Physically Feasible Dynamic Parameter Identification of the 7-DOF WAM Robot

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Abstract—This paper presents the dynamic parameter identification of the 7-DOF WAM™ Arm using a novel physically consistent regression technique. Due to model and data errors, physically impossible parameters can arise with classical estimations methods. Such infeasible estimations cannot be used in robot control or simulation. This paper proposes a semidefinite programming (SDP) reformulation of the classical ordinary least squares method. This enables the inclusion of constraints guaranteeing physically feasible solutions only. The SDP method efficiently finds the feasible solution with the lowest regression error. Regression data processing issues related to the WAM robot are also addressed.

Index Terms—Dynamics, Calibration and Identification.

I. INTRODUCTION

This paper presents the dynamic model and parameter identification of the anthropomorphic 7-DOF WAM™ Arm, a lightweight robot manipulator with seven revolute joints. Knowledge of robot dynamics enables the design of advanced control techniques and robot dynamic simulation. A common method to estimate the dynamic parameters is through linear regression techniques based on commanded torques and joint position data. These methods are prone to errors that can compromise the physical feasibility of the estimated parameters, resulting in parameter values that are impossible to be real, e.g., negative masses. In [1] the non-linear physical feasibility conditions are formulated and a recursive method to check base parameter feasibility is proposed. In [2] it is proposed a method to estimate feasible parameters through the correction of a previous regression estimation. A nonlinear Bayesian parameter identification method is presented in [3], where regression optimization is done over a physically feasible virtual parameter space which has a nonlinear projection onto the classical parameter space. In [4] it is proposed an identification method which guarantees physical feasibility by approximating the robot model with mass points, thus converting feasibility constraints into linear constraints.

In this paper we propose novel techniques to address the physically feasible estimation problem, and apply them to the identification of WAM robot parameters. The dynamic model and its classical identification method are introduced in Section II, while the physical feasibility conditions are introduced in Section III. In Section IV, we rewrite the feasibility conditions as a linear matrix inequality (LMI), enabling the use of semidefinite programming (SDP) techniques. Then the classical regression is also reformulated as an SDP problem, and the feasibility constraints are merged into it. SDP enables the estimation of feasible optimal parameters efficiently, which is comparable to linear programming. WAM robot identification is presented in Section V. Control design and sensor data processing are discussed. Unlike common industrial robots, the WAM is commanded in joint torque. Moreover, due to the high backdrivability, the static friction has a relatively high effect which we reduce through selective data elimination. Dynamic parameters computed by both classical and proposed methods are presented. The paper is concluded in Section VI.

II. DYNAMIC PARAMETER IDENTIFICATION

For a robot with \( N \) links, the inverse dynamic model, which relates joint position \( q \) with joint torque \( \tau \) (\( N \) sized vectors), is given by

\[
M(q)\ddot{q} + c(q,\dot{q}) + g(q) = \tau ,
\]

where \( M(q) \) is the inertia matrix, \( c(q,\dot{q}) \) is the Coriolis and centripetal forces term, \( g(q) \) is the gravity force term, and \( \tau \) is the generalized torque. Considering that \( \tau \) includes motor and friction torques, \( \tau_c \) and \( f(q,\dot{q}) \) respectively, (1) can be rewritten as

\[
M(q)\ddot{q} + c(q,\dot{q}) + g(q) + f(q,\dot{q}) = \tau_c .
\]

In this work, only viscous and Coulomb frictions are modeled. The friction of the \( k \)-th joint is modeled by

\[
f_k(q,\dot{q}) = f_{vk} \dot{q}_k + f_{ck} \text{sgn}(\dot{q}_k) ,
\]

where \( f_{vk} \) and \( f_{ck} \) are constants for viscous and Coulomb frictions, respectively. Besides friction parameters, the dynamic model is also linearly dependent on the inertial parameters. For each link \( k \), these parameters are the mass, \( m_k \), the first moment of inertia, \( I_k \), and the inertia tensor about link frame \( L_k \). The first moment of inertia is given by

\[
l_k = m_k r_k ,
\]

where \( r_k \) is the center of mass position relative to the link frame. By the Huygens–Steiner theorem, the tensor \( L_k \) is given by

\[
L_k = I_k + m_k S(r_k)^T S(r_k) ,
\]

