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Physically Feasible Dynamic Parameter Identification of the 7-DOF WAM Robot

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***Abstract*— This paper presents the dynamic parameter identification of the 7-DOF WAM™Arm using a novel physically consistent regression technique. Due to model and data errors, physically impossible parameters can arise with classical estimations methods. Such infeasible estimations cannot be used in robot control or simulation. This paper proposes a semidefinite programming (SDP) reformulation of the classical ordinary least squares method. This enables the inclusion of constraints guaranteeing physically feasible solutions only. The SDP method efficiently finds the feasible solution with the lowest regression error. Regression data processing issues related to the WAM robot are also addressed.**

***Index Terms*— Dynamics, Calibration and Identification.**

I. INTRODUCTION

This paper presents the dynamic model and parameter identification of the anthropomorphic 7-DOF WAM™Arm, a lightweight robot manipulator with seven revolute joints. Knowledge of robot dynamics enables the design of advanced control techniques and robot dynamic simulation. A common method to estimate the dynamic parameters is through linear regression techniques based on commanded torques and joint position data. These methods are prone to errors that can compromise the physical feasibility of the estimated parameters, resulting in parameter values that are impossible to be real, e.g., negative masses. In [1] the non-linear physical feasibility conditions are formulated and a recursive method to check base parameter feasibility is proposed. In [2] it is proposed a method to estimate feasible parameters through the correction of a previous regression estimation. A nonlinear Bayesian parameter identification method is presented in [3], where regression optimization is done over a physically feasible virtual parameter space which has a nonlinear projection onto the classical parameter space. In [4] it is proposed an identification method which guarantees physical feasibility by approximating the robot model with mass points, thus converting feasibility constraints into linear constraints.

In this paper we propose novel techniques to address the physically feasible estimation problem, and apply them to the identification of WAM robot parameters. The dynamic model and its classical identification method are introduced in Section II, while the physical feasibility conditions are introduced in Section III. In Section IV, we rewrite the feasibility conditions as a linear matrix inequality (LMI), enabling the use of semidefinite programming (SDP) techniques. Then the classical regression is also reformulated as an SDP problem, and the feasibility constraints are merged into it. SDP enables the estimation of feasible optimal parameters efficiently, which is comparable to linear programming. WAM robot identification is presented in Section V. Control design and sensor data processing are discussed. Unlike common industrial robots, the WAM is commanded in joint torque. Moreover, due to the high backdrivability, the static friction has a relatively high effect which we reduce through selective data elimination. Dynamic parameters computed by both classical and proposed methods are presented. The paper is concluded in Section VI.

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II. DYNAMIC PARAMETER IDENTIFICATION

For a robot with N links, the inverse dynamic model, which relates joint position q with joint torque τ (N sized vectors), is given by

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) = \tau \ , \tag{1}$$

where $M(q)$ is the inertia matrix, $c(q,\dot{q})$ is the Coriolis and centripetal forces term, $g(q)$ is the gravity force term, and τ is the generalized torque. Considering that τ includes motor and friction torques, τ_c and $f(q,\dot{q})$ respectively, (1) can be rewritten as

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) + f(q,\dot{q}) = \tau_c \ . \tag{2}$$

In this work, only viscous and Coulomb frictions are modeled. The friction of the k-th joint is modeled by

$$f_k(q,\dot{q}) = f_{vk}\,\dot{q}_k + f_{ck}\,\mathrm{sgn}\left(\dot{q}_k\right) \ , \tag{3}$$

where f_{vk} and f_{ck} are constants for viscous and Coulomb frictions, respectively. Besides friction parameters, the dynamic model is also linearly dependent on the inertial parameters. For each link k, these parameters are the mass, m_k, the first moment of inertia, l_k, and the inertia tensor about link frame L_k. The first moment of inertia is given by

$$l_k = m_k r_k \ , \tag{4}$$

where r_k is the center of mass position relative to the link frame. By the Huygens–Steiner theorem, the tensor L_k is given by

$$L_k = I_k + m_k \mathrm{S}\left(r_k\right)^T \mathrm{S}\left(r_k\right) \ , \tag{5}$$

