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Coordinating Mobile Manipulator's Motion to produce Stable Trajectories on Uneven Terrain based on Feasible Acceleration Count

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Abstract— **In this paper we consider the problem of coordinating the motion of the manipulator and the vehicle to produce stable trajectories for the combined mobile manipulator system on uneven terrain. These kinds of situations often arise in planetary exploration, where rovers equipped with a manipulator are required to navigate over general uneven terrain. Moreover the framework can also be used in situations where the mobile manipulator is required to transport objects on uneven terrain. We generate feasible trajectories for the vehicle between a given start and a goal point considering the dynamics of the manipulator. The framework proposed in the paper plans such motion profile of the manipulator that maximizes vehicle stability which is measured by a novel concept called Feasible Acceleration Count (FAC). We show that, from the point of view of motion planning of mobile manipulator on uneven terrains, FAC gives a better estimate of vehicle stability than more popular metrics like Tip-Over Stability. The trajectory planner closely resembles motion primitive based graph based planning and is combined with a novel cost function derived from FAC. The efficacy of the approach is shown through simulations of a mobile manipulator system on a 2.5D uneven terrain.**

I. INTRODUCTION

In this paper we address the problem of planning paths for an outdoor robotic vehicle equipped with a manipulator, between a given start and a goal location. The manipulator is not constrained to execute any specific trajectory. However the framework proposed is applicable in cases where the mobile manipulator is required to transport objects over uneven terrain. Although the objective is to generate a stable path for the vehicle, the manipulator joint space planning has to be included in the framework to ensure vehicle stability. As we show later that having the manipulator fixed while the vehicle is moving may compromise it's stability. This gains additional importance while operating over uneven terrains where the vehicle dynamics changes significantly with time and hence demands for a planning framework which can generate correct coordination between the vehicle and the manipulator.

Many researchers in the past have addressed the problem of coordinating the motion of the manipulator and the vehicle to maximize vehicle stability. Dubowsky et.al [1] did that for a stationary vehicle which cannot be used in the more general case of planning where the vehicle is moving and that too with significant speeds. The fact the manipulator's motion can be used for stabilizing the vehicle was used by Iagnemma et.al in [2] wherein the tip-over stability of the vehicle is improved by the manipulator motion. Huang et.al in their work [3] used the zero moment criteria (ZMP) for coordinating manipulator motion along a given vehicle trajectory. They later extended their work in [4],[5] to plan paths for the mobile manipulator system by first generating a path in the configuration space considering only the vehicle and then producing a time parametrization of the path considering the effects of the manipulator dynamics to deduce the velocity and acceleration profile. This approach suffers from the drawback that the configuration space planning would have to be repeated if there exists no feasible velocity and acceleration between two points, which we show later, could indeed be the case on a 3D uneven terrain. Similar problems arise in [6] where Tip-Over stability, originally introduced in [7] was proposed as a metric for generating manipulator motion to prevent or to recover from overturning situations. The procedure followed involved, first searching for specific configurations of the manipulator favourable to vehicle stability from Tip-over standpoint and then finding a suitable time parametrization between specific configurations. Authors in [8],[11] address a relatively simpler problem of analysing vehicle stability given a particular vehicle and manipulator trajectory on rough terrains. Since they don't provide any framework for 3D evolution of the vehicle which is necessary for motion planning, extending their method to a more general planning domain would be difficult.

The proposed work builds on our previous works [12],[13]. For a passive suspension vehicle on uneven terrain, it is possible to divide the configuration variables into active and passive category. The passive variables evolve according to the underlying terrain and are a function of the active variables. We use framework proposed in [13] to deduce the functional relationships between active and passive variables. These functional relationships allows us to derive the full 3D dynamics of the mobile manipulator. From the dynamics, we use [12] to deduce the concept of Feasible Acceleration Count (FAC). The key difference however in the current work is that the FAC has been extended to include the configuration variables of not only the vehicle but also the manipulator.