where \( I_k \) is the inertia tensor about the center of mass and \( S(\cdot) \) is the skew-symmetric matrix operator. The dynamic model (2) can be written in the linear to parameters form as

\[
H(q,\dot{q},\ddot{q}) \delta = \tau_c .
\]
The vector $\delta$ is a vector of size $n = 12N$ which combines all link dynamic parameters,
\[
\delta = \begin{bmatrix}
\delta_1^T & \delta_2^T & \cdots & \delta_k^T & \cdots & \delta_N^T
\end{bmatrix}^T ,
\] (7)
where each $\delta_k$ is composed by the unique elements of the inertia tensor, the first moment of inertia elements, the mass and the friction parameters,
\[
\delta_k = [L_{k,xx} \; L_{k,xy} \; L_{k,xz} \; L_{k,yx} \; L_{k,yz} \; L_{k,zx} \; l_{k,x} \; l_{k,y} \; l_{k,z} \; m_k \; f_{vk} \; f_{ck}]^T .
\] (8)
The vector $\delta$ can be estimated through a linear regression by minimizing the residual error $\epsilon$ in
\[
H_S \delta + \epsilon = \omega ,
\] (9)
where $H_S$ is the regressor matrix (size $NS \times n$) obtained by staking $H(q, \dot{q}, \ddot{q})$ matrices evaluated at a large $S$ number of robot postures (joint position, velocity and acceleration), and $\omega$ is obtained by stacking the $\tau_c$ vectors measured at those postures. The classical approach to solve this problem is the ordinary least squares (OLS) minimization whose optimal value $\hat{\delta}$ verifies
\[
(H_S^T H_S) \hat{\delta} = H_S^T \omega .
\] (10)

Since some parameters have no effect on the robot dynamics, and other parameters have linearly proportional effect, the matrix $H_S$ has null and linearly dependent columns, entailing that $H_S^T H_S$ is singular and that there are multiple $\delta$ solutions. To overcome this problem it is usual to eliminate and regroup parameters into the base parameter vector $\beta$,
\[
\beta = \delta_b + K_d \delta_d ,
\] (11)
where
\[
\delta_b = P_b^T \delta
\] (12)
and
\[
\delta_d = P_d^T \delta .
\] (13)
The matrices $P_b^T$ and $P_d^T$ are truncated permutation matrices which select the independent parameters, $\delta_b$, and the dependent parameters, $\delta_d$, from $\delta$. The dependent parameters are grouped into the independent ones by the dependencies matrix $K_d$. These matrices can be obtained by numerical methods [5] or by rule based methods [6], [7]. Equation (9) is then rewritten as
\[
W \beta + \epsilon = \omega ,
\] (14)
where $W$ is obtained by eliminating the dependent columns of $H_S$. The regression problem can be written as
\[
\text{minimize }_{(\beta, \delta)} \| \omega - W \beta \|^2 ,
\] (15)
whose optimal solution is given by
\[
\hat{\beta} = (W^T W)^{-1} W^T \omega .
\] (16)

III. DYNAMIC PARAMETER PHYSICAL FEASIBILITY

Dynamic parameters represent physical properties which are limited to physically feasible values. The use of physically infeasible estimations lead to unrealistic simulation and intrinsically unstable control [1]. Physically feasible masses shall be positive, inertia tensors shall be positive definite, and friction gains shall be positive,
\[
\begin{cases}
  m_k > 0 \\
  I_k > 0 \\
  f_{vk} > 0 \\
  f_{ck} > 0
\end{cases}
\text{ for } k = 1, \cdots , N .
\] (17)

From (4) and (5), the second condition\(^1\), $I_k > 0$, can be rewritten as
\[
L_k - S (l_k)^T m_k^{-1} S (l_k) > 0 ,
\] (18)
which also implies that $L_k$ is positive definite. The physical feasibility condition can be written in terms of $\delta$, hence the set of $\delta$ vectors which verify it can be defined by
\[
D = \{ \delta \in \mathbb{R}^n : L_k - S (l_k)^T m_k^{-1} S (l_k) > 0 , \\
  m_k > 0 , \; f_{vk} > 0 , \; f_{ck} > 0 , \\
  \text{ for } k = 1, \cdots , N \} .
\] (19)

Given a $\delta$ estimation, its feasibility can be directly checked with (17) and (18). Nevertheless, estimations are done in $\beta$ space and for each $\beta$ solution there are an infinite number of corresponding $\delta$ solutions (the map from $\delta$ to $\beta$ spaces through (11) is not bijective). If for a given $\beta$ estimation there is at least one feasible $\delta$ which maps to it, then such estimation is feasible [1].

