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Rolling a Dynamic Object with a Planar Soft-fingertip Robot Arm

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Abstract— **Force-position control through one or multiple robots, or fingers, typically assumes a rigid endpoint without rolling nor slipping. However, there are some interesting tasks where rolling is involved, such as turning a knob (object is pivoting at a fixed rotational axis) or rolling a wheel (object rotational axis is moving). In such a case, rigid endpoint force control becomes very difficult if not impossible, even for us humans. This stems from two facts, firstly, infinitesimally small rigid point does not yield a tangent force, therefore it is very difficult to control it indirectly; and secondly, the pair robot-object stands for a highly non-linear constrained underactuated dynamical systems. In this paper, we aim at exploring rolling of a rigid dynamic circular object with hemispherical deformable fingertip, then with area, not point, contact. The dynamic model and a control scheme are presented inspired in previous works, but regulation of normal and tangential forces, as well as position and orientation of the object are synthesized. In particular, tangential force control proves instrumental to regulate posture, and displacement of the object with a simple transpose Jacobian Cartesian *PDF+g* control. Regulation of rolling angle and displacement with stable normal and tangential forces are obtained without force sensing, neither any model of the deformation nor any dynamic parameter of the object. To entertain these control objectives, a redundant configuration is required so as to yield local regulation, based on the stability-in-the-manifold criteria, whose dimension is greater than the operational space. Illustrative simulations are discussed that provide insight into the closed-loop numerical performance, and finally, remarks on the structure and potential applications are addressed.**

I. INTRODUCTION

The problem of force control with *rigid point contact*, [10], [11], establishes enormous limitations to exert force onto a deformable environment clearly because it is not designed for *deformable area contact*, let alone for manipulation under rolling constraints, [1], [2]. As an alternative, soft end-effectors have been proposed, [3], [4], to deal explicitly, essentially, with deformation at contact. Variable contact area arises when soft end-effectors or soft fingertips are considered, then appear a variable deformation at contact, which brings issues yet to unveil for complex applications regarding modeling and control design, in particular when it is assumed that deformation is unknown, [5]. This leads to, and sounds intuitive for, rolling, either at contact between end-effector and rigid object or between manipulated rigid object and environment, [6]. Despite this compelling argument for dexterous manipulation, the study of position-force control of soft fingertips, the first contact models, and control design are very recent [5], [7], yet, so far, manipulation with soft fingertip is still a basic research problem, including how to deal with environmental kinematic uncertainties, [8]. However, for rigid infinitesimally small contact, tangential force is not explicitly modeled in the dynamics of the system, which is a primary source of problem when rolling is involved.

Contact mechanics at deformable area shows that normal and tangential forces arise in 2D, either with radial, [9], or parallel deformation, [7], models. That is, end-effector deformation brings into object dynamics a tangential force variable that can be controlled to deal with rolling, and then to treat and prevent slippage and sliding phenomena at contact. Handling rolling is a commonly accepted measure of dexterity in robotic hands, nevertheless, tangent force control has not really been explored for rolling a dynamic object through a deformable fingertip. In this case, assuming a hemispherical deformable fingertip, end-effector rolls onto the rigid dynamic circular object to give rise to a rolling constraint. Consequently, the control of deformable fingertip surpasses the limitations of classical point contact-based force control, [10], [11]. Moreover, a more subtle inspection on this suggests that deformation is a key characteristics for dexterity of robot arms, which cannot be taken for granted by simply applying known controllers developed for contact point with tangent friction. This claim is substantiated by the fact that not only the tangential forces are not dissipative ones as the tangent friction is, but tangent force must be controlled to keep watch of rolling and compensation of gradient of environmental potential energy. In this regard, a different approach is explored in [8] to compensate the uncertainties of kinematic of the environment.

In this paper, it is assumed hemispherical shape of the deformable fingertip of a planar redundant robot arm in contact to a dynamical circular object, and a controller is proposed for the regulation of position, orientation, tangential and normal forces. The coupled system stands for an under-actuated constrained differential algebraic system, [13]. The controller is a transpose Jacobian-based Cartesian $PDF+g$ scheme, [12] (hereby acronym *PDF* stands for the usual *PD* control term plus a *F*orce control term, and g is gravity vector) that does not require any knowledge of deformation, neither normal nor tangential force. Subsequently, two cases are studied, when the rigid object is constrained at its axis and when it is constrained to roll on the floor. Finally, representative simulation studies illustrate the performance under various operational conditions.

