Deformation-tracking Impedance Control in interaction with Uncertain Environments

Loris Roveda^{1,2}, Federico Vicentini¹ and Lorenzo Molinari Tosatti¹

Abstract—A deformation-tracking impedance control strategy is discussed for applications where a manipulator interacts with environments of unknown geometrical and mechanical properties, especially with stiffness comparable to a controlled robot stiffness. Based on force-tracking impedance controls, the deformation-tracking strategy allows the control of a desired deformation of the target environment, requiring the on-line estimation of the environment stiffness. An Extended Kalman Filter is used for the estimation of the environment because of measurement uncertainties and errors in compound interaction model. The tasks presented involve full body spatial interactions with a time-varying environment stiffness. The Extended Kalman Filter and the deformat ion-tracking impedance control are validated in simulation and with experiments. In particular, a cooperative assembly task is also performed with a human operator acting as varying environment, i.e. unpredictably changing the handling arm stiffness.

I. INTRODUCTION

The importance of compliance control in technological and manipulation tasks has long been demonstrated, since the milestones of sensor-based force/dynamics control [1], [2], [3], [4], [5]. Among those major classes, the hybrid velocity/force control remains one of the most effective solutions

¹ Institute of Industrial Technologies and Automation (ITIA) of Italian National Research Council (CNR), via Bassini, 15 - 20133 Milan, Italy

² Politecnico di Milano, Department of Mechanical Engineering, via La Masa 1, 20156 Milan, Italy

loris.roveda@itia.cnr.it



Fig. 1: Compliant KUKA LightWeightRobot inserting shapes (unknown to the controller) on partially unknown substrate. The environment is made by two different material with unknown mechanical properties and unpredictable shear.

when interacting bodies are well defined and their geometrical and mechanical features can be integrated into the control model, which is the case of many industrial applications. Although it is one of the primary control modes for tracking robot-environment forces along a subset of task directions, and the observation of model uncertainties has been investigated since early works [6], the position accuracy may not always be preserved in loosely structured or dynamically changing environments where the interaction model is prone to erroneous observation. Alternatively, impedance control [7] compounds a dynamic balance response by both the robot and the environment, regardless (to some extent [8]) the predictability of the environment dynamics. However, a control of explicit interaction forces or deformations cannot be directly obtained, undermining the suitability of impedance control in those technological tasks that require some degree of process control over the interaction. Many efforts have been made, in fact, to achieve a force/position tracking with impedance control despite the lack of knowledge of the environmental stiffness and location. In [9] the timevarying force is tracked starting from a position control law, scaling the trajectory as a function of the estimated environment stiffness. One of major approaches (as in [10], [11]) involves the generation of the reference motion as a function of the force-tracking error, under the condition that the environment stiffness is variously unknown, *i.e.* estimated as a function of the measured force. A contribution to the definition of an interaction model is given by the online estimation of the environment stiffness through persistent excitations [12] or some knowledge of bodies geometry [13]. The application of a persistent excitation allows of course the best accuracy in parameters estimation but is not always suitable during the execution of a technological task. Other approaches involve impedance control laws that modify the robot stiffness according to force measurements in order to cope with unmodeled variable target stiffness [14]. In this way the interaction dynamics are predominantly compensated at robot side absorbing all energy variations, whose promptness and accuracy in force tracking depend very much on the robot modeling and correlated robust control techniques. In some general purpose manipulation of objects of unknown shape and material, like in humanoids compliant motion [15], [16], the stiffness estimation can be discarded or loosely predefined.

The purpose of the presented work is to derive a deformation-tracking control, making use of the compliant behavior of a lightweight manipulator in interaction with a comparably soft or even softer environment, relying on non-



Fig. 2: Deformation-tracking with impedance control: reference trajectory $\Delta \mathbf{x}_0$ for Cartesian impedance controller id defined by a *control law* using a force-tracking derived from deformation-tracking on the basis of environment estimation by the *observer*.

restrictive signal conditions in force sensing. In particular, the goal of the developed control strategy is to define an impedance control set-point in order to track a desired deformation of the environment, indirectly controlling the interaction force. The used platform (KUKA lightweight robot in Fig. 1), in fact, allows w.l.o.g. the definition of a task space impedance behavior ranging from soft to mid-stiff settings, and provides a task space estimate of interaction forces and torques whose sensitivity could be in some cases insufficient in directly deriving the environment response. In particular, the generic case that is here discussed considers passive environments whose location and dynamic parameters are totally or partially u nknown. In or der to do this, it is necessary to on-line estimate the dynamic parameters of the environment without providing a persistent excitation. For this purpose an Extended Kalman Filter (EKF) is implemented based on a pure impedance contact model, and taking into account the sampling noise and the modeling inaccuracies.

