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Identification of Standard Dynamic Parameters of Robots with positive definite inertia matrix

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Abstract— **For any rigid robot, a set of 14 *standard parameters* characterises the dynamics of each of its links and joints. Only a subset of these *standard parameters*: the *base parameters* have unique values identified with the Inverse Dynamic Identification Model and linear least squares techniques (*IDIM-LS*). Moreover, some of the base parameters are poorly identified when their effect on the joint torques is too small. They can be eliminated, leading to a new subset of *essential (base) parameters*. However, the consistency of the identified values of the *base* or the *essential* parameters cannot be guaranteed, regarding to the loss of the positive definiteness of the robot inertia matrix. The past methods proposed to verify the physical consistency of the identified parameters, relies on complicated, time consuming computations and even leads to non-optimal *LS* parameters. We propose a method that overcomes these drawbacks, calculating the set of optimal *LS standard parameters* closest to a set of *a priori* consistent dynamic parameters obtained through CAD data given by the robot manufacturers. This is a straightforward method, which relies on the use of the Singular Value Decomposition (*SVD*), the Cholesky factorization and the linear least squares techniques. The method is experimentally validated on a Stäubli TX-40, which is a 6 Degrees of Freedom (DoF) industrial robot. This example enlighten a strong result: the *essential base* parameters, which have significant identified values with respect to their small relative standard deviation, are consistent.**

I. Introduction

DYNAMICS IDENTIFICATION has been widely studied in Robotics in the last decades. Yet several unanswered fundamental issues exist such as the consistency of the experimentally identified parameters with respect to the positivity of the inertia matrix of the robot, while this condition is crucial for simulation and model based control.

In [1] a method for finding a set of virtual standard inertial parameters that can be related to the base or essential parameters and that guaranty the positive definiteness of the inertial matrix is proposed. However, this method is based on a trial and error algorithm and is complicated and time-consuming due to its iterative process. In some of our previous work [2] and in other works [3]–[5] the approaches are based on adding numerical constraints to the system so that the inertia matrices are identified definite positive, the masses are positive and eventually that the center of mass are located into a convex hull representing the segment's shape, and make use of quadratic programming. These constraints, inherently lead to non-optimal *LS* results since the identified standard parameters do not minimize the norm of the model error anymore.

Each robot link can be defined by a set of 10 inertial parameters, and each joint and drive chain can be defined by 4 parameters. They define the set of *standard inertial parameters* [6]. The set of *base parameters* is defined as the minimum set of inertial parameters necessary for calculating the joint torque vector; they constitute also a set of unique (identifiable) values identified with the Inverse Dynamic Identification Model and linear least squares techniques (*IDIM-LS*) [7]–[9]. Some base parameters may almost be too small or are poorly excited to have a significant contribution to the joint torque/force. They are poorly identified and cancelled to keep a set of *essential (base) parameters* of a simplified dynamic model without loss of joint torque/force model accuracy [10].

The use of the *a priori* values of the standard parameters and some linear algebra can be of help to address this issue. It is now relatively easy for robots' manufacturers to obtain a reliable set of *a priori* values of the robot dynamic *standard parameters* from the *CAD* data. This information should be taken as an advantage for finding a set of updated standard parameters as close as possible to these *a priori* values, and corresponding to the actual robot parameters, taking into account its behaviour and the actual data that are not included in the nominal *CAD* data.

Here, we propose a method to calibrate the *standard parameters* with respect to the *a priori* known values using the Singular Value Decomposition (*SVD*) of the observation matrix. The solution, which minimizes the norm error, remains one of the best possible *LS* solutions. Moreover, if the *a priori* values of the parameters are physically consistent and well chosen, and if the measurement errors are small enough, then the calibrated *standard parameters* are physically consistent. The method has been validated on a 6 *DoF* industrial manipulator but can be extended to any type of robot or multi-body system.

II. The Inverse Dynamic Identification Model

The inverse dynamic model (*IDM*) of a rigid robot composed of *n* links and *n* joints calculates the ($n\times1$) motor torque vector τ_{IDM}, as a function of the generalized coordinates and their derivatives [6], [11]. It can be written as the following:

$$\tau_{idm}=M(q)\,\ddot{q}+N(q,\dot{q}) \tag{1}$$

where q, $\dot{q}$ and $\ddot{q}$ are respectively the ($n\times1$) vectors of generalized joint positions, velocities and accelerations, $M(q)$ is the ($n\times n$) robot inertia matrix, and $N(q,\dot{q})$ is the

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($n\times1$) vector of centrifugal, Coriolis, gravitational and friction forces/torques.