where I_k is the inertia tensor about the center of mass and $\mathrm{S}\left(\cdot\right)$ is the skew-symmetric matrix operator. The dynamic model (2) can be written in the linear to parameters form as

$$H(q,\dot{q},\ddot{q})\,\delta = \tau_c \ . \tag{6}$$

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The vector δ is a vector of size $n = 12N$ which combines all link dynamic parameters,

$$\delta = \begin{bmatrix} \delta_1^T & \delta_2^T & \cdots & \delta_k^T & \cdots & \delta_N^T \end{bmatrix}^T , \tag{7}$$

where each δ_k is composed by the unique elements of the inertia tensor, the first moment of inertia elements, the mass and the friction parameters,

$$\begin{aligned} \delta_k = [&L_{k,xx}\ L_{k,xy}\ L_{k,xz}\ L_{k,yy}\ L_{k,yz}\ L_{k,zz} \\ &l_{k,x}\ l_{k,y}\ l_{k,z}\ m_k\ f_{vk}\ f_{ck}]^T \ . \end{aligned} \tag{8}$$

The vector δ can be estimated through a linear regression by minimizing the residual error ϵ in

$$H_S\, \delta + \epsilon = \omega \ , \tag{9}$$

where H_S is the regressor matrix (size $NS \times n$) obtained by staking $H(q, \dot{q}, \ddot{q})$ matrices evaluated at a large S number of robot postures (joint position, velocity and acceleration), and ω is obtained by stacking the τ_c vectors measured at those postures. The classical approach to solve this problem is the ordinary least squares (OLS) minimization whose optimal value $\hat{\delta}$ verifies

$$({H_S}^T H_S)\, \hat{\delta} = {H_S}^T \omega \ . \tag{10}$$

Since some parameters have no effect on the robot dynamics, and other parameters have linearly proportional effect, the matrix H_S has null and linearly dependent columns, entailing that ${H_S}^T H_S$ is singular and that there are multiple $\hat{\delta}$ solutions. To overcome this problem it is usual to eliminate and regroup parameters into the base parameter vector β,

$$\beta = \delta_b + K_d \delta_d \ , \tag{11}$$

where

$$\delta_b = {P_b}^T \delta \tag{12}$$

and

$$\delta_d = {P_d}^T \delta \ . \tag{13}$$

The matrices ${P_b}^T$ and ${P_d}^T$ are truncated permutation matrices which select the independent parameters, δ_b, and the dependent parameters, δ_d, from δ. The dependent parameters are grouped into the independent ones by the dependencies matrix K_d. These matrices can be obtained by numerical methods [5] or by rule based methods [6], [7]. Equation (9) is then rewritten as

$$W\beta + \epsilon = \omega \ , \tag{14}$$

where W is obtained by eliminating the dependent columns of H_S. The regression problem can be written as

$$\underset{\beta}{\text{minimize}} \quad \|\omega - W\beta\|^2 \ , \tag{15}$$

whose optimal solution is given by

$$\hat{\beta} = \left(W^T W\right)^{-1} W^T \omega \ . \tag{16}$$

III. Dynamic Parameter Physical Feasibility

Dynamic parameters represent physical properties which are limited to physically feasible values. The use of physically infeasible estimations lead to unrealistic simulation and intrinsically unstable control [1]. Physically feasible masses shall be positive, inertia tensors shall be positive definite, and friction gains shall be positive,

$$\begin{cases} m_k > 0 \\ I_k \succ 0 \\ f_{vk} > 0 \\ f_{ck} > 0 \end{cases} \quad \text{for } k = 1, \cdots, N \ . \tag{17}$$

From (4) and (5), the second condition[1], $I_k \succ 0$, can be rewritten as

$$L_k - \mathrm{S}\left(l_k\right)^T m_k^{-1} \mathrm{S}\left(l_k\right) \succ 0 \ , \tag{18}$$

which also implies that L_k is positive definite. The physical feasibility condition can be written in terms of δ, hence the set of δ vectors which verify it can be defined by

$$\begin{aligned} \mathcal{D} = \{\delta \in \mathbb{R}^n :\ & L_k - \mathrm{S}\left(l_k\right)^T m_k^{-1} \mathrm{S}\left(l_k\right) \succ 0, \\ & m_k > 0,\ f_{vk} > 0,\ f_{ck} > 0, \\ & \text{for } k = 1, \cdots, N\} \ . \end{aligned} \tag{19}$$

Given a δ estimation, its feasibility can be directly checked with (17) and (18). Nevertheless, estimations are done in β space and for each β solution there are an infinite number of corresponding δ solutions (the map from δ to β spaces through (11) is not bijective). If for a given β estimation there is at least one feasible δ which maps to it, then such estimation is feasible [1].