The experimental testbed involves an assembly task, performed in impedance control through a simple insertion procedure. The assembly substrate (*i.e.* the environment) is of variable unknown stiffness. An assembly task is quite used as a test case because it requires the full characterization of the impedance of the environment along all directions of the interaction. The only hypothesis introduced here consists in limiting the dynamics of first contact to a single degree-offreedom (DoF) in order to partially assess the initial contact location, *e.g.* restricting the starting of the task to a typical vertex-surface interaction.

II. PROBLEM FORMULATION AND CONTROL MODEL

The deformation-tracking impedance control (Fig. 2) defines a reference trajectory, or just the reference pose update Δx^0 , as a function of the deformation of a target environment. Deformation-tracking rearranges force-tracking control laws and is useful when the penetration of a robot tool into a (generally) softer, yet unknown, environment is somehow a major specification of the task at hand. The core control law defines a target contact force f^d acting predominantly elastically on the environment during the execution of the task, for which it is required to estimate the stiffness $\hat{\mathbf{K}}_e$ of the environment, in order to track the desired deformation \mathbf{x}_e^d with rispect to the actual position of the environment x_e :

$$\mathbf{f}^d = \mathbf{K}_e(\mathbf{x}_e^d - \mathbf{x}_e) \tag{1}$$

$$\Delta \mathbf{x}^0 = \mathbf{K}_P \mathbf{K}^{-1} (\mathbf{f}^d - \mathbf{f}_e)$$
(2)

$$\mathbf{K}_e = f(\mathbf{f}_e, \mathbf{x}_{e,eq}, \mathbf{x}_e) \tag{3}$$

where \mathbf{K}_p is the proportional gain on the force-tracking error, \mathbf{K} is the diagonal stiffness matrix of the controlled robot, \mathbf{f}_e is the force vector acting on the environment and $\mathbf{x}_{e,eq}$ is the equilibrium position of the environment.

The main task space impedance loop is performed by the model-based control of the lightweight manipulator (see II-A) at mid/fast rate (1 - 5ms), synchronously with the environment estimation (*observer* in Fig. 2). A model of the multi-port robot-environment interaction is, in fact, needed in order to define the force setpoints in (1) through the environemt stiffness $\hat{\mathbf{K}}_e$, which in turn is estimated through the deformation of the environment and the full state of robot kinematics and exchanged forces. Interaction states and parameters are eventually observed by an EKF (see II-C). The deformation and the force tracking setpoints in (1) are updated (*control law* in Fig. 2) at slower rate (10 - 20ms)due to the nature of tracking at hand and ensuring the a steady state of the observer.

A. Controlled Robot Model

The dynamic behavior of the controlled robot is defined by the diagonal stiffness \mathbf{K} and damping \mathbf{D} matrices in task space impedance control [17], in interaction with a force \mathbf{f} , in this case essentially due to the environment deformation:

$$\mathbf{D}\dot{\mathbf{x}} + \mathbf{K}\Delta\mathbf{x} = \mathbf{f}_r \tag{4}$$

where $\Delta \mathbf{x} = \mathbf{x} - \mathbf{x}^0$ is the difference between the actual robot pose and the desired one \mathbf{x}^0 as generated in (2).

The Cartesian pure impedance behavior is obtained by the

control law [18]:

$$\mathbf{u} = -\mathbf{J}(\mathbf{q})^T \mathbf{f}_r + \mathbf{g}(\mathbf{q})) \tag{5}$$

where \mathbf{f}_r is the force vector acting on the controlled robot, \mathbf{J} is the Jacobian matrix of the robot and $\mathbf{g}(\mathbf{q})$ is the gravitational term of the robot, ideally decoupling [19] the joints dynamics in the task space.