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II. MODELING AND CONTROL OF A CIRCULAR OBJECT PINNED AT A PIVOT POINT

A. Kinematic Constraint and Force Model

Consider a redundant planar (2D) robot manipulator equipped with hemispherical soft fingertip with 3 degrees of freedom (DoF) in contact to a circular dynamic rigid object, which is pivoting at O_0, see Fig. 1. It is assumed initial

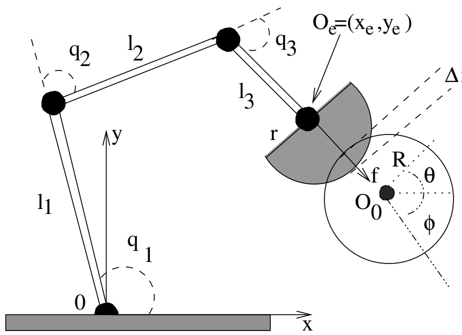

Fig. 1. Kinematic parameters of the circular object pivoting around O_0, where O is the origin of the world reference frame, $O_e = (x_e, y_e)$ is the center position of the base of the deformable soft fingertip of radius r. Furthermore, $\mathbf{q} = [q_1, q_2, q_3]^T$ stands for the generalized angular position coordinate, R is the radius of circular object and l_i is the length of each link i.

contact between the hemispherical lossless elastic fingertip and the rigid circular object. Then, the maximum radial deformation Δx of the tip arises when the force is applied along the line that connects O_o with O_e, hence it can be computed as follows

$$\begin{aligned} \Delta x &= r + R - R_0 \\ R_0 &= (x_0 - x_e)\cos(\theta) - (y_0 - y_e)\sin(\theta) \end{aligned} \tag{1}$$

In this way, force can be reasonably modeled as the square of the maximum radial deformation Δx of the fingertip, as it has been characterized and validated in experimental testing, [15], such that:

$$f(\Delta x) = k\Delta x^2, \tag{2}$$

where $k > 0$ stands for the stiffness depending on the soft material of the hemispherical fingertip. It is this shape that allows omnidirectional rolling onto the object, while a contact (ellipsoid) area is conformed to produce tangential forces as well as high contact friction force, which can be used to prevent slipping. These two forces stand for key characteristics for an improved manipulation, [4], [16], [7].

In order to guarantee that contact between the object and fingertip is maintained for all time, there arises a velocity constraint given by $R\dot{\theta} = -(r - \Delta x(t))\dot{\phi}$, [2], which can be written as as follows,

$$\dot{S} = R\dot{\theta} + (r - \Delta x(t))\dot{\phi} = 0 \tag{3}$$

where $\phi = \theta - \mathbf{q}^T\mathbf{e}$ and $\mathbf{e} = [1, -1, -1]^T$. However, notice that (3) is not a non-holonomic constraint because it is integrable, whose vector-valued solution in fact stands for the following *rolling* holonomic constraint,

$$S = R\theta + [r - \Delta x(t)]\phi - C_0(t) \equiv 0, \tag{4}$$

where $C_0(t) = R\theta(t_0) + [r - \Delta x(t)]\phi(t_0)$ represents the initial condition at contact. Thus, the configuration space $\mathscr{M}$ is conformed by the state of the pair robot-object given by $\mathbf{z} = (\mathbf{q}, \theta) \in R^4$ and $\dot{\mathbf{z}}$ that complies with two constraints $(S = 0, \dot{S} = 0) \in R^2$. In this manner, a 6 dimensional primary constrained manifold arises as follows

$$\mathscr{M}_6 = \{(\mathbf{z}, \dot{\mathbf{z}}) : S = 0, \dot{S} = 0\}$$

B. Constrained Dynamic Model

Let the constrained Lagrangian be $L_c = L_u + S^T\lambda$, where $L_u = K - P$ stands for the standard (unconstrained) Lagrangian, and λ for the Lagrangian multiplier, where K and P are the kinetic and potential energy, respectively. Each energy terms are composed of the summation of robot and object components, more precisely $K = 1/2\dot{\mathbf{q}}^T H(\mathbf{q})\dot{\mathbf{q}} + 1/2I\dot{\theta}^2$, for $H(\mathbf{q})$ and I the inertial positive definite (matrix) of the robot and the object inertia, respectively, and $P = P_g(\mathbf{q}) + P_c(\Delta x)$. Term $P_g(\mathbf{q})$ models the generalized energy induced by the Earth gravitational field and $P_d(\Delta x)$ stands the potential elastic deformation of the fingertip. Applying the variational principle, there arises the equation of motion of the robot manipulator as follows

$$\int_{t_1}^{t_2} \left[\delta L_u + S^T\lambda\,\delta\mathbf{q} - \mathbf{u}^T\delta\mathbf{q}\right] dt = 0$$

for $\mathbf{u}$ the generalized exogenous input forces. According to the Euler-Lagrange modeling formalism, one obtains

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\mathbf{q}}}\right] - \frac{\partial L}{\partial \mathbf{q}} + \frac{\partial S^T}{\partial \mathbf{q}}\lambda = \mathbf{u} \tag{5}$$