The key contribution of the proposed work is that it highlights the efficacy of FAC from motion planning standpoint. It specifically shows that the Tip-Over metric [7] does not realistically predict the stability of mobile manipulator systems moving with high speeds on uneven terrain. It also highlights the need for coordinating the motion of the manipulator and the vehicle because as shown later, having the manipulator fixed while navigating may lead to instability. The trajectory

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planner presented closely resembles the concept of motion primitives based sampling based planning [14]. We also contribute by showing that at the fundamental level the concept of FAC and motion primitives is closely interlinked in the sense that FAC actually represents the set of feasible motion primitives available at any given instant.

The rest of the paper has been organized as follows: Section II introduces the framework for vehicle 3D state evolution. Section III derives the vehicle and manipulator dynamics considering the posture information derived in section II. Section IV presents a comparison between FAC and Tip-Over Stability metric. Section V describes the motion planning framework. The Simulation results are discussed in Section VI.

II. Vehicle 3D States Derivation

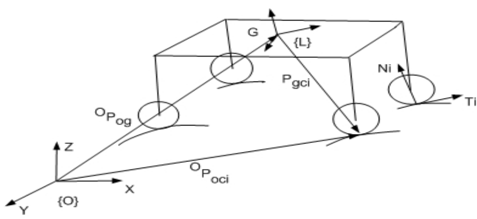

Fig. 1. A generic four-wheeled vehicle

We assume here that the terrain equation can be represented in the form

$$a = h(b,c) \tag{1}$$

where a represents the height at the $x-y$ coordinate (b,c). For a passive suspension vehicle operating on 3D uneven terrain, it is possible to represent every vehicle states in terms of some functions of the yaw plane parameters, which includes the position x,y and the heading angle α. We call these three variables as active variables. These functional relationships stem from the holonomic constraint defining the geometry of the vehicle, which can be written as refer fig.1)

$$\overrightarrow{P}_{OG} + \overrightarrow{P}_{Gci} = \overrightarrow{P}_{Oci} \tag{2}$$

where

$$\overrightarrow{P}_{Gci} = \overrightarrow{r}_f, \overrightarrow{P}_{Oci} = [x_{ci}, y_{ci}, z_{ci}]^T \tag{3}$$

$$\overrightarrow{r}_f = R \begin{bmatrix} \delta h & \frac{2.5-i}{|(2.5-i)|} w & -(l+r) \end{bmatrix}^T, \forall i = 1,2,3,4 \tag{4}$$
$$\delta = 1, i = 1,4$$
$$\delta = -1, i = 2,3$$

R is the rotation matrix describing the orientation of the frame $\{G\}$ with respect to $\{L\}$. Frame $\{G\}$ has the same orientation as the inertial frame $\{O\}$ but moves along with the vehicle. $\{L\}$ is the local body reference frame. $(2.5-i)$ and δ has been incorporated to ensure proper sign of w and h and corresponding to each vertex of the chassis. We assume that the suspension travel length of the vehicle is small and hence each leg length can be represented as l. h and w are half width and breadth of the chassis and r is the radius of the wheels. Equation 4 written for all the wheels represents 12 equations in 15 variables. They are twelve wheel ground contact points x_{ci},y_{ci},z_{ci}, roll β, pitch γ and z coordinate of the vehicle.

In our earlier work [13] we have linearised the terrain equation (1) and the holonomic constraint (2) about the current vehicle coordinate x, y and roll angle β and pitch γ. This allowed us to obtain the following good approximate relationships.

$$\gamma = k_2 \cos\alpha - k_1 \sin\alpha, \beta = -k_1 \cos\alpha - k_2 \sin\alpha \tag{5}$$

$$\begin{aligned} z &= 2k_1 w \sin\alpha - 2k_2 w \cos\alpha - k_1^2 l \\ &+2k_1^2 l \cos\alpha^2 - 2k_2^2 l \cos\alpha^2 + k_2 y + k_3 + l \\ &+4k_1 k_2 l \sin\alpha \cos\alpha + k_2^2 l + k_1 x \end{aligned} \tag{6}$$

where $k_3 = h(x,y)$,$k_1 = \frac{\partial(h)}{\partial b}, b = x, c = y$, $k_2 = \frac{\partial(h)}{\partial c}, b = x, c = y$.