B. Properties of the Environment

Colgate and Hogan [8] considered classes of linear time invariant (LTI) environments models, highlighting the worst case conditions for the stability of interactions. They extended the closed loop stability conditions for the interaction ports also to non-linear and non-passive environments. The critical configuration is when stiffnesses of both the robot and of the environment nearly match, which is the case of interest in this paper where the control law in (4)-(5) generates a relatively soft manipulator dynamics, in presence of non-LTI evironments. However, as long as the controlled robot displays passive behavior [18] and low frequencies are considered for the task, *i.e.* preserving the link-side rigid body properties, the nature of (unknown) environments to be coupled on interaction ports remains fairly general. The model of the unknown environment is somehow restrained to be worst-case compliant, introducing some damping terms (see Fig. 1). Under the hypothsis that exchanged forces at interaction ports remain unhaltered by the port, *i.e.* $\mathbf{f}_e = \mathbf{f}_r =$ f in (2), (3), (4) and (5), the simplest way to describe the impedance port is the linear KelvinVoigt contact model [20] (mass M_e - spring K_e - damper D_e model). Considering soft environments, diagonally-dominant natural frequencies $\omega_e = \sqrt{\mathbf{M}_e^{-1} \mathbf{K}_{\mathbf{e}}}$ could display resonances in the operating bandwith of the linearly decoupled impedance control. However, considering a reasonable task bandwidth limited at 5Hz, the worst case - e.g. undamped - minimum ratio $\min_i \frac{K_e}{M_e}$, $\forall i$ DoFs, is about $15^2 \frac{N^2}{m^2 k g^2}$, so that in the damped case the masses of the environment model can be neglected. Accordingly, the environment pure impedance model results in

$$\sum_{i} (\mathbf{D}_{e}^{i} \dot{\mathbf{x}}_{e}^{i} + \mathbf{K}_{e}^{i} \Delta \mathbf{x}_{e}^{i}) = \mathbf{f}, \forall i = 1, \cdots, N$$
(6)

for all the finite number N of interaction ports. Nonetheless, although critical damping is set by the controller and the environment is realistically damped, some excitation of natural frequencies of the environment may arise during the initial contact phase. Therefore the initial exploratory phase in the task is considered to be far from resemble severe impacts.

C. Environment Observer

Due to the unknown/partially known geometry of a generic interacting environment, the model of the N interaction ports as used in [21] happens to be unfeasible. The environment in (6) is reduced to a translational lumped impedance model with diagonal \mathbf{K}_e and \mathbf{D}_e matricesto be used in the EKF dynamics. Under the mild hypothesis that the contact is preserved once established and simplification hypothesis that the contact(s) are elastic, *i.e.* $(\mathbf{x}, \dot{\mathbf{x}})_{tool} = (\mathbf{x}, \dot{\mathbf{x}})_e$, the



Fig. 3: Unidimensional model of interaction with unknown environment stiffness.

robot-environment interaction is defined by the filter state, augmented with the environment properties:

$$\mathbf{x}_a = [\Delta \mathbf{x}_e, \mathbf{K}_e, \mathbf{D}_e, \mathbf{f}]^T.$$
(7)

Substituting the augmented state (7) in model (6) the filter dynamics result in:

$$\mathbf{f}(\mathbf{x}_{\mathbf{a}}, \boldsymbol{\nu}) = \begin{bmatrix} \dot{\mathbf{x}}_{e} \\ \dot{\mathbf{K}}_{e} \\ \dot{\mathbf{D}}_{e} \\ \dot{\mathbf{f}} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_{e}^{-1} \left(-\mathbf{K}_{e} \mathbf{x}_{e} + \mathbf{f} + \boldsymbol{\nu}_{\mathbf{x}_{e}} \right) \\ \boldsymbol{\nu}_{\mathbf{K}_{e}} \\ \boldsymbol{\nu}_{\mathbf{D}_{e}} \\ \boldsymbol{\nu}_{\mathbf{f}} \end{bmatrix}$$
(8)

where the vector $\boldsymbol{\nu} = [\boldsymbol{\nu}_{\mathbf{x}_e}, \boldsymbol{\nu}_{\mathbf{K}_e}, \boldsymbol{\nu}_{\mathbf{D}_e}, \boldsymbol{\nu}_{\mathbf{f}_e}]^T$ accounts for uncertainties in models parameters/estimates.