IV. Physically Feasible Parameter Estimation

We can constrain the estimation (15) to the physically feasible solution space doing

$$\begin{aligned} \underset{(\beta,\delta)}{\text{minimize}} \quad & \|\omega - W\beta\|^2 \\ \text{subject to} \quad & \beta = K\delta \\ & \delta \in \mathcal{D} \ , \end{aligned} \tag{20}$$

where

$$K = {P_b}^T + K_d {P_d}^T \ , \tag{21}$$

thus guaranteeing that the solution is the feasible one which best fits the regression. In a practical view point, (20) has no simple solution. However, as we will show in the sequel, the set $\mathcal{D}$ can be defined by a linear matrix inequality (LMI), proving that it is a convex set ready to be used in semidefinite programming (SDP). This entails optimal global solutions and enables efficient solving methods.

[1] $M \succ 0$ means that matrix M is positive definite.

A. LMI Formulation of the Physically Feasible Conditions

The left-hand side of (18) represents the Schur complement of $m_k\mathcal{I}$ in the block matrix $D_k(\delta_k)$ defined as

$$D_k(\delta_k) = \begin{bmatrix} L_k & \mathrm{S}\left(l_k\right)^T \\ \mathrm{S}\left(l_k\right) & m_k\mathcal{I} \end{bmatrix} , \tag{22}$$

where $\mathcal{I}$ is the identity matrix [8]. Making use of Schur complement condition for positive definite matrices we know that (18) is in fact equivalent to

$$D_k(\delta_k) \succ 0 , \tag{23}$$

which implicitly entails the condition $m_k > 0$. Extending $D_k(\delta_k)$ to include the friction gains into a matrix $E_k(\delta_k)$ defined as

$$E_k(\delta_k) = \begin{bmatrix} D_k(\delta_k) & 0 \\ 0 & \begin{bmatrix} f_{vk} & 0 \\ 0 & f_{ck} \end{bmatrix} \end{bmatrix} , \tag{24}$$

we can write the set $\mathcal{D}$ as

$$\mathcal{D} = \{\delta \in \mathbb{R}^n :\ E_k(\delta_k) \succ 0,\ \text{for } k = 1, \cdots, N\} . \tag{25}$$

Collapsing all $E_k(\delta_k)$ matrices into a single block matrix $E(\delta)$,

$$E(\delta) = \begin{bmatrix} E_1(\delta_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & E_N(\delta_N) \end{bmatrix} , \tag{26}$$

we get

$$\mathcal{D} = \{\delta \in \mathbb{R}^n :\ E(\delta) \succ 0\} . \tag{27}$$

Since all elements of $E(\delta)$ are linear combinations of δ, the condition

$$E(\delta) \succ 0 \tag{28}$$

is an LMI. LMIs define a class of convex sets which can be used in SDP.

B. LMI Formulation of the Regression Error

The OLS optimization function of (20) can also be put in an LMI perspective. Defining a scalar u as being an upper limit to the regression error,

$$u \geq \|\omega - W\beta\|^2 , \tag{29}$$

one can write

$$u - (\omega - W\beta)^T \mathcal{I}^{-1} (\omega - W\beta) \geq 0 , \tag{30}$$

which, by Schur complements, can be written in LMI form[2],

$$\begin{bmatrix} u & (\omega - W\beta)^T \\ \omega - W\beta & \mathcal{I} \end{bmatrix} \succeq 0 . \tag{31}$$

In this form, the LMI matrix has a size as big as the number of data points. However, the size can be reduced performing the QR decomposition of W. Knowing that

$$W = QR = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} R_1 \\ 0 \end{bmatrix} = Q_1 R_1 , \tag{32}$$