The dynamic model of any manipulator with n actuators can be linearly written in terms of a ($n\times1$) vector of *standard parameters* χ_{st} [6], [12], [13]:

$$\tau_{idm}(q,\dot{q},\ddot{q},\chi_{st}) = IDM_{st}(q,\dot{q},\ddot{q})\chi_{st} \tag{2}$$

where IDM_{st} is the ($n\times n_{st}$) matrix of τ_{idm}, with respect to the ($n_{st}\times1$) vector χ_{st} of the *standard parameters* given by:

$$\chi_{st} = \left[\chi_{st}^{1\,T}\;\chi_{st}^{2\,T}\;\ldots\;\chi_{st}^{n\,T}\right]^T.$$

For rigid robots the parameters can be regrouped into the (14×1) vector χ_{st}^j [1]:

$$\chi_{st}^j = \left[XX_j\;XY_j\;XZ_j\;YY_j\;YZ_j\;ZZ_j\;MX_j\;MY_j\;MZ_j\;M_j\;Ia_j\;Fv_j\;Fc_j\;\tau_{off_j}\right]^T \tag{3}$$

Where XX_j, XY_j, XZ_j, YY_j, YZ_j, ZZ_j are the 6 components of the inertia matrix of link j at the origin of frame j. MX_j, MY_j, MZ_j are the 3 components of the first moment of link j, M_j is the mass of link j, Ia_j is a total inertia moment for rotor and gears of actuator j. Fv_j, Fs_j are the viscous and Coulomb friction coefficients of the transmission chain, respectively, $\tau_{off_j} = \tau_{offFS_j} + \tau_{off\tau_j}$ is an offset parameter which regroups the amplifier offset $\tau_{off\tau_j}$ and the asymmetrical Coulomb friction coefficient τ_{offFS_j}.

Because of perturbations due to noise measurement and modelling errors, the actual force/torque τ differs from τ_{idm} by an error, e, such that:

$$\tau = \tau_{idm} + e = IDM_{st}(q,\dot{q},\ddot{q})\chi_{st} + e \tag{4}$$

where τ is calculated with the drive chain relations:

$$\tau = v_\tau\, g_\tau = \begin{bmatrix} v_\tau^1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & v_\tau^n \end{bmatrix}\begin{bmatrix} g_\tau^1 \\ \vdots \\ g_\tau^n \end{bmatrix} \tag{5}$$

v_τ is the ($n\times n$) matrix of the actual motor current references of the current amplifiers (v_τ^j corresponds to actuator j) and g_τ is the ($n\times1$) vector of the joint drive gains (g_τ^j corresponds to actuator j) that is given by *a priori* manufacturer's data or identified [14][15]. Equation (4) represents the Inverse Dynamic Identification Model (*IDIM*).

III. Weighted Least Squares Identification of Essential Base Parameters with QR factorization (*IDIM-WLS*)

The general identification problem (4) after being sampled and usually low-pass filtered to remove undesirable noise can be written as follows:

$$Y = W_{st}^a \chi_{st} + \rho_a \tag{6}$$

Where, for r samples over all the DoF, Y is the ($r\times1$) sampled vector of motor torques τ, W_{st}^a is the ($r\times n_{st}$) sampled regressor IDM_{st}, ρ_a is the ($r\times1$) of errors due to noise measurement and modelling error.

The identification problem consists in finding χ_{st} that minimizes the square norm of the error ρ_a:

$$\min_{\chi_{st}}\|\rho_a\|^2 = \min_{\chi_{st}}\left\|Y - W_{st}^a\chi_{st}\right\|^2 \tag{7}$$

Usually, the vector of standard parameters is not calculated directly when solving (7), as there is a structural rank deficiency of W_{st}^a because the n_{st} columns of the regressor IDM_{st} are not independent: $rank(W_{st}^a) = n_b$ such that $n_b \le n_{st}$. Consequently, there exists infinity of solutions for χ_{st} from which only some are physically consistent. It is thus common to identify the base parameters χ_b, which are the minimal set of parameters that calculates the motor torque with the *IDIM* (2), and which can be identified using linear least squares (*LS*) techniques. They are obtained by linear combinations of the standard parameters which depend on the choice of the independent columns in W_{st}^a and which can be determined using simple closed-form rules [7], or by numerical method based on the *QR* or *SVD* decomposition [8]–[9]. This leads to a non-unique minimal model, and a non-unique set of base parameters depending on the choice of W_b^a, such that (6) becomes:

$$Y = W_b^a\chi_b + \rho \Rightarrow \hat{\chi}_b = W_b^{a+}Y \tag{8}$$

where W_b^{a+} is the pseudo-inverse of W_b^a and $\hat{\chi}_b$ is the unique least squares (*LS*) solution of (8) which is computed using the *QR* factorization of W_b^a.