The observer of the augmented state is therefore defined as:

$$\begin{cases} \dot{\mathbf{x}}_{a} = \mathbf{f}(\mathbf{x}_{a}, \boldsymbol{\nu}) + \mathbf{K}_{EKF}(\mathbf{y} - \mathbf{C}_{a}\hat{\mathbf{x}}_{a}) \\ \hat{\mathbf{y}} = \mathbf{h}(\mathbf{x}_{a}, \mathbf{w}) \end{cases}$$
(9)

where $\widehat{\mathbf{x}}_a$ are estimates, \mathbf{K}_{EKF} is the gain matrix:

$$\mathbf{K}_{EKF} = \mathbf{P}\mathbf{C}_a\mathbf{R}^{-1} \tag{10}$$

with C_a as the observation matrix for the pose x and force f measurements, and R as the measurement noise matrix defined as

$$\mathbf{R} = \mathbf{H} E\{\mathbf{w} \mathbf{w}^T\} \mathbf{H}^T = \mathbf{H} \mathbf{W} \mathbf{H}^T$$
(11)

where the observation function \mathbf{h} linearly maps the sample inaccuracies, due to measurement noise \mathbf{w} , through the matrix \mathbf{H} :

$$\mathbf{H} = \left. \frac{\partial \mathbf{h}}{\partial \mathbf{w}} \right|_{\widehat{\mathbf{x}}_a}.$$
 (12)

The covariance matrix **P** and its rate, as in:

$$\dot{\mathbf{P}} = \mathbf{A}_a \mathbf{P} - \mathbf{P} \mathbf{C}_a^T \mathbf{R}^{-1} \mathbf{C}_a \mathbf{P} + \mathbf{G}_a \mathbf{Q} \mathbf{G}_a^T + \mathbf{P} \mathbf{A}_a^T \quad (13)$$

are based on the dynamics of the state and the model uncertainties, defined with matrix A_a and matrix G_a respectively:

$$\mathbf{A}_{a} = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}_{a}} \right|_{\widehat{\mathbf{x}}_{a}} \qquad \qquad \mathbf{G}_{a} = \left. \frac{\partial \mathbf{f}}{\partial \boldsymbol{\nu}_{a}} \right|_{\widehat{\mathbf{x}}_{a}} \qquad (14)$$



Fig. 4: Stiffness estimation in simulation experiment in 1 DoF, along a vertical axis.

and on matrix ${\bf Q}$ used for the estimation of the parameters, which is defined as:

$$\mathbf{Q} = \mathbf{G}_a E\{\boldsymbol{\nu}\boldsymbol{\nu}^T\}\mathbf{G}_a^T = \mathbf{G}_a \mathbf{V}\mathbf{G}_a^T.$$
 (15)

Vector **w** is defined based on an analysis of the noise content in the signal of position and force measurements. In particular $[\mathbf{w}_p \mathbf{w}_f] = [10 \ 10^2]^T$ are the noise scalar values for position and force sampling, correspondingly replicated for all applied DoFs. Vector $\boldsymbol{\nu}$ is defined from experimental tests. In particular $\boldsymbol{\nu}$ base scalars are $[10^2 \ 10^6 \ 10^5 \ 10^2]^T$. Values of the components associated to the estimation of the stiffness $\hat{\mathbf{K}}_e$ and damping $\hat{\mathbf{D}}_e$ are higher than the components associated to the estimation of the deformation of the environment $\Delta \hat{\mathbf{x}}_e$ and force $\hat{\mathbf{f}}$ due to the fact that no measures are available for these quantities and a higher dynamic is needed in order to have a faster convergence.

III. SIMULATION AND EXEPRIMENTAL RESULTS FOR 1DOF STIFFNESS ESTIMATION

Characterization, tuning and evaluation of the observer in (9) are performed in simulation and real experiments for the stimation of the environment stiffnes \hat{K}_e . The initial assessment of the environment location is done exploring the task space along the approach direction until the surface contact $x_{e,eg}$ is



Fig. 5: Experimental configuration in 1 DoF: right-hand side robot is providing a variation of stiffness (softening) at the single point contact along the y axis of the left-hand side robot.



Fig. 6: Stiffness estimation in real experiment in 1 DoF, along a horizontal axis.

detected with a suitable force threshold. Such transient phase applies the force tracking in (2) where the desired contact force f^d is kept as small as possible, w.r.t. sensitivity, in order to limit the environment deformation. It is assumed that the contact behaves as a 1-port with known location w.r.t. the robot pose. Then the on-line estimation of environment stiffness can be executed using the the 1-DoF formulation of (8) for the filter states update. In Fig. 4 a variable pattern of simulated environment stiffness is observed, while in Fig. 5 a companion manipulator in impedance control is used to generate a (non-shared) reference stiffness for the observer, whose results are shown in Fig. 6. The estimaded stiffness has a delay in the estimation of approximately $0.5 \ s$ and a maximum steady state error of less than 1% and 3% w.r.t. the nominal known values in simulated and real experiments, respectively.