[2] $M \succeq 0$ means that matrix M is positive semidefinite.

it is possible to write the equivalence

$$\|\omega - W\beta\|^2 = u_o + \|\rho_1 - R_1\beta\|^2 , \tag{33}$$

where

$$\rho_1 \equiv Q_1^T \omega , \tag{34}$$

and u_o is the optimization function value at the optimum,

$$u_o = \|Q_2^T \omega\|^2 . \tag{35}$$

Equation (31) is then equivalent to

$$\begin{bmatrix} u - u_o & (\rho_1 - R_1\beta)^T \\ \rho_1 - R_1\beta & \mathcal{I} \end{bmatrix} \succeq 0 . \tag{36}$$

This LMI is nonstrict (the matrix is positive semidefinite) while the LMI for the physical feasibility constraint (28) is strict (positive definite). In a practical sense, it is possible to write a nonstrict version of (28) as

$$\bar{E}(\delta) \succeq 0 , \tag{37}$$

by subtracting an infinitesimally small positive scalar ε to each $E(\delta)$ diagonal element,

$$\bar{E}(\delta) = E(\delta) - \varepsilon\mathcal{I} . \tag{38}$$

The nonstrict formulation is advantageous since it is the standard for SDP.

C. SDP Formulation of the Constrained Regression

Both LMIs can now be merged into a single one. Letting the matrix $F(u, \beta, \delta)$ be defined as

$$F(u, \beta, \delta) = \begin{bmatrix} U(u, \beta) & 0 \\ 0 & \bar{E}(\delta) \end{bmatrix} , \tag{39}$$

where

$$U(u, \beta) = \begin{bmatrix} u - u_o & (\rho_1 - R_1\beta)^T \\ \rho_1 - R_1\beta & \mathcal{I} \end{bmatrix} , \tag{40}$$

we can write problem (20) as

$$\begin{aligned} \underset{(u,\beta,\delta)}{\text{minimize}} \quad & u \\ \text{subject to} \quad & \beta = K\delta \\ & F(u, \beta, \delta) \succeq 0 . \end{aligned} \tag{41}$$

This problem includes both β and δ solution spaces, and the map $\beta = K\delta$ between them. Rewriting F in term of u and δ only,

$$F(u, \delta) = \begin{bmatrix} U(u, \delta) & 0 \\ 0 & \bar{E}(\delta) \end{bmatrix} , \tag{42}$$

where

$$U(u, \delta) = \begin{bmatrix} u - u_o & (\rho_1 - R_1 K\delta)^T \\ \rho_1 - R_1 K\delta & \mathcal{I} \end{bmatrix} , \tag{43}$$

we can turn the original problem (20) into

$$\begin{aligned} \underset{(u,\delta)}{\text{minimize}} \quad & u \\ \text{subject to} \quad & F(u, \delta) \succeq 0 , \end{aligned} \tag{44}$$

which is a typical SDP problem. For this problem, as for (10), there are multiple δ optimal solutions. Being $\delta^\star$ one of the optimal solutions, we know that it is a physically feasible solution with minimal regression error. Such $\delta^\star$ has only meaning as a whole, since some of its elements can take arbitrary values. Nevertheless, all the $\delta^\star$ multiple solutions map to single optimal base parameter vector $\beta^\star$,

$$\beta^\star = K\delta^\star \ . \tag{45}$$

Vector $\beta^\star$ is the physically feasible base parameter solution which minimizes the regression error.

V. 7-DOF WAM ROBOT DYNAMIC PARAMETER IDENTIFICATION

In this section we present the methodologies and results obtained in the 7-DOF WAM robot parameter identification. The robot dynamic model (2) and the base dynamic parameters have been computed using the SageRobotics open-source software [9]. The base parameter combinations are shown in the first column of Table I.