The standard deviations $\sigma_{\hat{\chi}_i}$ are estimated assuming that W_b^a is a deterministic matrix and ρ_a is a zero-mean additive independent Gaussian noise, with a covariance matrix $C_{\rho\rho}$:

$$C_{\rho\rho} = E(\rho_a\rho_a^T) = \sigma_\rho^2 I_r \tag{9}$$

E is the expectation operator and I_r, ($r\times r$) identity matrix. An unbiased estimation of the standard deviation σ_ρ is:

$$\hat{\sigma}_\rho^2 = \left\|Y - W_b^a\hat{\chi}_b\right\|^2 / (r - n_b) \tag{10}$$

The covariance matrix of the estimation error is then given by:

$$C_{\hat{\chi}\hat{\chi}} = E[(\chi_b - \hat{\chi}_b)(\chi_b - \hat{\chi}_b)^{\mathrm{T}}] = \hat{\sigma}_\rho^2(W_b^{aT}W_b^a)^{-1} \tag{11}$$

$\sigma_{\hat{\chi}_i}^2 = C_{\hat{\chi}\hat{\chi}}(i,i)$ is the i^{th} diagonal coefficient of $C_{\hat{\chi}\hat{\chi}}$

Finally the relative standard deviation $\%\sigma_{\hat{\chi}_{ri}}$ for each identified parameters is given by:

$$\%\sigma_{\hat{\chi}_{ri}} = \sigma_{\hat{\chi}_i} / |\hat{\chi}_i| \text{ for } |\hat{\chi}_i| \ne 0 \text{ (i-th coefficient of } \hat{\chi}_b) \tag{12}$$

Data in Y and W_b^a of (8) are sorted and weighted with the inverse of the standard deviation of the error calculated from ordinary *LS* solution of the equations of joint j [16].

As mentioned in the introduction, some parameters are poorly identifiable because they do not significantly contribute to the joint torques. These parameters can be cancelled to simplify the dynamic model. Parameters such that the relative standard deviation $\%\sigma_{\hat{\chi}_{ri}}$ is too high are suppressed to keep a set of *essential (base) parameters* χ_e of a simplified dynamic model without loss of accuracy. The *essential parameters* are calculated using an iterative procedure according to [10].

IV. Standard essential consistent parameters identification with SVD factorization

A. Standard parameters identification with SVD

A solution of a system, such as the identification model (6), can be obtained using the *SVD*. As the standard identification model is considered, from the n_{st} columns of W_{st}^a a distinction is made between the n_b independent columns and the others. Thus the following decomposition is obtained:

$$W_{st}^a V_a = U_a \Sigma_a \, , \Sigma_a = \begin{bmatrix} \Sigma_1^a & 0_{n_b, n_{st}-n_b} \\ 0_{n_{st}-n_b, n_b} & \Sigma_2^a \end{bmatrix} \tag{13}$$

where $V_a \in R^{n_{st}\times n_{st}}, V_a = \left[V_1^a, V_2^a\right]$ is a matrix composed of two .sub-matrices V_1^a and V_2^a of respective dimensions $n_{st}\times n_b$ and $n_{st}\times(n_{st}-n_b)$. $U_a \in R^{r\times n_{st}}, U_a = \left[U_1^a, U_2^a\right]$ is a matrix composed of two .sub-matrices U_1^a and U_2^a of respective dimensions $r\times n_b$ and $r\times(n_{st}-n_b)$. $\Sigma_a \in R^{n_{st}\times n_{st}}$ is a diagonal matrix composed of the singular values of W_{st} sorted in decreasing order; Σ_a is decomposed into two .sub-matrices Σ_1^a and Σ_2^a of respective dimensions $n_b \times n_b$ and $(n_{st}-n_b)\times(n_{st}-n_b)$.

In the ideal case, i.e. without noise and perturbations on the data, W_{st}^a must be rank-deficient and Σ_2^a a zero matrix. However, with measured data, this is not the case but the values of Σ_2^a are very small and can be set to zero. The system (13) thus becomes:

$$W_{st} V = U \begin{bmatrix} \Sigma_1 & 0_{n_b,(n_{st}-n_b)} \\ 0_{(n_{st}-n_b),n_b} & 0_{(n_{st}-n_b),(n_{st}-n_b)} \end{bmatrix}, \tag{14}$$

where W_{st} is the rank deficient matrix closest to W_{st}^a with respect to the Frobenius norm and is given by [15]:

$$W_{st} = W_{st}^a - \sum_{k=n_b+1}^{n_{st}} s_k U_k^a V_k^{aT} \, , \tag{15}$$

with s_k is the k^{th} value on the diagonal of Σ_a and U_k^a (V_k^a, resp.) the k-th column of U_a (V_a, resp.) corresponding to s_k, and $V \in R^{n_{st}\times n_{st}}, V = \left[V_1, V_2\right]$ is composed of two sub-matrices V_1 and V_2 of respective dimensions $n_{st}\times n_b$ and $n_{st}\times(n_{st}-n_b)$; $U \in R^{r\times n_{st}}, U = \left[U_1, U_2\right]$ is composed of two sub-matrices U_1 and U_2 of respective dimensions $r\times n_b$ and $r\times(n_{st}-n_b)$.

The rank-deficient system closest to the actual one (8) is thus given by:

$$Y = W_{st}\chi_{st} + \rho \; , \; \|\rho\| > \|\rho_a\| \tag{16}$$

with a rather small increasing between $\|\rho\|$ and $\|\rho_a\|$.