(5)-(6) represents the vehicle's posture evolution in terms of the active states x, y, α. These relationships allows deeper insights into the following 3D kinematics of the vehicle.

$$\begin{bmatrix} V_x & V_y & V_z \end{bmatrix}^T = R \begin{bmatrix} v & 0 & 0 \end{bmatrix}^T \tag{7}$$

$$\dot{V_x} = a_x = \dot{v} c\beta c\alpha - v s\alpha c\beta \dot{\alpha} - v c\alpha s\beta \dot{\beta} \tag{8}$$

$$\dot{V_y} = a_y = \dot{v} s\alpha c\beta + v c\alpha c\beta \dot{\alpha} - v s\alpha s\beta \dot{\beta} \tag{9}$$

$$\dot{V_z} = a_z = -\dot{v} s\beta - v c\beta \dot{\beta} \tag{10}$$

where $c\alpha = \cos\alpha$, $s\alpha = \sin\alpha$ and similarly others. v is the velocity of the robot along the longitudinal axis of the robot. It is assumed that due to the non-holonomic constraint, robot's velocity in the local reference $\{L\}$ lies only along the longitudinal axis.

Similar to the structure of (7)-(10), the components of angular velocity $\overrightarrow{\Omega}$ and angular acceleration $\dot{\overrightarrow{\Omega}}$ can be easily expressed in terms of derivatives of α, β, γ and has been omitted here because of lack of space. This in turn means that by utilising (5)-(6), the entire 3D kinematics of the vehicle can be converted to functions of three variables x, y, α, their derivatives and the control inputs of vehicle's motion $\dot{v}$ and $\ddot{\alpha}$. These two control inputs decide the complete 6 dof evolution of the vehicle on a given 2.5D uneven terrain.

III. Mobile Manipulator Dynamics

The model of the mobile manipulator used in the paper is shown in figure 2. The manipulator consists of a 2 dof non-planar arm. It consists of an elbow joint(θ_1) for moving in the $X-Y$ plane while the shoulder joint(θ_2) produces motion in the vertical plane The philosophy behind deriving the vehicle dynamics is to express the traction and normal forces acting on the wheel ground contact point as a function of linear and angular velocity and acceleration of the chassis. To do this we divide the entire system into two parts comprising of vehicle and the manipulator respectively. While there are numerous works which models the mobile manipulator as one identity, separating them into two parts provides a unique advantage and more so in the current case, where the objective is to analyse the effect the manipulator motion on the vehicle stability. Dividing the system into two parts gives access to the reaction force between the vehicle and the manipulator, which would become an internal force if the system is analysed as one whole system and would not appear explicitly in the equations of motion.

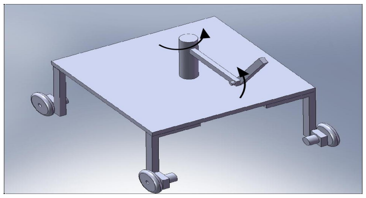

Fig. 2. Model of Mobile Manipulator

The approach followed here to derive the dynamics is to consider the reaction forces at the base of the manipulator as an external force on the vehicle while the effect of vehicle's motion on manipulator is taken into account by superimposing the base of the manipulator with the velocity and acceleration of the vehicle. Similar decoupled analysis can be found in [8] and [9]. [8] concerns with mobile manipulator itself while in [9] the reaction forces at the moving base of the manipulator is computed. The D-H parameter for the manipulator is given in table 1. Newton-Euler formulation is used to derive the dynamics of the manipulator which involves computing the linear and angular velocities of each link recursively through outward iterations starting from the base link and then with it's help computing the forces and moments at each link through inward iterations starting from the last link. The equation for recursively computing the velocity, acceleration, forces and moments is given by [10]. The reaction forces exerted by the manipulator on the chassis of the vehicle (base link) in the global frame can be written as