IV. PHYSICALLY FEASIBLE PARAMETER ESTIMATION

We can constrain the estimation (15) to the physically feasible solution space doing
\[
\text{minimize }_{\beta} \| \omega - W \beta \|^2 \\
\text{subject to } \beta = K \delta
\] (20)
where
\[
K = P_b^T + K_d P_d^T ,
\] (21)
thus guaranteeing that the solution is the feasible one which best fits the regression. In a practical view point, (20) has no simple solution. However, as we will show in the sequel, the set $D$ can be defined by a linear matrix inequality (LMI), proving that it is a convex set ready to be used in semidefinite programming (SDP). This entails optimal global solutions and enables efficient solving methods.

\(^1\) $M \succ 0$ means that matrix $M$ is positive definite.
A. LMI Formulation of the Physically Feasible Conditions

The left-hand side of (18) represents the Schur complement of \( m_k \mathcal{I} \) in the block matrix \( D_k(\delta_k) \) defined as

\[
D_k(\delta_k) = \begin{bmatrix}
L_k & S(l_k)^T \\
S(l_k) & m_k \mathcal{I}
\end{bmatrix},
\]

(22)

where \( \mathcal{I} \) is the identity matrix \([8]\). Making use of Schur complement condition for positive definite matrices we know that (18) is in fact equivalent to

\[
D_k(\delta_k) \succ 0,
\]

(23)

which implicitly entails the condition \( m_k > 0 \). Extending \( D_k(\delta_k) \) to include the friction gains into a matrix \( E_k(\delta_k) \) defined as

\[
E_k(\delta_k) = \begin{bmatrix}
D_k(\delta_k) & 0 \\
0 & f_{kk}\end{bmatrix},
\]

(24)

we can write the set \( \mathcal{D} \) as

\[
\mathcal{D} = \{ \delta \in \mathbb{R}^n : E_k(\delta_k) \succ 0, \text{ for } k = 1, \cdots, N \}.
\]

(25)

Collapsing all \( E_k(\delta_k) \) matrices into a single block matrix \( E(\delta) \),

\[
E(\delta) = \begin{bmatrix}
E_1(\delta_1) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & E_N(\delta_N)
\end{bmatrix},
\]

(26)

we get

\[
\mathcal{D} = \{ \delta \in \mathbb{R}^n : E(\delta) \succ 0 \}.
\]

(27)

Since all elements of \( E(\delta) \) are linear combinations of \( \delta \), the condition

\[
E(\delta) \succ 0
\]

(28)

is an LMI. LMIs define a class of convex sets which can be used in SDP.

B. LMI Formulation of the Regression Error

The OLS optimization function of (20) can also be put in an LMI perspective. Defining a scalar \( u \) as being an upper limit to the regression error,

\[
u \geq \| \omega - W\beta \|^2,
\]

(29)

one can write

\[
u - (\omega - W\beta)^T \mathcal{I}^{-1} (\omega - W\beta) \geq 0,
\]

(30)

which, by Schur complements, can be written in LMI form²,

\[
\begin{bmatrix}
u \\
\omega - W\beta
\end{bmatrix} \begin{bmatrix}
(\omega - W\beta)^T \\
\mathcal{I}
\end{bmatrix} \geq 0.
\]

(31)

In this form, the LMI matrix has a size as big as the number of data points. However, the size can be reduced performing the QR decomposition of \( W \). Knowing that

\[
W = QR = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} R_1 \\
0 \end{bmatrix} = Q_1 R_1,
\]

(32)

it is possible to write the equivalence

\[
\| \omega - W\beta \|^2 = u_o + \| \rho_1 - R_1\beta \|^2,
\]

(33)

where

\[
\rho_1 \equiv Q_1^T \omega,
\]

(34)

and \( u_o \) is the optimization function value at the optimum,

\[
u_o = \|Q_2^T \omega\|^2.
\]

(35)