Accordingly, (5) becomes[1]

$$H(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + g_r = \mathbf{u} - \lambda J_S \tag{6}$$

$$I\ddot{\theta} + fY = u_\theta - \lambda(R + r - \Delta x) \tag{7}$$

where $g_r(\mathbf{q}) = g(\mathbf{q}) + g_c$, for $g(\mathbf{q}) = \partial P_g(\mathbf{q})/\partial \mathbf{q}$ and $g_c = \partial P_c(\Delta x)/\partial \Delta x = fJ^T(\mathbf{q})r_X$. Term $J_S = J(\mathbf{q})^T r_Y - (r - \Delta x)\mathbf{e}$, $r_X = [\cos(\theta), -\sin(\theta)]^T$, $r_Y = [\sin(\theta), \cos(\theta)]^T$ and $C(\mathbf{q}, \dot{\mathbf{q}}) \in R^{3\times 3}$ represents the matrix of centrifugal and Coriolis forces. The non-square Jacobian matrix of the manipulator $J(\mathbf{q}) \in R^{3\times 2}$ is evaluated at (x_e, y_e), scalar $Y = (x_e - x_o)\sin(\theta) + (y_e - y_0)\cos(\theta)$, and $\mathbf{u} \in R^3$ is the exogenous control input applied to each link of the robot manipulator. Notice that in the formulation (6)-(7) is completed with the holonomic constraint (4), for $u_\theta = 0$. System (6)-(7) yields an under actuated differential algebraic systems of equations of index 2, or DAE-2, [13][2].

We find useful to express it in a vector-matrix equation as follows

$$\mathbf{H}\ddot{\mathbf{z}} + \mathbf{C}\dot{\mathbf{z}} + \mathbf{G} + B\Lambda = \mathbf{u}_z \tag{8}$$

[1]Notice that nor joint neither object dissipative friction affine in velocity are included, but those can be modeled without altering the result of this paper.

[2]Equivalently to the notion of relative degree, where (4) is required to be derived twice to appear $\mathbf{u}$.

where

$$\mathbf{H} = diag(H(\mathbf{q}), I), \tag{9}$$
$$\mathbf{C} = diag(C(\mathbf{q}, \dot{\mathbf{q}}), 0), \tag{10}$$
$$\mathbf{G} = [g(\mathbf{q}), 0]^T, \tag{11}$$
$$\mathbf{u}_z = [\mathbf{u}, 0]^T, \tag{12}$$
$$\mathbf{z} = [\mathbf{q}^T, \theta]^T \tag{13}$$
$$\Lambda = [f, \lambda]^T, \tag{14}$$
$$B = \begin{bmatrix} J(\mathbf{q})^T r_X & J_S \\ Y & (R+r-\Delta x) \end{bmatrix}, \tag{15}$$

Surprisingly, [9], the integral of inner product between $\dot{\mathbf{z}}^T$ and (8) shows the passivity properties of the underactuated system (8), more precisely,

$$\int_0^t (\dot{\mathbf{z}}^T \mathbf{u}_z) d\tau = E(t) - E(0) \geq -E(0), \tag{16}$$

in virtue of $\frac{d}{dt}S = \dot{\mathbf{q}}^T \frac{\partial}{\partial \mathbf{q}} S + \dot{\theta} \frac{\partial}{\partial \theta} S = 0$, where $E(t) = K + P$. Thus, the passivity condition is satisfied in open loop[3]. Precisely this fact motivates [5], [9] to design a simple regulator, where damping injection is used to shape the one dimensional basic manifold of stability $\mathscr{M}_1 \subseteq \mathscr{M}_6$, however zeroing the tangent force.

C. Control Design

Let the control law be

$$\mathbf{u} = -K_v \dot{\mathbf{q}} + f_d J(\mathbf{q})^T r_X + \lambda_d J_S + g(\mathbf{q}) \tag{17}$$

where $\lambda_d = \frac{\beta}{R+r-\Delta x}\Delta\theta$, and β, α, K_v are positive feedback gains. Notice that by assumption the robot is in contact regime at any initial condition with $\Delta x > 0$, thus $R + r > \Delta x$, therefore (17) is well posed. Now, we have the following result.

Theorem 1. Consider the robot dynamics (6) in closed loop with control law (17). For small error on initial conditions, if the non-degeneracy condition of the following interaction matrix

$$\mathbf{D} = \begin{bmatrix} J^T(\mathbf{q}) r_X & J_S & 0_{3\times 1} \\ \omega & (R+r-\Delta x) & \beta \end{bmatrix}, \tag{18}$$

is satisfied for all time, then the convergence to the following basic manifold

$$\mathscr{M}_1 = \{(\mathbf{z}, \dot{\mathbf{z}} = 0) : S = 0, \dot{S} = 0, \Delta f = 0, \Delta\theta = 0\}$$

establishes the locally asymptotically convergence of $(\Delta_f, \Delta\theta, \Delta\theta) \to (0,0,0)$, with all closed-loop signals bounded.

Proof: Let a bounded region $\Omega = \{\mathbf{z} |\ \Delta x = \sqrt{\frac{f}{k}}, \mathbf{z} \in \mathscr{M}_3, q_2 \in]0, \pi[, q_3 \in]0, \pi[\}$ where $\Delta x > 0$, $\sqrt{\frac{f}{k}} \leq r$, $\theta \in]\frac{\pi}{4}, -\frac{\pi}{4}[$ and the Jacobian of the robot manipulator is well-defined. Now, substituting (17) in (8), we obtain

$$\mathbf{H}\ddot{\mathbf{z}} + \mathbf{C}\dot{\mathbf{z}} + \mathbf{D}\Lambda_f = \mathbf{F} \tag{19}$$
$$S = 0 \tag{20}$$