A. Exciting Trajectory

Parameter identification requires a joint trajectory to generate data points (position and torque) and a criterion to evaluate data robustness. To achieve good estimations, trajectories must excite dynamic parameters as much as possible. The condition number (i.e., the ratio between maximum and minimum singular values) of the regressor matrix W can be used as a trajectory evaluation criterion [10]. Well conditioned trajectories entail small condition numbers. In this work, we choose to generate the exciting trajectory by the method proposed by Swevers et al. [11], using regressor matrix condition number as excitation measure. Each joint k trajectory is defined by

$$q_k(t) = \sum_{l=1}^{L} \frac{a_{k,l}}{\omega_f\, l} \sin(\omega_f\, l\, t) - \frac{b_{k,l}}{\omega_f\, l} \cos(\omega_f\, l\, t) + q_{k0} \ , \tag{46}$$

where t is the time variable and ω_f is the fundamental angular frequency. Parameters $a_{k,l}$, $b_{k,l}$ and q_{k0} must minimize the reference trajectory condition number, entailing a nonlinear optimization problem with $2L+1$ free variables per joint. In our setup, parameter L has been set to 5 and parameter ω_f to 0.1π. Since robot joint positions, velocities and accelerations have limited ranges, cost constraint functions have been used to confine the generated trajectory. To generate the exciting trajectory a constrained nonlinear optimization by linear approximation (COBYLA) has been used.

B. Control for Trajectory Tracking

Unlike common industrial robots, the WAM enables the design of computed torque controllers. Since the robot CAD model is publicly available, an a priori estimation of inertial parameters (m_k, l_k and I_k) can be obtained (see second column of Table I). Such information has been used to design and implement a trajectory tracking control to perform the excitation trajectory. The dynamic model enables feedback linearization techniques in joint space,

$$\tau_c = \tilde{g}(q) + \tilde{c}(q, \dot{q}) + \tilde{M}(q)\alpha \ , \tag{47}$$

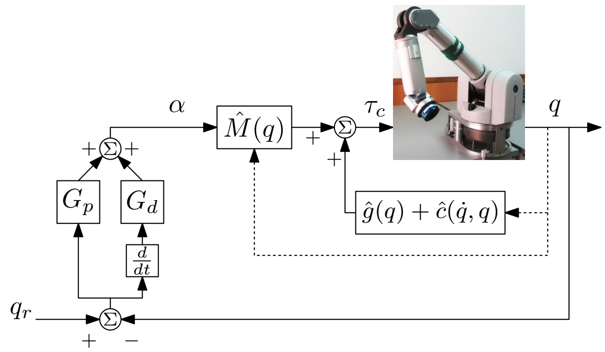

Fig. 1. Joint position control scheme. Vector q_r is the reference joint position. G_p and G_d are diagonal matrices with proportional and derivative gains, respectively.

where τ_c is the computed torque sent to joint actuators, $\tilde{g}(q)$, $\tilde{c}(q, \dot{q})$ and $\tilde{M}(q)$ are estimations of $g(q)$, $c(q, \dot{q})$ and $M(q)$ using CAD inertial parameters, respectively. Vector α is the desired acceleration. Neglecting friction and estimation errors, the free space plant for each joint is given by (see (2) and (47))

$$\ddot{q}_k = \alpha_k \ . \tag{48}$$

This is equivalent to a double integrator over which a controller can be designed. No complete knowledge of how the command is transformed into real motor torque is available, however high frequencies are likely to be heavily filtered. By this reason a proportional and derivative (PD) controller with low gains has been chosen rather than a controller with higher dynamic response. Although trajectories are followed with less accuracy, the frequency in torque command is decreased entailing better regression data. The chosen control scheme is depicted in Fig. 1. Proportional and derivative gains have been tuned for critically damped response.

C. Experimental Data Processing

Having the designed control and the generated excitation trajectory, we have performed a 60 seconds experiment (repeating the trajectory 3 times) for which we have recorded actuator and sensor data. The WAM provides joint position measurements but no acceleration nor velocity explicit data. First and second orders derivatives of position have been computed so that the regressor matrix could be calculated. Signals have been filtered with third order low-pass Butterworth filters with cut frequency $f_c = 10\omega_f L/(2\pi)$. Since this process has been done offline, phase distortion has been compensated. With this process a regression data set (W and ω, see (15)) has been obtained.

In the proposed model, static friction is not taken into account, thus the regression is affected by this unmodeled effect. Nevertheless, the static friction effect shows up only when velocity is zero or near zero. If data points with close to zero velocities are removed from the data set, such reduced data set is likely to entail better structural parameter estimation. We have tested the elimination of data points below several velocity thresholds, comparing the

TABLE I

7-DOF WAM ROBOT BASE PARAMETERS.