By multiplying Y and $W_{st}\,\chi$, respectively, on the left by U^T, the following relations are obtained:

$$U^T Y = G = \begin{bmatrix} U_1^T Y \\ U_2^T Y \end{bmatrix} = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, \tag{17}$$

$$U^T W_{st}\,\chi_{st} = \begin{bmatrix} \Sigma_1 & 0_{n_b,(n_{st}-n_b)} \\ 0_{(n_{st}-n_b),n_b} & 0_{(n_{st}-n_b),(n_{st}-n_b)} \end{bmatrix} \begin{bmatrix} V_1^T \chi_{st} \\ V_2^T \chi_{st} \end{bmatrix} = \begin{bmatrix} \Sigma_1 V_1^T \chi_{st} \\ 0_{(n_{st}-n_b),1} \end{bmatrix}. \tag{18}$$

Let us define vector Z as:

$$Z = V^T \chi_{st} = \begin{bmatrix} V_1^T \chi_{st} \\ V_2^T \chi_{st} \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \text{ or } \chi_{st} = V Z \, . \tag{19}$$

(18) can be rewritten as:

$$U^T W_{st}\,\chi_{st} = \begin{bmatrix} \Sigma_1 Z_1 \\ 0_{(n_{st}-n_b),1} \end{bmatrix}. \tag{20}$$

As the product by U^T keeps the norm unchanged, the identification problem (16) can be expressed as follows:

$$\begin{aligned} \|\rho\|^2 &= \|Y - W_{st}\chi_{st}\|^2 = \|U^T Y - U^T W_{st}\chi_{st}\|^2 \\ &= \|G - \Sigma Z\|^2 = \|G_1 - \Sigma_1 Z_1\|^2 + \|G_2\|^2 \end{aligned} . \tag{21}$$

The unique solution $\hat{Z}_1$ to this problem is given by:

$$\hat{Z}_1 = \Sigma_1^{-1} G_1 = \Sigma_1^{-1} U_1^T Y \, , \tag{22}$$

and the family of all optimal solution $\hat{Z}$ is, for any Z_2

$$\hat{Z} = \begin{bmatrix} \hat{Z}_1 \\ Z_2 \end{bmatrix} \tag{23}$$

Thus, an optimal solution $\hat{\chi}_{st}$ to (21) is given by:

$$\hat{\chi}_{st} = V\,\hat{Z} = V_1 \Sigma_1^{-1} U_1^T Y + V_2 Z_2 \tag{24}$$

Introducing (24) in to (21), it is shown that, for any optimal solution, the minimal norm of the error ρ is:

$$\|\rho\|_{min} = \|G_2\| = \|U_2^T Y\| \, . \tag{25}$$

Finally, the optimal solution $\hat{\chi}_{st}^{opt}$ that minimizes both norms of $\hat{\chi}_{st}$ and ρ at the same time is obtained for $Z_2 = 0$, i.e.:

$$\hat{\chi}_{st}^{opt} = V_1 \Sigma_1^{-1} U_1^T Y \tag{26}$$

With $V_1 \Sigma_1^{-1} U_1^T$ is the Moore-Penrose pseudo-inverse of W_{st}.

B. Standard parameters closest to a priori values

The minimal norm solution obtained by (26) is optimal in term of the error norm (25). However the consistency of the parameters, with respect to its physical meaning is not guaranteed. Here a new approach is proposed that takes benefits of the *a priori* values χ_{st}^{ref} of the inertial parameters calculated with *CAD* data from the manufacturers' data.

Let us denote as Y^{ref} the joint torques estimated with the *a priori* values χ_{st}^{ref}:

$$Y^{ref} = W_{st}\chi_{st}^{ref} \tag{27}$$

Substracting (27) to (6), it comes

$$Y - Y^{ref} = W_{st}(\chi_{st} - \chi_{st}^{ref}) + \rho \Leftrightarrow \Delta Y = W_{st}\Delta\chi_{st} + \rho \tag{28}$$

where the error ρ is the same as that of the system (6).

Similarly to (26), the optimal solution $\Delta\hat{\chi}_{st}^{opt}$ that minimizes the norm of $\Delta\hat{\chi}_{st}$ is given by:

$$\Delta\hat{\chi}_{st}^{opt} = V_1 \Sigma_1^{-1} U_1^T \Delta Y \tag{29}$$

which leads to

$$\hat{\chi}_{st}^{opt} = \chi_{st}^{ref} + V_1 \Sigma_1^{-1} U_1^T (Y - Y^{ref}) \, . \tag{30}$$

$\hat{\chi}_{st}^{opt}$ simultaneously minimizes the norm of ρ given in (16) and the norm of $\chi_{st} - \chi_{st}^{ref}$. $\hat{\chi}_{st}^{opt}$ is the optimal standard solution closest to a consistent solution χ_{st}^{ref}, then it is the best optimal standard solution that can keep the physical consistency of χ_{st}^{ref} if the minimal norm of $\chi_{st}-\chi_{st}^{ref}$ is small and if the measurement errors are small.