where

$$\mathbf{F} = \begin{bmatrix} -K_v \dot{\mathbf{q}} \\ 0 \end{bmatrix} \tag{21}$$

for $\Lambda_f = [\Delta f, \Delta\lambda, \Delta\theta]^T$, $\Delta f = f - f_d$, $\Delta\lambda = \lambda - \frac{\beta}{R+r-\Delta x}\Delta\theta$. For stability analysis purposes, a passivity analysis suggests to consider the following energy-based quadratic function

$$V_1(\mathbf{z}) = \frac{1}{2}\dot{\mathbf{q}}^T H(\mathbf{q})\dot{\mathbf{q}} + \frac{1}{2} I \dot{\theta}^2 + P_{\Delta f} + \frac{1}{2}\beta \Delta x^2$$

that its time derivative along the system (19) becomes

$$\dot{V}_1(\mathbf{z}) = -\dot{\mathbf{q}}^T K_v \dot{\mathbf{q}} + f_d Y \dot{\theta} \tag{22}$$

Since V_1 does not qualify as a Lyapunov function in the 8-dimensional space $(\mathbf{z}, \dot{\mathbf{z}})$, or in the primary constrained manifold M_6. Then, stability in the sense of Lyapunov cannot be concluded, however, certainty, V_1 is useful to analyse the convergence properties of the autonomous system (19). Applying the maximum invariance set to (22), at $\dot{V}_1(\mathbf{z}) = 0$, it leads to $(\dot{\mathbf{q}}, \dot{\theta}) = (0,0) \Rightarrow (\ddot{\mathbf{q}}, \ddot{\theta}) = (0,0)$, which means that there exists a maximum connected set, henceforth, (19) becomes

$$\mathbf{D}\Lambda_f = 0 \tag{23}$$

Thus, if exists a 3×3 sub-determinant that is not zero, it is easy to prove that matrix $\mathbf{D}$ is non-degenerate. Then, locally the solution of (19) is uniquely locally $\Lambda_f \to 0$. ■

III. Modeling and Control of a Circular Object Constrained by a Rigid Surface

A. Kinematic Constraint

Consider a rigid circular object rolling onto a rigid surface, as if it were a wheel, as depicted in Fig. 2. In this case, the rotational object axis is free to move along the plane, whose

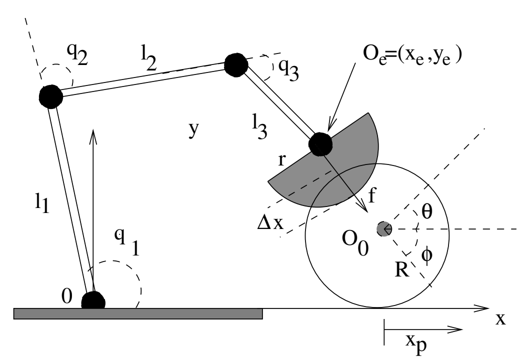

Fig. 2. Circular object manipulation over a rigid surface.

object translational velocity, which depends on its angular velocity, can be modeled simply as $\dot{x}_p = R\dot{\Theta}$. This gives rise to the following holonomic constraint

$$S_x = R\Theta - x_p + C_\theta = 0 \tag{24}$$

for C_θ the initial condition.

[3] It becomes dissipative if friction terms were present, as it is usually the case in practice.

B. Constrained Dynamic Model

Let the constrained Lagrangian be

$$L_c = L_u + S^T\lambda_1 + S_x^T\lambda_x \quad (25)$$

where $K = \frac{1}{2}\dot{\mathbf{q}}^T H(\mathbf{q})\dot{\mathbf{q}} + \frac{1}{2}I\theta^2 + \frac{1}{2}M\dot{x}^2$, $P = P_g(\mathbf{q}) + P_c(\Delta y)$ and λ_x as the Lagrangian multiplier representing physically the opposition force to the movement of the object onto the surface. Applying the variational principle to (25), and using the Euler-Lagrange formalism, we have that the dynamical equations of the system in fact constitutes an under actuated DAE-2 system as follows, that is

$$\begin{aligned} H(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + g(\mathbf{q}) + f_x J^T(\mathbf{q}) r_X & \\ +\lambda_1\left[J^T(q)r_Y - (r-\Delta x)\mathbf{e}\right] &= \mathbf{u}, & (26)\\ M\ddot{x}_p &= -\lambda_x & (27)\\ I\ddot{\theta} + f_x Y + \lambda_1(R+r-\Delta x) &= \lambda_x R & (28)\\ S_x &= 0 & (29)\end{aligned}$$

where λ_1 models the tangential force arising from the rolling contact constraint defined in (4). Notice that (26) models robot dynamics, (27) and (28) model translational and rotational object dynamics, respectively, and finally (29) yields the solution manifold as a holonomic constraint, that relates x_p and θ by a constant, see (24). Thus, we can choose either x_p or θ as a generalized coordinate such that (27)-(28) can be written, in virtue of (24),

$$\begin{aligned} H(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + g(\mathbf{q}) + f_x J^T(\mathbf{q}) r_X & \\ +\lambda_1\left[J^T(q)r_Y - (r-\Delta x)\mathbf{e}\right] &= \mathbf{u}, & (30)\\ (MR^2+I)\ddot{\theta} + f_x Y + \lambda_1(R+r-\Delta x) &= 0, & (31)\\ S_x &= 0 & (32)\end{aligned}$$

At this point, the control problem is to design $\mathbf{u}$ such that the triplet $(\Delta f_x, \Delta x_p, \Delta\lambda_x) \to (0.0.0)$ asymptotically when deformation, contact area and contact force are unknown.