β (parameter combination)	CAD β_{CAD}	OLS $\hat{\beta}$	Feasible LS $\beta^{\star}$
$L_{1yy} + L_{2zz}$	0.132848	0.177710	0.107308
fv_1	—	1.494895	1.459656
fc_1	—	1.862094	1.904568
$L_{2xx} - L_{2zz} + L_{3zz} - 1.1\, l_{3y} + 0.300475\,(m_3 + m_4 + m_5 + m_6 + m_7)$	1.180983	1.121856	1.185275
L_{2xy}	0.000013	-0.080853	-0.046713
L_{2xz}	0.000117	-0.012934	-0.005783
$L_{2yy} + L_{3zz} - 1.1\, l_{3y} + 0.300475\,(m_3 + m_4 + m_5 + m_6 + m_7)$	1.188985	1.036276	1.029939
L_{2yz}	-0.000037	0.086959	0.028633
l_{2x}	-0.009183	-0.004968	-0.002468
$l_{2z} - l_{3y} + 0.55\,(m_3 + m_4 + m_5 + m_6 + m_7)$	2.332826	2.445735	2.437263
fv_2	—	2.518510	2.521836
fc_2	—	0.630382	0.628597
$L_{3xx} - L_{3zz} + 0.002025\, m_3 + L_{4zz}$	0.003856	0.200800	0.164187
$L_{3xy} - 0.045\, l_{3y}$	-0.000025	-0.055185	-0.024346
L_{3xz}	-0.000005	0.026428	-0.000118
$L_{3yy} - 0.002025\, m_3 + L_{4zz} - 0.00405\,(m_4 + m_5 + m_6 + m_7)$	-0.006954	-0.014311	0.026154
L_{3yz}	0.000005	0.115010	0.095539
$l_{3x} + 0.045\,(m_3 + m_4 + m_5 + m_6 + m_7)$	0.147625	0.159042	0.157155
$l_{3z} + l_{4y}$	-0.000491	-0.033893	-0.028228
fv_3	—	1.199353	1.231224
fc_3	—	0.290818	0.281070
$L_{4xx} - L_{4zz} + 0.002025\, m_4 + L_{5zz} - 0.6\, l_{5y} + 0.092025\,(m_5 + m_6 + m_7)$	0.114905	-0.014376	0.067822
$L_{4xy} + 0.045\, l_{4y}$	-0.000039	0.066292	0.024353
L_{4xz}	-0.000082	0.019630	0.002997
$L_{4yy} - 0.002025\, m_4 + L_{5zz} - 0.6\, l_{5y} + 0.087975\,(m_5 + m_6 + m_7)$	0.105689	0.102886	0.112864
L_{4yz}	0.000094	-0.005328	-0.006727
$l_{4x} - 0.045\,(m_4 + m_5 + m_6 + m_7)$	-0.123510	-0.097343	-0.099383
$l_{4z} - l_{5y} + 0.3\,(m_5 + m_6 + m_7)$	0.501024	0.503525	0.506568
fv_4	—	0.383216	0.373214
fc_4	—	0.952803	0.941412
$L_{5xx} - L_{5zz} + L_{6zz}$	0.000565	-0.019741	0.000946
L_{5xy}	-0.000000	0.005445	0.002364
L_{5xz}	0.000000	0.003058	-0.000556
$L_{5yy} + L_{6zz}$	0.000653	-0.002980	0.002389
L_{5yz}	0.000001	0.000282	0.001821
l_{5x}	0.000011	-0.014509	-0.014906
$l_{5z} + l_{6y}$	-0.006580	-0.013968	-0.013277
fv_5	—	0.229574	0.220504
fc_5	—	-0.071959	-0.058760
$L_{6xx} - L_{6zz} + L_{7yy} + 0.12\, l_{7z} + 0.0036\, m_7$	0.000615	0.022288	0.014335
L_{6xy}	-0.000001	-0.004926	-0.004153
L_{6xz}	0.000002	0.000150	-0.003967
$L_{6yy} + L_{7yy} + 0.12\, l_{7z} + 0.0036\, m_7$	0.000758	0.001841	0.004626
L_{6yz}	0.000222	-0.004198	0.001502
l_{6x}	-0.000051	0.000686	0.003062
$l_{6z} + l_{7z} + 0.06\, m_7$	0.014214	0.015821	0.014443
fv_6	—	0.161594	0.153190
fc_6	—	0.041832	0.043348
$L_{7xx} - L_{7yy}$	-0.000000	0.005331	0.006061
L_{7xy}	0.000000	0.000603	-0.000133
L_{7xz}	-0.000000	0.005237	0.003771
L_{7yz}	0.000000	0.002409	-0.000120
L_{7zz}	0.000074	0.002256	0.003213
l_{7x}	-0.000005	0.009813	0.007886
l_{7y}	0.000011	-0.004071	-0.001545
fv_7	—	0.020896	0.026854
fc_7	—	0.089916	0.075723