$$\overrightarrow{f}_o = -RK_1^0\overrightarrow{f}_1 = g_1(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2, \ddot{\theta}_1, \ddot{\theta}_2) \tag{11}$$

$$\overrightarrow{\eta}_o = -RK_1^0\overrightarrow{\eta}_1 = g_2(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2, \ddot{\theta}_1, \ddot{\theta}_2) \tag{12}$$

f_1 and η_1 are the forces and moments exerted on the first link by the base link and K_1^0 is the transformation from the first link to the base link. While computing the outward iterations for the base link, we put the base velocity and acceleration as $\overrightarrow{\omega}_0 = \overrightarrow{\Omega}$, $\dot{\overrightarrow{\omega}}_0 = \dot{\overrightarrow{\Omega}}$ and $\dot{\overrightarrow{v_0}} = \overrightarrow{a} + \overrightarrow{g}$.

$\overrightarrow{\Omega}, \dot{\overrightarrow{\Omega}}, \overrightarrow{a}$, are the angular velocity and acceleration and linear acceleration of the vehicle respectively described in the previous section. $\overrightarrow{g}$ is the acceleration due to gravity.

A. Equations of Motion

The equations of motion can be written in the following form

$$A * C = D \tag{13}$$

where $C = \begin{bmatrix} T_i & N_i \end{bmatrix}^T_{2n\times 1}$ $D = \begin{bmatrix} m\overrightarrow{a} & I\overrightarrow{\Omega} \end{bmatrix}^T_{6\times 1}$ T_i,N_i, are the traction and normal forces acting at the wheel ground contact points. m is the mass of the vehicle. $I_{3\times 3}$ is the vehicle inertia matrix. n represents the number of wheels and in our case $n = 4$. Inverting the matrix A we get the following expressions.

$$\begin{aligned} T_i \;\; = \;\; & a_{i1}(ma_x + f_{ox}) + a_{i2}(ma_y + f_{oy}) \\ & + a_{i3}(mg + ma_z + f_{oz}) + a_{i4}(I_{xx}\dot{\Omega}_x + \eta_{ox}) \\ & + a_{i5}(I_{yy}\dot{\Omega}_y + \eta_{oy}) + a_{i6}(I_{zz}\dot{\Omega}_z + \eta_{oz}) \end{aligned} \tag{14}$$

$$\begin{aligned} N_i \;\; = \;\; & a_{j1}(ma_x + f_{ox}) + a_{j2}(ma_y + f_{oy}) \\ & + a_{j3}(mg + ma_z f_{oz}) + a_{j4}(I_{xx}\dot{\Omega}_x + \eta_{ox}) \\ & + a_{j5}(I_{yy}\dot{\Omega}_{y+\eta_{oy}}) + a_{j6}(I_{zz}\dot{\Omega}_z + \eta_{oz}) \end{aligned} \tag{15}$$

The stability constraints for the mobile manipulator system can be expressed in similar lines to [15] in terms of the following constraints

$$N_i > 0 \tag{16}$$

$$|(T_i)| < \rho N_i \tag{17}$$

$\forall \; i = \{1, 2, 3, 4\}$, $\forall \; j = \{5, 6, 7, 8\}$. m is the mass of the vehicle and g is acceleration due to gravity. a_{i1},a_{i3},a_{j2}..a_{in},a_{jn},are the elements of the pseudo inverse matrix of A.

(16) signifies that the mobile manipulator should always remain in contact with the ground i.e should not topple. (17) is the friction cone constraint. Based on these two constraints, the concept of Feasible Acceleration Count (FAC) for the mobile manipulator system is derived, which has the following definition

Definition Given the current state of the mobile manipulator i.e $(x, y, z, \alpha, \dot{x}, \dot{y}, \dot{z}, \dot{\alpha}, v, \theta_i, \dot{\theta}_i)$, the set of feasible acceleration control inputs $(\dot{v}, \ddot{\alpha}, \ddot{\theta}_i)$ is defined as the Feasible Acceleration Count (FAC). The variation of these feasible acceleration set along a trajectory gives an estimate of vehicle's stability. For example a vehicle state corresponding to which we can find 10 feasible acceleration satisfying stability constraints is more stable than a state for which only 5 feasible accelerations could be obtained.