Equation (31) is then equivalent to

\[
\begin{bmatrix}
u - u_o \\
\rho_1 - R_1\beta
\end{bmatrix} \begin{bmatrix}
\mathcal{I} \\
\mathcal{I}
\end{bmatrix} \geq 0.
\]

(36)

This LMI is nonstrict (the matrix is positive semidefinite) while the LMI for the physical feasibility constraint (28) is strict (positive definite). In a practical sense, it is possible to write a nonstrict version of (28) as

\[
E(\delta) \geq 0,
\]

(37)

by subtracting an infinitesimally small positive scalar \( \varepsilon \) to each \( E(\delta) \) diagonal element,

\[
\bar{E}(\delta) = E(\delta) - \varepsilon \mathcal{I}.
\]

(38)

The nonstrict formulation is advantageous since it is the standard for SDP.

C. SDP Formulation of the Constrained Regression

Both LMIs can now be merged into a single one. Letting the matrix \( F(u, \beta, \delta) \) be defined as

\[
F(u, \beta, \delta) = \begin{bmatrix} U(u, \beta) & 0 \\
0 & E(\delta) \end{bmatrix},
\]

(39)

where

\[
U(u, \beta) = \begin{bmatrix} u - u_o \\
\rho_1 - R_1\beta \\
\mathcal{I}
\end{bmatrix},
\]

(40)

we can write problem (20) as

\[
\begin{array}{ll}
\text{minimize} & u \\
\text{subject to} & \beta = K\delta \\
& F(u, \beta, \delta) \geq 0.
\end{array}
\]

(41)

This problem includes both \( \beta \) and \( \delta \) solution spaces, and the map \( \beta = K\delta \) between them. Rewriting \( F \) in term of \( u \) and \( \delta \) only,

\[
F(u, \delta) = \begin{bmatrix} U(u, \delta) & 0 \\
0 & E(\delta) \end{bmatrix},
\]

(42)

where

\[
U(u, \delta) = \begin{bmatrix} u - u_o \\
\rho_1 - R_1 K \delta \\
\mathcal{I}
\end{bmatrix},
\]

(43)

we can turn the original problem (20) into

\[
\begin{array}{ll}
\text{minimize} & u \\
\text{subject to} & F(u, \delta) \geq 0,
\end{array}
\]

(44)

which is a typical SDP problem. For this problem, as for (10), there are multiple \( \delta \) optimal solutions. Being \( \delta^* \) one of

\[
\]

² \( M \succeq 0 \) means that matrix \( M \) is positive semidefinite.
the optimal solutions, we know that it is a physically feasible solution with minimal regression error. Such $\delta^\ast$ has only meaning as a whole, since some of its elements can take arbitrary values. Nevertheless, all the $\delta^\ast$ multiple solutions map to single optimal base parameter vector $\beta^\ast$,

$$\beta^\ast = K\delta^\ast .$$  \hfill (45)

Vector $\beta^\ast$ is the physically feasible base parameter solution which minimizes the regression error.

V. 7-DOF WAM ROBOT DYNAMIC PARAMETER IDENTIFICATION

In this section we present the methodologies and results obtained in the 7-DOF WAM robot parameter identification. The robot dynamic model (2) and the base dynamic parameters have been computed using the SageRobotics open-source software [9]. The base parameter combinations are shown in the first column of Table I.

A. Exciting Trajectory

Parameter identification requires a joint trajectory to generate data points (position and torque) and a criterion to evaluate data robustness. To achieve good estimations, trajectories must excite dynamic parameters as much as possible. The condition number (i.e., the ratio between maximum and minimum singular values) of the regressor matrix $W$ can be used as a trajectory evaluation criterion [10]. Well conditioned trajectories entail small condition numbers. In this work, we choose to generate the exciting trajectory by the method proposed by Swevers et al. [11], using regressor matrix condition number as excitation measure. Each joint $k$ trajectory is defined by

$$q_k(t) = \sum_{l=1}^{L} \frac{a_{k,l}}{\omega_f} \sin(\omega_f l t) - \frac{b_{k,l}}{\omega_f} \cos(\omega_f l t) + q_{k0}, \hfill (46)$$

where $t$ is the time variable and $\omega_f$ is the fundamental angular frequency. Parameters $a_{k,l}, b_{k,l}$ and $q_{k0}$ must minimize the reference trajectory condition number, entailing a nonlinear optimization problem with $2L + 1$ free variables per joint. In our setup, parameter $L$ has been set to 5 and parameter $\omega_f$ to 0.1\pi. Since robot joint positions, velocities and accelerations have limited ranges, cost constraint functions have been used to confine the generated trajectory. To generate the exciting trajectory a constrained nonlinear optimization by linear approximation (COBYLA) has been used.