C. Control Design

Let the following control law be

$$\mathbf{u} = -K_x\dot{\mathbf{q}} + f_d J(\mathbf{q})^T r_X + \lambda_{1d} J_S - g(\mathbf{q}) \quad (33)$$

where K_x, $\beta_x > 0$ are positive feedback gains, $f_d > 0$, $\lambda_{xd} = \frac{\beta_x}{R+r-\Delta x}\Delta x_p$ is the desired value for λ_1, $J_S = J^T(q)r_Y - (r-\Delta x)\mathbf{e}$ and $\Delta x_p = x - x_{pd}$ stands for the error position, with x and x_{pd} the real and desired position, respectively. Substituting (33) into (30), one obtains

$$H(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + K_x\dot{\mathbf{q}} = -\Delta f_x J^T(\mathbf{q}) r_X - \Delta\lambda_x J_S \quad (34)$$

$$(MR^2+I)\ddot{\theta} = -\Delta f_x Y - \Delta\lambda_x(R+r-\Delta x) - f_d Y - \beta_x\Delta x_p \quad (35)$$

where $\Delta f_x = f_x - f_{xd}$, $\Delta\lambda_x = \lambda_x - \lambda_{xd}$. We have now the following result.

Theorem 2. Consider the under actuated dynamics of the pair robot-object given in (30)-(31), subject to (32), in closed loop with control law (33). A stability on the basic manifold is obtained if the initial conditions start near of the desired equilibrium point $(\mathbf{z}_x, \mathbf{z}_{xd})$ under assumption that interaction matrix $\mathbf{D}_x \in R^{4\times 4}$

$$\mathbf{D}_x = \begin{bmatrix} J^T(\mathbf{q})r_X & J_S & 0_{3\times 1} \\ Y & (R+r-\Delta x) & \beta_x \end{bmatrix}, \quad (36)$$

is non-degenerate. Then, the convergence of $\Delta f_x \to 0$, $\Delta x_p \to 0$ and $\Delta\lambda_x \to 0$ is guaranteed locally asymptotically.

Proof: The passivity balance of (34) and (35) considering $\dot{\mathbf{z}}_x = [\dot{\mathbf{q}}, \dot{\theta}]^T$ as the output, shows up that

$$\dot{V}_x = -\dot{\mathbf{q}}^T K_x\dot{\mathbf{q}} - f_d Y\dot{\theta} \quad (37)$$

where $V_x = K + P_c(\Delta x) + \frac{1}{2}\beta\Delta x_p^2$. Notice that V_2 does not qualify as a Lyapunov function because it does not fulfill two axioms for that purposes ($V_2(0) \neq 0$ and the dependency of all the state of (34)), then no stability in the sense of Lyapunov can be concluded. In any case, since (34) is an autonomous system, the maximum invariance set shows that $\dot{V}_x = 0$ only at $(\dot{\mathbf{q}}, \dot{\theta}) = (0,0) \Rightarrow (\ddot{\mathbf{q}}, \ddot{\theta}) = (0,0)$, giving rise to the basic constrained manifold, by using (34) and (35),

$$\mathbf{D}_x\Lambda_x = 0 \quad (38)$$

where $\Lambda x = [\Delta f_x, \Delta\lambda_x, \Delta x_p]^T$, for the interaction matrix $\mathbf{D}_x \in R^{4\times 4}$. Clearly, if interaction matrix $\mathbf{D}_x$ is non-degenerate and under the assumption of small error in initial conditions, system trajectories converge locally to the basic manifold

$$\mathscr{M}_1 = \{(\mathbf{z}_x, \dot{\mathbf{z}}_x = 0) : S_x = 0, \Delta f_x = 0, \Delta x_p = 0\}$$

where $f_x \to f_d$, $x_p \to x_{pd}$, $\lambda_x \to \lambda_{xd} \Rightarrow 0$. ∎

IV. Simulation

A. The Simulator

A digital simulator implements a stiff numerical solver on Matlab 7.14, under a $1ms$ of sampling time, with a Constrained Stabilization Method (CSM) [17]. The physical parameters of the robot manipulator are shown in Table I where l_i, m_i and I_i are the length, mass and the inertia moment of the link, respectively. The parameters of the object are: $M = 0.1[Kg]$, $I = 5.67\times 10^{-6}[kgm^2]$ and $R = 0.022[m]$, representing mass, moment of inertia and radius, respectively. Moreover, the hemispherical finger parameters $k = 500[N/m^2]$, and $r = 0.02[m]$ are the stiffness of the soft material and the radius, respectively.