B. Computing Feasible Acceleration Set

Given the current state of the mobile manipulator search for $\dot{v}$ in the region ($\dot{v}_{min}$,$\dot{v}_{max}$), $\ddot{\alpha}$ in the region ($\ddot{\alpha}_{min}$, $\ddot{\alpha}_{max}$) and $\ddot{\theta}_i$ in the region ($\ddot{\theta}_{imin}$, $\ddot{\theta}_{imax}$) to find those values which satisfies constraint (16) and (17). These values constitute the feasible acceleration set. Here the subscript min and max stands for maximum negative and positive accelerations respectively. A negative acceleration can signify a decrease in the velocity or increment in the negative direction, depending on the current velocity of the vehicle.

IV. Comparisons Between Tip-Over and FAC stability metric

Tip-Over metric [7] was originally proposed for stability prediction for mobile manipulators. Like FAC, it also includes all the generalised forces acting on the system to predict the stability. But the key difference between FAC and Tip-Over metric is that unlike Tip-Over metric, FAC explicitly depends on the conditions of the underlying terrain. As shown in the previous section that FAC depends on the satisfaction of the constraint (16) and (17) which in turn depends on the coefficients of the pseudo inverse of the matrix A i.e a_{i1},a_{i3},a_{j2}..a_{in},a_{jn}..etc. By the virtue of surface contact normals and tangents at the wheel ground contact points, these coefficients model the topology of the

TABLE I

INETIAL PROPERTIES OF THE MANIPULATOR

Joint	$I_{xx}(N-m^2)$	$I_{yy}=I_{zz}(N-m^2)$	$mass(kg)$	$d_i(m)$	$l_i(m)$
1	$75*10^-6$	$2.5*10^-3$	0.5	0.40	0
2	$75*10^-6$	$2.5*10^-3$	0.5	0	0.75
3	$75*10^-6$	$2.5*10^-3$	0.5	0	0.75

TABLE II

D-H PARAMETER OF THE MANIPULATOR

Joint	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	θ_1
2	$\frac{\pi}{2}$	l_1	0	θ_2
3	0	$l2$	0	0

underlying terrain. Moreover the effect of terrain friction is also included in the vehicle stability. In other words Tip-Over metric depends only on the state of the mobile manipulator while FAC depends not only on the state of the vehicle but also on the external environment in contact the mobile manipulator. The readers are requested to follow [12] for mathematical details on how the pseudo inverse coefficients model the effect of terrain geometry.

These fundamental difference makes FAC a better choice for stability metric from the point of view of motion planning on uneven terrains. As shown in the results section optimizing FAC optimizes Tip-Over as well. However the reverse is not seen to be true.

V. MOTION PLANNING

The planner used in the current work closely resembles motion primitives based search based planning [14]. This similarity arises naturally out of the definition of FAC. In [14] a set of pre-defined motion primitives are used to search the entire state space in order to generate a feasible trajectory from the start to the goal. In the current work a feasible motion primitive is one resulting from a feasible acceleration control input satisfying constraints (16) and (17). Hence FAC ,which is the actually the number of feasible acceleration control inputs for the current state, represents the number of motion primitives that can be used to expand the current mobile manipulator state. The trajectory planner has the following main steps

A. Forward Evolution of Mobile Manipulator State

Given the current state of the mobile manipulator, feasible accelerations are obtained. If p number of feasible accelerations exists, then for the small amount of time δt the mobile mobile manipulator state could be expanded to as many branches . For the vehicle first the yaw plane parameters i.e x, y, α are evolved and then the rest of the states are evolved with the help of framework described in section II.

B. Selection of Next Node

For the p number of states resulting from the evolution described above, the metric $M = \frac{FAC}{d}$ is evaluated, where d represents the geodesic distance to the goal. A state which maximises the metric M is chosen and is updated as the current state. Metric M is maximised when FAC is maximised and the distance to the goal is minimised. The process is repeated till the goal location is reached.