C. Standard essential and consistent parameters

The previous method does not take into account the fact that some parameters may almost be null and thus have no contribution to the system dynamics; or that some parameters can be identified with a very small confidence and have no significant values that lead to the loss of consistency of some standard parameters. To overcome this problem, let us take advantage of the correct knowledge that it is possible to have on the identified essential parameters denoted as $\hat{\chi}_e$ ($\hat{\chi}_e$ is composed of the n_e values of the essential parameters χ_e calculated with the *IDIM-WLS* method proposed in section III and of $(n_{st}-n_e)$ zeros). Weighting the matrix W_{st} in (6) by this vector leads to the new system:

$$Y=\left(W_{st}\,diag\left(\hat{\chi}_e\right)\right)\chi_{st}^{e}+\rho=W_{st}^{e}\chi_{st}^{e}+\rho \qquad (31)$$

where χ_{st}^{e} is a vector of standard parameters weighted by χ_e, i.e. $\chi_{st}=diag(\hat{\chi}_e)\chi_{st}^{e}$.

The *SVD* of the weighted matrix W_{st}^{e} allows to calculate the null-space of W_{st}^{e}, corresponding to the parameters with small contribution on the joint torques in (31), and the image of the transformation which allows to identify the essential parameters which are significant wrt their confidence interval, adding a small increase of the norm error of ρ [10].
Thus, solving the system (31) with *SVD* and applying the previous method for the calibration of the standard parameters, the optimal solution becomes:

$$\hat{\chi}_{st}^{opt}=\chi_{st}^{ref}+diag(\hat{\chi}_e)V_{1e}\Sigma_{1e}^{-1}U_{1e}^{T}(Y-Y^{ref}). \qquad (32)$$

where U_{1e}, V_{1e} and Σ_{1e} are obtained from the *SVD* of W_{st}^{e}:

$$W_{st}^{e}V_e=U_e\begin{bmatrix}\Sigma_{1e} & 0_{n_e,(n_{st}-n_e)}\\ 0_{(n_{st}-n_e),n_e} & 0_{(n_{st}-n_e),(n_{st}-n_e)}\end{bmatrix}, \qquad (33)$$

where $V_e\in R^{n_{st}\times n_{st}}$, $V_e=\left[V_{1e},V_{2e}\right]$ is composed of two sub-matrices V_{1e} and V_{2e} of respective dimensions $n_{st}\times n_e$ and $n_{st}\times(n_{st}-n_e)$; $U_e\in R^{r\times n_{st}}$, $U_e=\left[U_{1e},U_{2e}\right]$ is composed of two sub-matrices U_{1e} and U_{2e} of respective dimensions $r\times n_e$ and $r\times(n_{st}-n_e)$; $\Sigma_{1e}\in R^{n_e\times n_e}$ is a diagonal matrix composed of the singular values of W_{st}^{e} ranked in decreasing order.

In the next section, the identification of the standard parameters of an industrial Stäubli TX-40 robot is presented. It will be shown that the best results are obtained when using the calibration that takes into account the essential parameters.

V. Case Study

A. Description of the Stäubli TX 40

The Stäubli TX-40 robot (Fig. 1) has a serial structure with six rotational joints. Its kinematics is defined using the modified Denavit Hartenberg notation (*MDH*) [17]. The geometric parameters describing the robot frames are given in Table I.

The TX-40 robot is characterized by a coupling between the joints 5 and 6 such that:

$$\begin{bmatrix}\dot{q}r_5\\ \dot{q}r_6\end{bmatrix}=\begin{bmatrix}N_5=45 & 0\\ N_6=32 & N_6=32\end{bmatrix}\begin{bmatrix}\dot{q}_5\\ \dot{q}_6\end{bmatrix},\ \begin{bmatrix}\tau_{c_5}\\ \tau_{c_6}\end{bmatrix}=\begin{bmatrix}N_5 & N_6\\ 0 & N_6\end{bmatrix}\begin{bmatrix}\tau_{r_5}\\ \tau_{r_6}\end{bmatrix} \qquad (34)$$

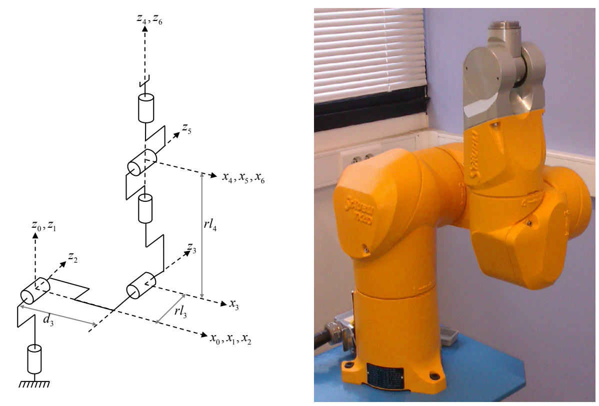

Fig. 1. Link frames of the TX-40 robot

TABLE I
GEOMETRIC PARAMETERS OF THE TX-40 ROBOT WITH THE PAYLOAD

j	σ_j	α_j	d_j	θ_j	r_j
1	0	0	0	q_1	0
2	0	$-\pi/2$	0	$q_2-\pi/2$	0
3	0	0	$d_3=0.225$ (m)	$q_3+\pi/2$	$rl_3=0.035$ (m)
4	0	$+\pi/2$	0	q_4	$rl_4=0.225$ (m)
5	0	$-\pi/2$	0	q_5	0
6	0	$+\pi/2$	0	$q_{6}+\pi$	0