Table I. Physical parameters of the robot

(l_1, l_2, l_3)	$(0.05, 0.04, 0.03)$ $[m]$
(l_{c1}, l_{c2}, l_{c3})	$(0.025, 0.02, 0.015)$ $[cm]$
(m_1, m_2, m_3)	$(0.05, 0.03, 0.02)$ $[Kg]$
$(I_{11} I_{22} I_{33})$	$(14.167, 6.25, 3)\times 10^{-6}$ $[kgm^2]$

Initial conditions are such that the fingertip is in contact to the object, consistent to the DAE-2 formulation at $t = t_0$, and then $\Delta x > 0, f(t_0) > 0$. Two scenarios are considered now, depending on the constraint of the object, either its axis is pinned at a pivot point or free to move along the plane x, as if the object moves in the floor.

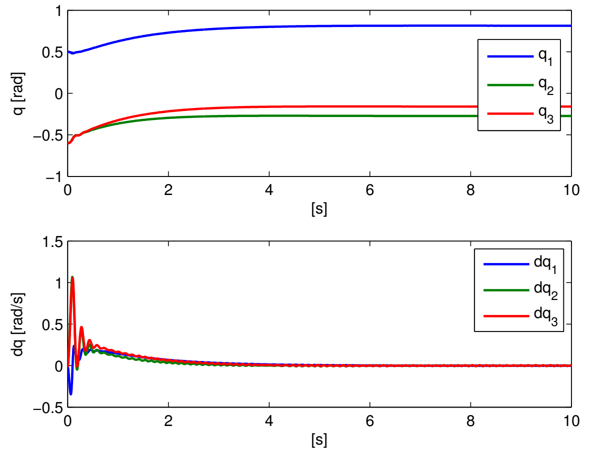

Fig. 3. Case 1: Joint position and velocity of the robot manipulator

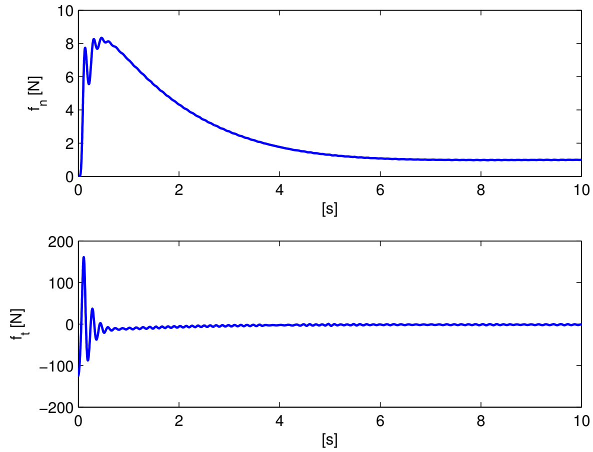

Fig. 5. Case 1: Convergence of normal and tangential forces. Notice that the shape of these forces are, as expected, highly correlated to deformation and angle transient, see Fig. 6.

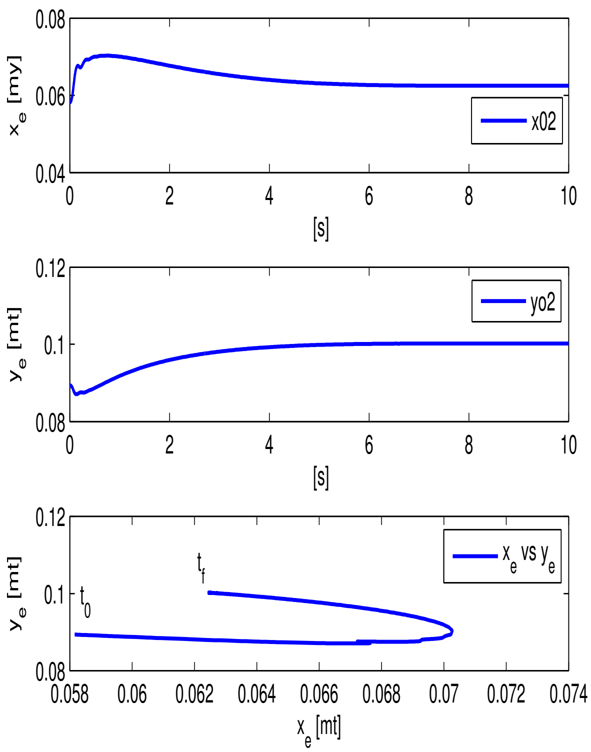

Fig. 4. Case 1: Displacement of rolling angle can be seen through how the center position of the base of the deformable soft finger-tip is moving, with the phase plane (x_e, y_e) is shown in the bottom.

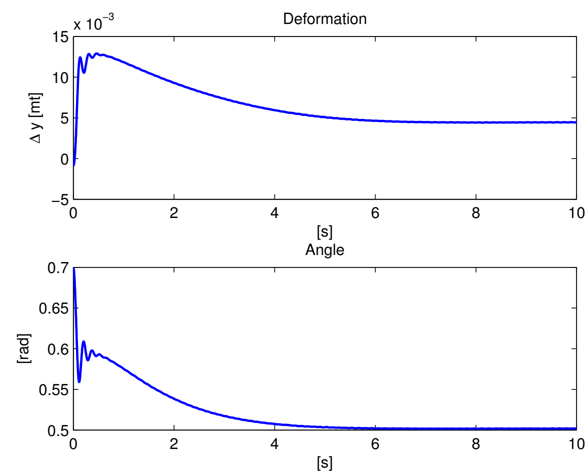

Fig. 6. Case 1: Soft finger-tip deformation (or penetration along its maximum radial deformation), top, and rolling angle convergence, bottom.