This two steps are illustrated in figure 3. Four motion primitives are generated at the shown step. Out of those four, the motion primitive shown in blue leads to a state which has highest value for the metric M. Hence that motion primitive is chosen and the corresponding state is updated as the current state and the process is repeated. One key point to be noted is that in case of uneven terrain motion planning, set of motion primitives are not fixed but rather directly dependent on how dynamically stable the vehicle is. A stable state will have more motion primitives which in turn helps the planner to quickly converge to the goal.

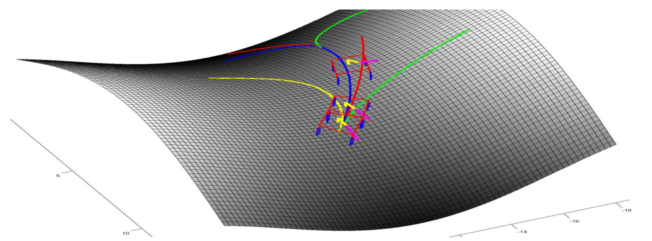

Fig. 3. Motion primitives for expanding mobile manipulator states. The number of motion primitives available for expanding states is not constant. In-fact the number of motion primitives is equal to Feasible Acceleration Count (FAC)

VI. RESULTS AND DISCUSSION

The concepts derived in the previous sections were applied to plan trajectories for a mobile manipulator. A relatively small vehicle was chosen for simulation with the dimension of the chassis being $1m^2$ and height $0.40m$. The mass of the vehicle was kept as m = 10 kg and coefficient of friction between wheel ground contact point as 0.8. The mass and geometric properties of the links of the manipulator are given in table II. Two set of paths were obtained for the mobile manipulator between the same start and goal location with different starting configuration of the manipulator. The simulation results consist of the following major parts: (i).Comparisons of the paths derived for the mobile manipulator and only vehicle in terms of FAC to analyze the effect of manipulator motion on FAC (ii).Analysis of the manipulator and vehicle's motion . (iii). Comparison of stability of the vehicle from the viewpoint of tip over and FAC.

For the sake of comparison, in all the presented results, the normalised version of FAC is shown, which is the ratio of FAC obtained for the current state to the maximum obtained on a perfectly flat terrain.

A. Comparison of Mobile Manipulator and only Vehicle's Motion in terms of FAC

Figure 4(a) shows the final paths on a 3D terrain. Two paths P_1,P_2 are generated for the mobile manipulator starting from the same initial vehicle configuration but different manipulator configuration. A path for only the vehicle without the manipulator is also planned for comparative purposes which will be referred to as the path V. The path V is significantly different from P_1 and P_2 and to understand the underlying cause behind this, we analyze the FAC for the vehicle without the manipulator when evolved along the trajectory of P_1 and P_2. The FAC plots are shown in figure 4(b) (top) and 4(b) (bottom). As stated earlier, the normalised version of FAC is presented. It can be seen that while only vehicle has higher FAC at some places, mobile manipulator shows on an average better performance. In fact, FAC for only vehicle goes to zero at some places along $P_1$and P_2. For example consider figure4(b) (top) where around 58th simulation step($x = 2.24$,$y = 3.6$) the FAC for the vehicle goes to zero for the first time. Hence at this point, the path V and P_1 which were very similar prior to this point, bifurcates. The role of the manipulator in increasing the FAC at this particular point can be inferred from table III. As can be seen from the table that the vehicle without the manipulator violates both constraint (16) and (17), while utilizing the reaction forces of the manipulator, the vehicle with the manipulator is able to improve upon the no-slip and normal force constraint. This further reiterates the fact that planning an appropriate motion for the manipulator is necessary for planning stable trajectories of the vehicle. This can also be inferred from figure 4(c) which shows the FAC plots along $P_1$when the manipulator is kept fixed. The plot shows the FAC for various possible fixed positions and it can be seen that no fixed position maintains a non-zero FAC along the entire path.