where $\dot{q}r_j$ is the velocity of the rotor of motor j, $\dot{q}_j$ is the velocity of joint j, N_j is the transmission gain ratio of axis j, τ_{cj} is the motor torque of joint j, taking into account the coupling effect on the motor side, τ_{rj} is the electro-magnetic torque of motor j. The coupling between joints 5 and 6 also adds the effect of the inertia of rotor 6 and new viscous and Coulomb friction parameters Fvm_6 and Fcm_6, to both τ_{c5} and τ_{c6}.

$\tau_{c_5}=\tau_5+Ia_6\,\ddot{q}_6+Fvm_6\,\dot{q}_6+Fcm_6\,\mathrm{sign}(\dot{q}_6)$ and

$\tau_{c_6}=\tau_6+Ia_6\,\ddot{q}_5+Fvm_6\,\dot{q}_5+Fcm_6\left(\mathrm{sign}(\dot{q}_5+\dot{q}_6)-\mathrm{sign}(\dot{q}_6)\right)$

where τ_j already contains the terms $(Ia_j\ddot{q}_j+Fv_j\dot{q}_j+Fc_j sign(\dot{q}_j))$, for j=5 and 6 respectively, and with Ja_j is the moment of inertia of rotor j comes (35).

$$Ia_5=N_5^2Ja_5+N_6^2Ja_6 \text{ and } Ia_6=N_6^2Ja_6 \qquad (35)$$

Finally,(35) is introduced into (4) to obtain the *IDIM*.

B. Experimental Identification results

As it is a calibration procedure, the choice of the *a priori* value χ_{st}^{ref} is crucial. However, in the manufacturer's datasheets, the friction parameters and the drive chain inertia Ia_j taking into account the gear box inertias are not given.

These values are extracted from a first identification of the dynamic parameters using the *IDIM-WLS* procedure described in section III. They are given in bold font in Table II, with the *a priori* parameter values χ_{st}^{ref} . The TX40 has n_{st}=86 *standard parameters*, n_b=61 *base parameters* and n_e=31 *essential parameters*.

priori value is also shown. It can be clearly observed that the difference between the *a priori* parameters and those estimated using the essential parameters is 4 times smaller than those estimated using the base parameters.

We carried out a cross validation, comparing the joint torques calculated with (5) from the measure of the current reference

TABLE II

EXPERIMENTALLY IDENTIFIED STANDARD PARAMETERS OF THE STAUBLI TX-40.