B. Case 1: Circular Object Pinned at a Pivot Point

1) Simulation conditions: The object is pinned at its rotation axis, and the objective is to roll the circular object to desired angle $\theta_d = 0.5[rad]$ from $\theta(t_0) = 0.7[rad]$, that is a precision rolling angle of 11.46 *deg*, while applying a desired force of $f_d = 1[N]$. The robot start motionless, with an initial angular position of $\mathbf{q}(0) = [0.5; -0.6, -0.6]^T[rad]$, for $diag(K_v) = 1.0 * I_{3\times3}$, $\beta = 25$.

2) Simulation results: Figure (3) shows smooth convergence of joint coordinates to the desired constant value, with a settling time of joint velocities to zero, in a very short time, about 4*sec*. Notice that Fig. 4 represents the soft finger-tip rolling onto the object surface to reach the desired angle while simultaneously is converging to the tangential and normal desired forces, see Fig 5. Finally, Fig. 6 show the penetration along the maximum deformation with a fast settling toward the desired angle. In accordance with these results, firstly the robot moves the object to the desired angle while a desired force is applied to avoid any movement. At the same time, the center position of the base of the deformable soft finger-tip is moving.

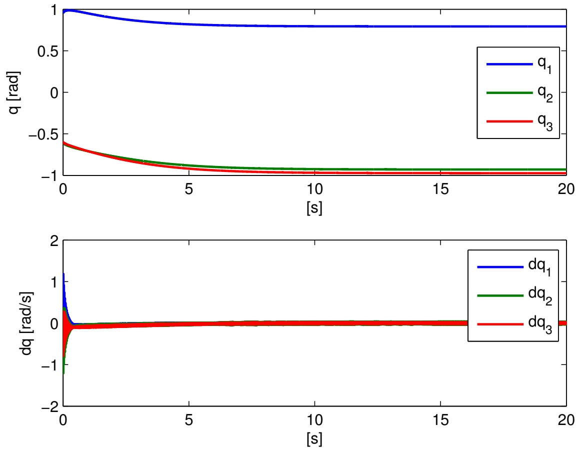

Fig. 7. Case 2: Joint position and velocities, the latter showing effort because the passive dynamic object is under actuated, as expected.

C. Case 2: Circular Object Constrained by a Rigid Surface

1) Simulation conditions: The fingertip manipulate the object that moves freely onto the floor. The objective is to roll the circular object to desired position $x_d = 0.028[mt]$ with a desired force $f_d = 3[N]$ applied to the object. The robot and object start motionless, with an initial angular position of $\mathbf{q}(0) = [1.2; -0.6, -0.5]^T[rad]$ and the initial and desired angle is defined as $\theta_d = 0.7[rad]$ from $\theta(t_0) = 0.6[rad]$.

2) Simulation results: As in the previous case, the movement of the center position of the base of the deformable soft finger-tip allows to roll the object onto the rigid surface, Fig 8. As Theorem 2 establishes, regulation of f and λ is obtained, see Fig. 9. Notice the effect of the tangential force in this more complex task, because now it contributes to rolling as well as preventing slipping. Finally, Fig. 10 shows the position error which is near to zero and the angle performance.

V. Final Remarks

Several concrete remarks are in order now:

- When deformation at end-effector is considered, it enables to manipulate dexterously a circular object in rolling tasks. In this paper, we extend previous judicious observations of [9] and [18], to show that stable rolling is possible with one finger robot with a simple regulator. The regulator structure is particularly simple in view of the complex non-linear under-actuated DAE-2 structure of the system. It stems essentially from the fact that passivity property, typical of robot manipulators in open loop, is remarkably preserved for this case, an fundamental property that plays a crucial role as intuitively expected and analyzed in the stability analysis, [5].
- Area contact enlarges the convex hull for easier inequality constraints of motion planning, [20], however it deserves a more detailed discussion whether the variable area contact is required for a more efficient planning.