B. Control for Trajectory Tracking

Unlike common industrial robots, the WAM enables the design of computed torque controllers. Since the robot CAD model is publicly available, an a priori estimation of inertial parameters $(m_k, l_k$ and $I_k)$ can be obtained (see second column of Table I). Such information has been used to design and implement a trajectory tracking control to perform the excitation trajectory. The dynamic model enables feedback linearization techniques in joint space,

$$\tau_c = \hat{g}(q) + \hat{c}(q, \dot{q}) + \hat{M}(q) \alpha , \hfill (47)$$

where $\tau_c$ is the computed torque sent to joint actuators, $\hat{g}(q)$, $\hat{c}(q, \dot{q})$ and $\hat{M}(q)$ are estimations of $g(q)$, $c(q, \dot{q})$ and $M(q)$ using CAD inertial parameters, respectively. Vector $\alpha$ is the desired acceleration. Neglecting friction and estimation errors, the free space plant for each joint is given by (see (2) and (47))

$$\hat{\ddot{q}}_k = \alpha_k , \hfill (48)$$

This is equivalent to a double integrator over which a controller can be designed. No complete knowledge of how the command is transformed into real motor torque is available, however high frequencies are likely to be heavily filtered. By this reason a proportional and derivative (PD) controller with low gains has been chosen rather than a controller with higher dynamic response. Although trajectories are followed with less accuracy, the frequency in torque command is decreased entailing better regression data. The chosen control scheme is depicted in Fig. 1. Proportional and derivative gains have been tuned for critically damped response.

C. Experimental Data Processing

Having the designed control and the generated excitation trajectory, we have performed a 60 seconds experiment (repeating the trajectory 3 times) for which we have recorded actuator and sensor data. The WAM provides joint position measurements but no acceleration nor velocity explicit data. First and second orders derivatives of position have been computed so that the regressor matrix could be calculated. Signals have been filtered with third order low-pass Butterworth filters with cut frequency $f_c = 10\omega_f L/(2\pi)$. Since this process has been done offline, phase distortion has been compensated. With this process a regression data set ($W$ and $\omega$, see (15)) has been obtained.