Fig. 7: Compliant KUKA LightWeightRobot inserting shapes (unknown to the controller) while an operator is holding the substrate. The environment preserve passivity to some extents and is naturally time-variant.



Fig. 8: Assembly task results (translational components). (top) commanded (dotted line) and measured position (solid line) of controlled TCP; (mid-top) desired band of (dotted line) and obtained (solid line) deformation of environment; (mid-bottom) desired band of (dotted line) and obtained (solid line) interaction force; (bottom) estimated environment stiffness.

IV. ENVIRONMENT STIFFNESS ESTIMATION IN MULTI-PORT INTERACTION.

Full state observation as in (9) is performed in a test case of deformation-tracking that presents peculiar features for assembly tasks: the environment location is only coarsely known, the coumpound stiffess is either unknown or timevariant, compliant contact is desired along all directions. The assembly task displays therefore a plain preparatory phase for contact engagement, followed by a seeking phase for insertion. The insertion is made according to an intuitive strategy. When the assembly is done, the environment changes while the controller is able to track the acting force and the resulting deformation also in case of a inherently time-variant (as in Fig. 7) environment.

In detail, 4 phases are recognized (see Fig. 8 and Fig. 9): **Phase A.** Identification of the position of the environment

(approach in free space).

Phase B. Exploration along translation components and online estimation of the environment stiffness orthogonally to the surface of contact, using the EKF model in (8). Rotational components of the impedance control set-point in (2) are kept constant. The impedance control set-point is computed as a function of force-tracking error $\Delta \mathbf{f} = \mathbf{f}_{task}^d - \mathbf{f}$ as in (2), where $\mathbf{f}_{task}^d = \left[f_x^d f_y^d f_z^d 0 0 0\right]^T$. In exploration, \mathbf{f}_{task}^d is masked such as only f_z^d is defined according to (1), while f_x^d and f_y^d are just set equal to f_z^d due to the fact that estimation of $\hat{K}_{e,x}$ and $\hat{K}_{e,y}$ is quite unreliable if/when no contact takes place beforehand. This setting, in turn, allows that the first inserted edge remain in contact with the surface of the environment for the entire exploration phase.

Phase C. Execution of the assembly task, enabling rotations for insertion, relying on $\hat{\mathbf{K}}_e$ observed along the



Fig. 9: Assembly execution: rotations about Cardan axes (enabled only during insertion phase).

searching directions. The impedance control set-point in (2) uses the full vector of nominal tracking forces $\mathbf{f}_{task}^d = \left[f_x^d f_y^d f_z^d \tau_x^d \tau_y^d \tau_z^d\right]^T$, where $\tau_x^d \tau_y^d \tau_z^d$ are experimentally determined.

Phase D. After tight assembly, on-line estimation of the variable $\hat{\mathbf{K}}_{e}$. The estimations $\hat{K}_{e,x}$, $\hat{K}_{e,yz}$ and $\hat{K}_{e,z}$ enable the manipulated component to maintain the inserted location and a desired deformation using the control strategy in (1).

Task phases in Fig. 8 and Fig. 9 are easily identified from measured pose/force data: surface contact is detected as in single DoF experiments (see III) from pose error due to the engaged environment, while the insertion is completed once the rotational components reach a steady state (see Fig. 9) due to inserted rigid body full constraining. Is important to underline that in the third phase (gray lines in Fig. 8) the roto-translation of the TCP does not allow fully reliable environment observation.

V. CONCLUSIONS

Based on force-tracking control laws, the deformationtracking control of a soft unknown environment has been implemented and tested in a full rigid body assembly real task. The force set-point has been generated according to a task execution strategy based on a nominal distribution of forces during the assembly/insertion, and on the online estimate of the stiffness of the interacting environmen through an EKF. The developed control strategy and the EKF have been in fact applied to assembly tasks where the geometry of manipulated components is not completely known. The delay in stiffness estimation is due to the update frequency of the EKF. However, this delay does not introduces interaction forces overshuts because of the low dynamics of the task and the softness of the environment in interaction. The stiffness variations of the environment are therefore limited. In order to improve performaces of the EKF a non-linear model of the environment is considered for upgrade, while the performances of the task execution could benefit from a model of the controlled robot at midhigh frequencies.

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