TABLE III

COMPARISON OF ONLY VEHICLE AND MOBILE MANIPULATOR

	$\min N_i$	$\max(\lvert T_i\rvert - \rho N_i)$
$OnlyVehicle$	-0.79	22.15
$MobileManipulator$	14.39	-10.67

B. Analysis of Mobile Manipulator's Motion

Figure 4(d) shows the 3D evolution of the mobile manipulator along P_1 where it moves from right to left towards the goal (Please refer to the video submitted with the paper). Common intuition dictates that when the vehicle is moving up the slope, the most appropriate position for the manipulator is towards the front and vice-versa while coming down the slope. The mobile manipulator tries to follow this as closely as possible provided it finds a feasible acceleration. For example consider the initial part of the mobile manipulator's trajectory shown in a magnified view in figure 4(e) . It can be seen from the figure the vehicle is moving up the slope in the encircled part C_1 but due to the lack of appropriate feasible acceleration, there is very little movement of the elbow joint(θ_1) (shown in yellow) and the shoulder joint (θ_2)(shown in pink). This can be confirmed by the plot of joint angles shown in figure 4(f) (sim step 0-20). However during the encircled part C_2, when the vehicle is moving down the slope, the elbow joint is able to rotate backwards and shoulder joint downwards(sim step 20-50). Moreover the manipulator moves in a way so as to compensate for the centripetal forces acting on the vehicle. The vehicle velocity profiles are shown in 4(g).

C. Comparison between FAC and Tip Over Stability Metric

Here we compare between the Tip over and FAC stability metric with respect to the trajectory planning problem considered in this work. During planning of trajectories P_1 and P_2, two sets of manipulator trajectories were evaluated. One set maximizes FAC while the other maximizes Tip Over. Figure 4(h) gives the FAC and Tip-Over plot along the trajectories P_1andP_2 for the set of manipulator motions aimed at maximizing FAC. It can be seen from the figure that maximizing FAC ensures that both Tip-Over and FAC remain above zero. However the situation is quite different in figure 4(i) which shows the results for the set of manipulator motions aimed at maximizing Tip-Over. It can be seen that while the Tip-Over has improved as compared to figure 4(h), FAC has drastically deteriorated. This shows that FAC is a more conservative metric and its satisfaction generally ensures stability beyond what is predicted by tip over.

VII. CONCLUSIONS AND FUTURE WORKS

The paper presented a framework for coordinating the motion of the mobile manipulator on uneven terrains. The concept of stable coordination is described in terms of acceleration control inputs, rather than configuration variables, which is different from other existing works in this field. A full 3D vehicle evolution and dynamics were considered on a generic 2.5D terrain. Concept of Feasible Acceleration Count (FAC) was used to judge the stability of the vehicle and it was shown that it reflects the dynamic stability of the vehicle better than the Tip-Over metric. FAC as stability metric was shown to be more conservative in the sense that maximizing FAC ensured that Tip-Over metric also remains high. While attempts to maximize Tip-Over led to such vehicle states for which FAC was zero. Hence the paper establishes the efficacy of FAC, as a stability metric from the standpoint of motion planning. Further the concept of FAC blends naturally with motion primitives based search based planning as FAC represents the number of motion primitives available at any given instant.

The future efforts are directed towards: 1. Studying the effect of terrain uncertainty on the fidelity of the planned stable paths. A framework which accounts for terrain uncertainty and gives a probabilistic measure of mobile manipulator safety will improve the practicality of the framework. 2. The concept of non-linear time scaling proposed in our earlier works [13] can be used to devise an on-line mobile manipulator navigation and coordination strategy for uneven terrains.