In the proposed model, static friction is not taken into account, thus the regression is affected by this unmodeled effect. Nevertheless, the static friction effect shows up only when velocity is zero or near zero. If data points with close to zero velocities are removed from the data set, such reduced data set is likely to entail better structural parameter estimation. We have tested the elimination of data points below several velocity thresholds, comparing the
<table>
<thead>
<tr>
<th>β (parameter combination)</th>
<th>CAD β_{CAD}</th>
<th>OLS β</th>
<th>Feasible LS β*</th>
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<td>( L_{1yy} + L_{2zz} )</td>
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<td>-0.046713</td>
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<td>0.000063</td>
<td>-0.002980</td>
<td>0.002389</td>
</tr>
<tr>
<td>( L_{5yz} )</td>
<td>0.000001</td>
<td>0.001821</td>
<td>0.000815</td>
</tr>
<tr>
<td>( l_{5x} )</td>
<td>0.000011</td>
<td>-0.014509</td>
<td>-0.014906</td>
</tr>
<tr>
<td>( l_{5z} + l_{6y} )</td>
<td>-0.006580</td>
<td>-0.013968</td>
<td>-0.013277</td>
</tr>
<tr>
<td>( f_{v5} )</td>
<td>—</td>
<td>0.229574</td>
<td>0.220504</td>
</tr>
<tr>
<td>( f_{c5} )</td>
<td>—</td>
<td>-0.071959</td>
<td>-0.058760</td>
</tr>
<tr>
<td>( L_{6xx} - L_{6zz} + L_{7yy} + 0.12 l_{7z} + 0.0036 m_7 )</td>
<td>0.000615</td>
<td>0.022288</td>
<td>0.014335</td>
</tr>
<tr>
<td>( L_{6xy} )</td>
<td>-0.000001</td>
<td>-0.004926</td>
<td>-0.004153</td>
</tr>
<tr>
<td>( L_{6xz} )</td>
<td>0.000002</td>
<td>0.000150</td>
<td>-0.003967</td>
</tr>
<tr>
<td>( L_{6yz} + L_{7yy} + 0.12 l_{7z} + 0.0036 m_7 )</td>
<td>0.007578</td>
<td>0.001841</td>
<td>0.004626</td>
</tr>
<tr>
<td>( L_{6yz} )</td>
<td>0.000222</td>
<td>-0.004198</td>
<td>0.001502</td>
</tr>
<tr>
<td>( l_{6x} )</td>
<td>-0.000051</td>
<td>0.000686</td>
<td>0.003062</td>
</tr>
<tr>
<td>( l_{6z} + l_{7z} + 0.06 m_7 )</td>
<td>0.014214</td>
<td>0.015821</td>
<td>0.014443</td>
</tr>
<tr>
<td>( f_{v6} )</td>
<td>—</td>
<td>0.161594</td>
<td>0.153190</td>
</tr>
<tr>
<td>( f_{c6} )</td>
<td>—</td>
<td>0.041832</td>
<td>0.043348</td>
</tr>
<tr>
<td>( L_{7xx} - L_{7yy} )</td>
<td>-0.000000</td>
<td>0.005531</td>
<td>0.000601</td>
</tr>
<tr>
<td>( L_{7xy} )</td>
<td>0.000000</td>
<td>0.000603</td>
<td>-0.000133</td>
</tr>
<tr>
<td>( L_{7xz} )</td>
<td>-0.000000</td>
<td>0.005237</td>
<td>0.003771</td>
</tr>
<tr>
<td>( L_{7yz} )</td>
<td>0.000000</td>
<td>0.002409</td>
<td>-0.000120</td>
</tr>
<tr>
<td>( L_{7zz} )</td>
<td>0.000074</td>
<td>0.002256</td>
<td>0.003213</td>
</tr>
<tr>
<td>( l_{7x} )</td>
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<td>0.009813</td>
<td>0.007886</td>
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<tr>
<td>( l_{7y} )</td>
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<td>-0.004071</td>
<td>-0.001545</td>
</tr>
<tr>
<td>( f_{v7} )</td>
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<td>0.020896</td>
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<tr>
<td>( f_{c7} )</td>
<td>—</td>
<td>0.089916</td>
<td>0.075723</td>
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</table>
Table I. This estimation has been used to compute inertia estimations are intrinsically unstable.

\[ \beta_0 \] is a base parameter solution in less than one second. The physically feasible available SDPA software [13]. Such software finds the inertia matrix is enough to prove physical infeasibility. In fact, a single non positive definite \( \beta^\star \) of them, the inertia matrix is not positive definite, therefore the error of \( \beta^\star \) parameter is not physically feasible. In fact, a single non positive definite inertia matrix is enough to prove physical infeasibility.

An estimation has then been performed by the “Feasible Least Squares” method proposed in Section IV (SDP problem (44)). The solution \( \delta^\star \) has been obtained using the freely available SDPA software [13]. Such software finds the solution in less than one second. The physically feasible base parameter solution \( \beta^\star \), shown in the rightmost column of Table I, is then obtained from \( \delta^\star \) through (45). The empirical evaluation at random postures has always given positive definite inertia matrices. The regression MSE of \( \beta^\star \) is a well conditioned [12].

D. Results

Given the reduced regression data set, the dynamic base parameter \( \beta^\star \) estimated by (15), i.e., unconstrained with respect to physical feasibility, is presented in the third column of Table I. This estimation has been used to compute inertia matrix values at thousands of random robot postures. For all of them, the inertia matrix is not positive definite, therefore \( \beta^\star \) is not physically feasible. In fact, a single non positive definite inertia matrix is enough to prove physical infeasibility.

In this work the dynamic parameters of the WAM robot have been identified. Physical feasibility of the estimated parameters is guaranteed by a new proposed regression method. Such method reformulates both the ordinary least squares and the feasibility constraints into the LMI–SDP framework. This enables efficient estimation of dynamic parameters, providing the physically feasible solution which better fits regression data. Practical identification issues, including the elimination of static friction effects from the data set have been discussed.

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REFERENCES