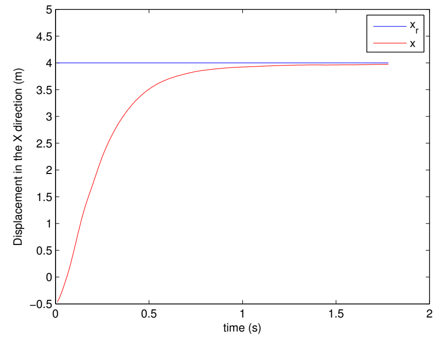

Fig. 9. Tracking of Position X of stabilization

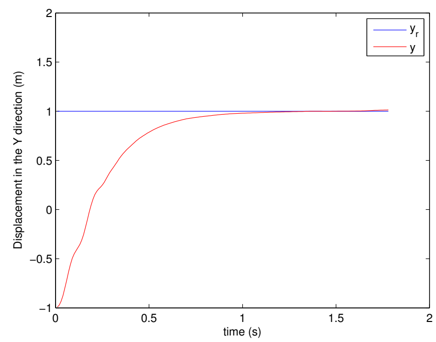

Fig. 10. Tracking of Position Y of stabilization

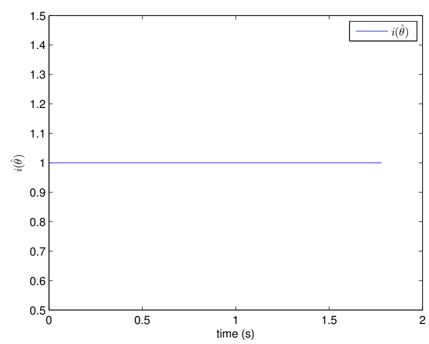

Fig. 11. $i(\hat{\theta})$ of tabilization

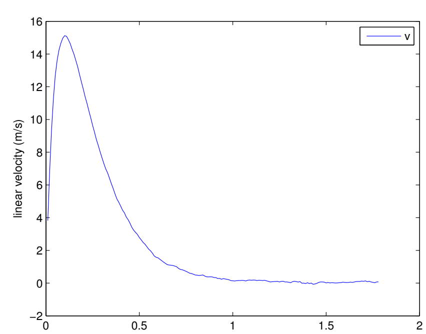

Fig. 12. Linear velocity control of stabilization

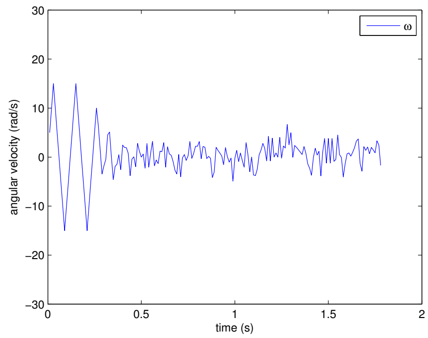

Fig. 13. Angular velocity control of stabilization

V. CONCLUSION

This paper presents the i-PID controller applied to the nonholonomic wheeled mobile robot. After the study of the system, the parameter α in the controller is selected as a switching function according to the information of the system. The presented i-PID controller is robust to the measurement disturbance of the robot, and it can even stabilize the robot at a static point with the robot velocity equals to zero with the proposed parameter α. The effectiveness and robustness of the designed controller were shown thereafter via several different simulations.

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