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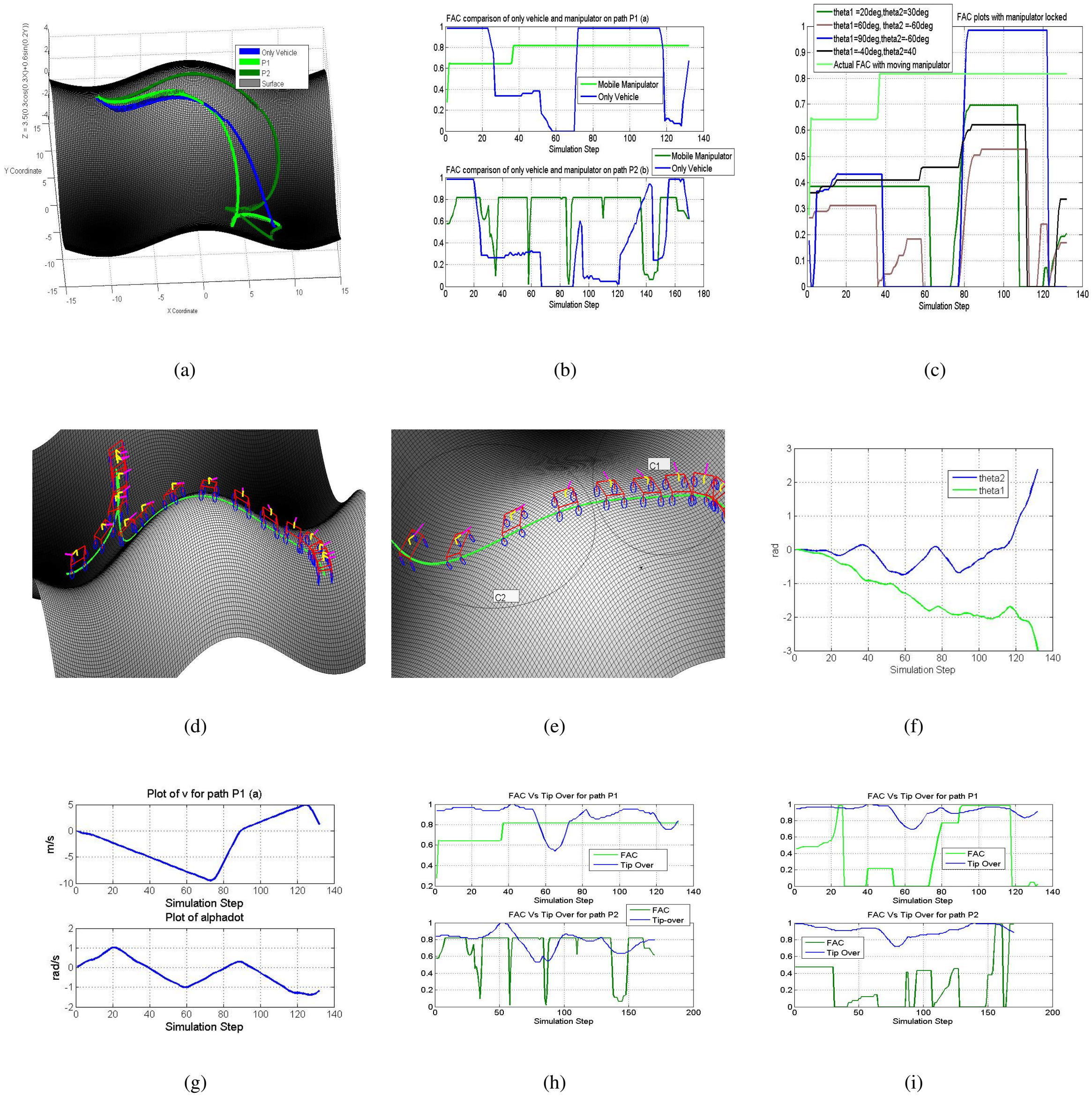

Fig. 4. (a): Final planned paths. (b): FAC of only vehicle and mobile manipulator. (c): FAC with mobile manipulator locked. (d): Evolution of Mobile Manipulator on uneven terrain. (e): Magnified view of mobile manipulator's evolution. (f): Variation of Manipulator's joint angles. (g): Plot of magnitude of vehicle velocity (v) and $\dot{\alpha}$. (h): Vehicle stability when manipulator's motion is maximising FAC. (i): Vehicle stability when manipulator's motion is maximising Tip-Over

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