Param.	χ_{st}^{ref}	$\hat{\chi}_{st}^{0b}$	$\hat{\chi}_{st}^{0e}$	e_{bi}	e_{ei}	Param.	χ_{st}^{ref}	$\hat{\chi}_{st}^{0b}$	$\hat{\chi}_{st}^{0e}$	e_{bi}	e_{ei}
ZZ_1	3.92e-02	1.24e+00	3.15e-01	*1.20*	*0.28*	MX_4	1.45e-02	-3.26e-02	-1.27e-02	0.05	0.03
Ia_1	**3.62e-01**	3.62e-01	3.62e-01	*0.00*	*0.00*	MY_4	-7.24e-03	-7.55e-03	-7.24e-03	0.00	0.00
Fv_1	**7.96e+00**	7.96e+00	7.93e+00	*0.00*	*0.03*	MZ_4	-5.86e-01	-5.86e-01	-5.86e-01	0.00	0.00
Fs_1	**6.79e+00**	6.81e+00	6.86e+00	*0.02*	*0.07*	M_4	3.62e+00	3.62e+00	3.62e+00	0.00	0.00
τ_{off1}	**0.00e+00**	3.14e-01	3.14e-01	*0.31*	*0.31*	Ia_4	**3.41e-02**	3.15e-02	3.03e-02	0.00	0.00
XX_2	1.63e-02	-4.71e-01	2.71e-02	*0.49*	*0.01*	Fv_4	**1.06e+00**	1.07e+00	1.07e+00	0.01	0.01
XY_2	7.85e-04	1.03e-02	6.19e-03	*0.01*	*0.01*	Fs_4	**2.72e+00**	2.60e+00	2.67e+00	0.12	0.05
XZ_2	-1.57e-02	-1.49e-01	-4.05e-02	*0.13*	*0.02*	τ_{off4}	**0.00e+00**	-6.06e-02	0.00e+00	0.06	0.00
YY_2	8.81e-02	8.81e-02	8.81e-02	*0.00*	*0.00*	XX_5	1.01e-03	1.92e-03	1.01e-03	0.00	0.00
YZ_2	3.24e-04	5.56e-03	3.24e-04	*0.01*	*0.00*	XY_5	0.00e+00	-9.30e-04	0.00e+00	0.00	0.00
ZZ_2	8.28e-02	1.08e+00	1.31e-01	*1.00*	*0.05*	XZ_5	0.00e+00	-2.37e-03	-1.58e-04	0.00	0.00
MX_2	3.92e-01	2.14e+00	9.59e-02	*1.75*	*0.30*	YY_5	1.00e-03	1.00e-03	1.00e-03	0.00	0.00
MY_2	-7.20e-03	1.05e-01	6.79e-02	*0.11*	*0.08*	YZ_5	-3.06e-06	6.73e-05	-3.06e-06	0.00	0.00
MZ_2	1.62e-01	1.62e-01	1.62e-01	*0.00*	*0.00*	ZZ_5	1.01e-03	3.93e-03	1.01e-03	0.00	0.00
Ia_2	**5.07e-01**	5.07e-01	5.07e-01	*0.00*	*0.00*	MX_5	0.00e+00	8.24e-03	0.00e+00	0.01	0.00
Fv_2	**5.92e+00**	5.93e+00	5.92e+00	*0.01*	*0.00*	MY_5	-3.06e-03	-1.05e-02	2.39e-03	0.01	0.01
Fs_2	**7.38e+00**	7.47e+00	7.42e+00	*0.09*	*0.04*	MZ_5	-1.02e-03	-1.02e-03	-1.02e-03	0.00	0.00
τ_{off2}	**0.00e+00**	8.60e-01	0.00e+00	*0.86*	*0.00*	M_5	1.02e+00	1.02e+00	1.02e+00	0.00	0.00
XX_3	2.23e-02	1.30e-01	6.91e-02	*0.11*	*0.05*	Ia_5	**3.61e-02**	3.22e-02	3.46e-02	0.00	0.00
XY_3	-1.95e-04	-6.97e-03	-1.95e-04	*0.01*	*0.00*	Fv_5	**1.24e+00**	1.24e+00	1.24e+00	0.00	0.00
XZ_3	-1.16e-02	2.20e-03	-1.16e-02	*0.01*	*0.00*	Fs_5	**2.62e+00**	2.60e+00	2.62e+00	0.02	0.00
YY_3	2.24e-02	2.24e-02	2.24e-02	*0.00*	*0.00*	τ_{off5}	**0.00e+00**	1.05e-01	0.00e+00	0.11	0.00
YZ_3	-2.22e-03	4.53e-03	-2.22e-03	*0.01*	*0.00*	XX_6	3.53e-04	4.71e-04	3.53e-04	0.00	0.00
ZZ_3	4.41e-03	1.08e-01	4.20e-02	*0.10*	*0.04*	XY_6	0.00e+00	8.46e-04	0.00e+00	0.00	0.00
MX_3	3.26e-02	8.41e-02	3.08e-02	*0.05*	*0.00*	XZ_6	0.00e+00	3.53e-04	0.00e+00	0.00	0.00
MY_3	2.44e-02	-6.31e-01	-1.15e-01	*0.66*	*0.14*	YY_6	3.53e-04	3.53e-04	3.53e-04	0.00	0.00
MZ_3	2.65e-01	2.65e-01	2.65e-01	*0.00*	*0.00*	YZ_6	0.00e+00	-4.41e-04	0.00e+00	0.00	0.00
M_3	4.07e+00	4.07e+00	4.07e+00	*0.00*	*0.00*	ZZ_6	0.00e+00	7.04e-04	0.00e+00	0.00	0.00
Ia_3	**8.29e-02**	1.02e-01	9.14e-02	*0.02*	*0.01*	MX_6	0.00e+00	1.07e-03	0.00e+00	0.00	0.00
Fv_3	**1.98e+00**	1.99e+00	2.01e+00	*0.01*	*0.03*	MY_6	0.00e+00	-3.71e-03	0.00e+00	0.00	0.00
Fs_3	**6.43e+00**	6.41e+00	6.37e+00	*0.02*	*0.06*	MZ_6	8.40e-03	8.40e-03	8.40e-03	0.00	0.00
τ_{off3}	**0.00e+00**	4.48e-01	0.00e+00	*0.45*	*0.00*	M_6	2.00e-01	2.00e-01	2.00e-01	0.00	0.00
XX_4	1.09e-01	5.60e-03	1.09e-01	*0.10*	*0.00*	Ia_6	**1.14e-02**	1.10e-02	1.12e-02	0.00	0.00
XY_4	2.90e-05	-3.66e-03	2.90e-05	*0.00*	*0.00*	Fv_6	**6.94e-01**	6.40e-01	6.37e-01	0.05	0.06
XZ_4	1.35e-03	-2.60e-03	1.35e-03	*0.00*	*0.00*	Fs_6	**0.00e+00**	4.20e-01	4.08e-01	0.42	0.41
YY_4	1.08e-01	1.08e-01	1.08e-01	*0.00*	*0.00*	τ_{off6}	**0.00e+00**	1.88e-01	1.74e-01	0.19	0.17
YZ_4	-1.17e-03	-6.64e-03	-1.17e-03	*0.01*	*0.00*	Fvm_6	**5.92e-01**	6.04e-01	5.98e-01	0.01	0.01
ZZ_4	4.07e-03	3.78e-03	4.07e-03	*0.00*	*0.00*	Fsm_6	**1.88e+00**	1.78e+00	1.81e+00	0.10	0.07