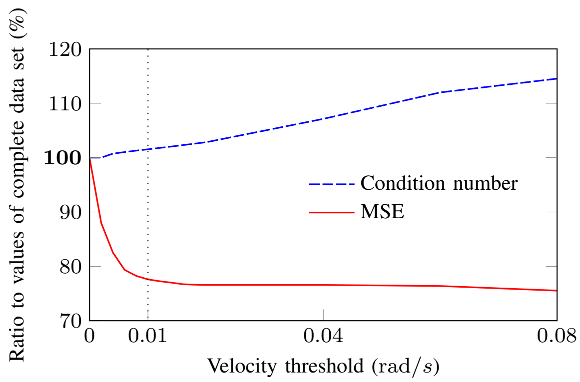

Fig. 2. Variation on the regressor condition number and on the regression mean squared error for the removal of data set points with absolute velocities below different threshold values.

regressor matrix condition number and the regression mean squared error (MSE) variations. As it can be seen in Fig. 2, removing data points corresponding to near zero velocities abruptly decreases the MSE while only slightly increasing the condition number. This has been asserted by comparing with the elimination of random data points and with the elimination of data points within other ranges of velocity: these increase the condition number but gave no reduction in the MSE. For the final reduced data set a velocity threshold of 0.01 rad/s has been chosen, entailing a reduction of about 22% on the MSE and an increase of less than 2% on the condition number. The final condition number is about 48.4, hence it is well conditioned [12].

D. Results

Given the reduced regression data set, the dynamic base parameter $\hat{\beta}$ estimated by (15), i.e., unconstrained with respect to physical feasibility, is presented in the third column of Table I. This estimation has been used to compute inertia matrix values at thousands of random robot postures. For all of them, the inertia matrix is not positive definite, therefore $\hat{\beta}$ is not physically feasible. In fact, a single non positive definite inertia matrix is enough to prove physical infeasibility.

An estimation has then been performed by the "Feasible Least Squares" method proposed in Section IV (SDP problem (44)). The solution $\delta^\star$ has been obtained using the freely available SDPA software [13]. Such software finds the solution in less than one second. The physically feasible base parameter solution $\beta^\star$, shown in the rightmost column of Table I, is then obtained from $\delta^\star$ through (45). The empirical evaluation at random postures has always given positive definite inertia matrices. The regression MSE of $\hat{\beta}$ and $\beta^\star$ are 0.148925 and 0.155154, respectively. As expected, the error of $\beta^\star$ is higher. This does not entail that $\beta^\star$ is a worse estimation, rather it can be considered to entail a better structural model since physical feasibility is guaranteed while the regression error is kept as low as possible. Control or simulation performance comparisons between $\hat{\beta}$ and $\beta^\star$ are not useful and even not possible since physically unfeasible estimations are intrinsically unstable.

VI. Conclusion

In this work the dynamic parameters of the WAM robot have been identified. Physical feasibility of the estimated parameters is guaranteed by a new proposed regression method. Such method reformulates both the ordinary least squares and the feasibility constraints into the LMI–SDP framework. This enables efficient estimation of dynamic parameters, providing the physically feasible solution which better fits regression data. Practical identification issues, including the elimination of static friction effects from the data set have been discussed.

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