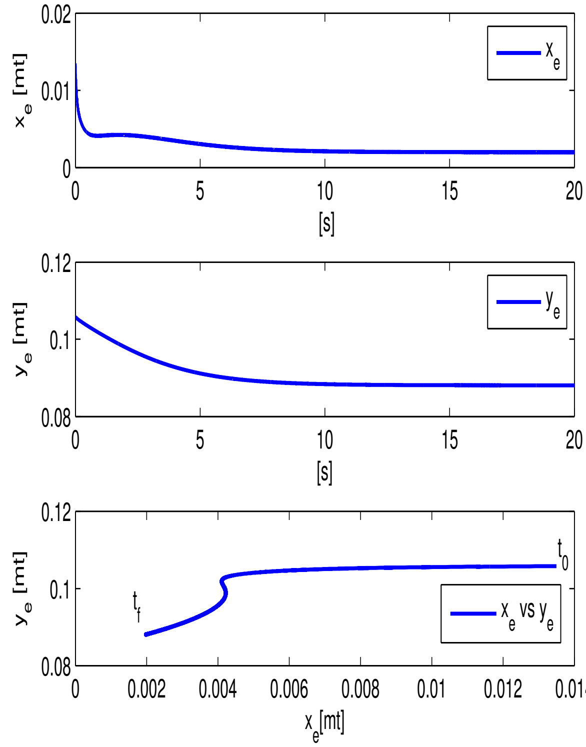

Fig. 8. Case 2: Smooth convergence to the regulation point by the deformable soft fingertip which is interpreted also as the hemispherical fingertip is rolling, with the phase plane (x_e, y_e) shown in the bottom.

- Under-actuation of the object is resolved similar to standard case of robotic hands, nonetheless the paradigm of virtual control can also be used, popular for underactuated quadrotors. In this case, normal and tangent forces can be declared as control inputs of the underactuated object. As such, the physical (real) controller guarantees convergence to the virtual controllers so as to the actuated robot dynamics is surrogated to the underactuated object dynamics. A subject that remains open in the literature.
- The potential impact of deformable fingertips to handle unilateral constraints and impacts with lesser complexity, as well as dealing with stable transition from/to free- to/from constrained-motion regimes, also for safe interaction, are open subjects. All these seem feasible because kinetic energy is converted into potential energy at deformation, [19], which under some conditions may lead to preserve analyticity of the ODE formulations.
- Two illustrative scenarios are studied in this paper, fixed and moving rotational dynamic object. The interplay of tangential force with rolling angle plays a relevant role to regulate stably posture since the desired tangent force depends on angle error or displacement error. Simulations are conceptually parametrized for a small robot and small rolling angles, though similar results

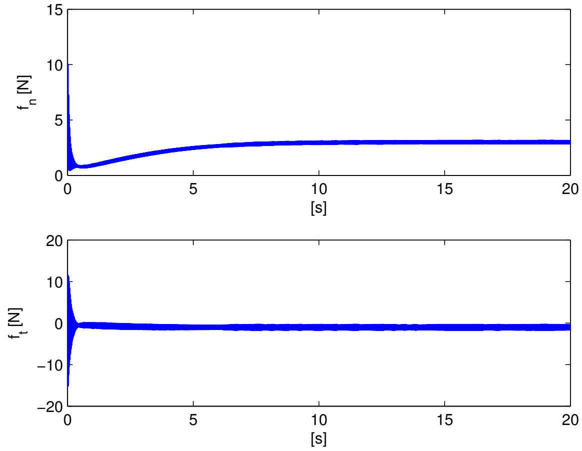

Fig. 9. Case 2: Normal force converges in about $10s$ while the tangential forces converges in few seconds to prevent slipage of the freely moving dynamic object. For the same reason, it shows a very small high frequency value coming from its corresponding position error along the tangent direction, see bottom of Fig. 10.

can be obtained for larger robots and larger object.

- Our proposal can be extended easily to other difficult tasks. Superior view of Fig. 1, from $\vec{z}$ perspective ($\vec{z}$ pointing outward the plane $(\vec{x},\vec{y})$ where Fig. 1 is depicted) can be interpreted similar to opening a door by pushing along the radial axis, as if touching a slender section of the circular object. Also, Figure 2 in 3D can be seen as if rolling a tube, however in this case it is clear that a non-holonomic Pfaffian-type constraint will be enforced, which can be addressed similar to [18].

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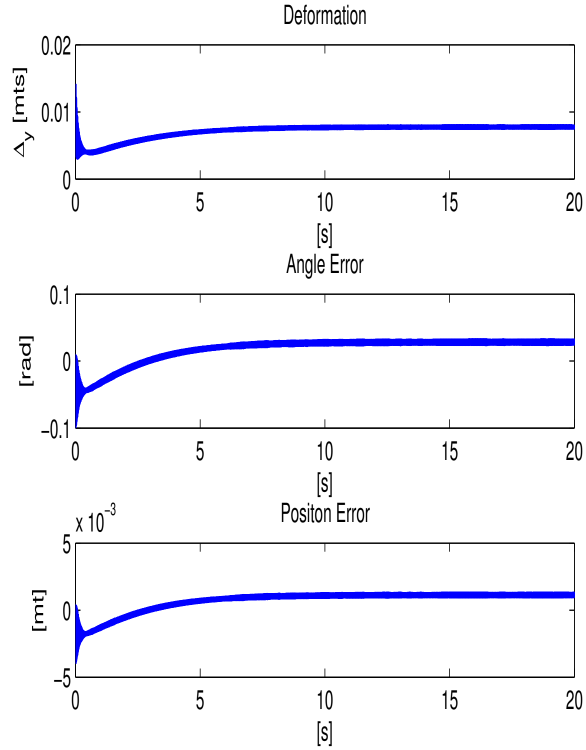

Fig. 10. Case 2: Soft finger-tip penetration along its maximum radial deformation, top, with the convergence of rolling angle, middle, and position error, bottom.

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