$e_{bi} = \left| \hat{\chi}_{st}^{0b} - \chi_{st}^{ref} \right|$, $e_{ei} = \left| \hat{\chi}_{st}^{0e} - \chi_{st}^{ref} \right|$. *norm*($e_{bi}$)/*norm*(χ_{st}^{ref})=0.1591. *norm*(e_{ei})/*norm*(χ_{st}^{ref})=0.0416.

The standard parameters are calibrated using the approach presented above. The trajectory used for identification consists of 11 through points linked by a trajectory using the trapezoidal acceleration interpolation function of the controller CS8C of the Stäubli robots. In Table II, the parameters $\hat{\chi}_{st}^{0b}$ are those computed using the matrix W_{st} (16) defined with the n_b=61 independent columns of the base parameters and the parameters $\hat{\chi}_{st}^{0e}$ are those calculated using the matrix W_{st}^{e} (31) defined with the n_e=31 independent columns of the essential parameters. The difference with respect to the *a*

with the *IDIM* (2) computed with the parameters $\hat{\chi}_{st}^{0e}$, on trajectories different from those used for the identification. The relative norm error remains less than 10%, which shows that the joint torques are well estimated.

Let us now verify the physical consistency of the identified parameters. The identified parameters are computed at the joint centre position of each link. They are physically consistent if the identified mass is positive and the inertia matrix calculated at the center of mass (CoM) of each link is positive definite. We use the Huygens theorem matrix transformation formula to compute the inertia matrix J_j at the CoM, from the identified parameters according to:

$$J_j = \begin{bmatrix} XX_j & XY_j & XZ_j \\ XY_j & YY_j & YZ_j \\ XZ_j & YZ_j & ZZ_j \end{bmatrix} - \frac{1}{M_j} \begin{bmatrix} MY_j^2 + MZ_j^2 & -MX_jMY_j & -MX_jMZ_j \\ -MX_jMY_j & MX_j^2 + MZ_j^2 & -MY_jMZ_j \\ -MX_jMZ_j & -MY_jMZ_j & MX_j^2 + MY_j^2 \end{bmatrix} \quad (36)$$

The positive definitiveness of J_j can be tested either with eigenvalue decomposition, with the Sylvester theorem, or a Cholesky decomposition. Each method is equivalent; however, as noted in [1], the Sylvester theorem allows us to find conditions that the parameters must verify to obtain the positive definitiveness. In the case of a failed test, these conditions make it possible to adjust the parameters to obtain a positive definite matrix by modifying the inertial parameters that are in the null-space of the regressor, i.e. the non-base parameters. The parameters are not independent, thus modifying one parameter results in the modification of all the non-base parameters and manipulations need precautions. The Cholesky decomposition presents the advantage that a tolerance $\varepsilon \leq 0$ can be set in the algorithm and allows for taking into account noise and measurement error, which in the case of experimental data is of importance. It is similar to setting the tolerance that defines a numerical rank in the *SVD* or *QR* decomposition. The tolerance is chosen according to the error and the level of noise in the collected data. Results on the positivity of inertia matrices using the Cholesky decomposition are shown in Table III. The parameters obtained with the base parameters $\hat{\chi}_{st}^{0b}$ need a tolerance $|\varepsilon| \geq 0.04$ to obtain definitive positive matrices for all the links, while the use of essential parameters $\hat{\chi}_{st}^{0e}$ needs only the zero tolerance.

TABLE III
TOLERANCE OF THE CHOLESKY FACTORIZATION AND NUMBER OF DEFINITE POSITIVE INERTIA MATRICES IN THE DIFFERENT CASES

Tolerance	$\hat{\chi}_{st}^{0b}$	$\hat{\chi}_{st}^{0e}$
0 strict	2	6
-0.01	3	6
-0.02	5	6
-0.04	6	6

VI. CONCLUSION

A new method for computing a set of standard essential and consistent dynamic parameters closest to *a priori CAD* values, using *SVD* factorization and *LS* techniques, was presented. This method was experimentally validated on an industrial Stäubli TX-40 robot and give extremely conclusive results. The positivity of inertia matrices using the Cholesky decomposition have shown that the standard parameters identified on the space spanned by the n_e columns of W_{st}^e corresponding to the essential parameters and closest to *a priori* consistent values, are consistent for all the links with a zero Cholesky tolerance. This is a strong result, which means that the essential parameters, which have significant identified values with respect to their small standard deviation (de pending on measurement and modelling errors), are consistent because they lead to identify a set of standard essential consistent parameters. The base parameters which are not well identified are inconsistent because they lead to inconsistent standard parameters.

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