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Application of a micro-genetic algorithm for gait development on a bio-inspired robotic pectoral fin*

Jeff C Kahn, Jr.[1] and James L. Tangorra,[2] *Member, IEEE*

***Abstract*— Biologically-inspired robotic (biorobotic) platforms have been successfully adapted for engineering use, but it is difficult to extend these platforms' locomotive gaits to meet optimization goals. The gait spaces of biorobotic platforms can be very large, with multiple local optima and intractable numerical models. Further, the time cost of empirical exploration is often prohibitive. Micro-genetic algorithms have been successful in developing inverse kinematics in simulation, optimizing in spaces with numerous local optima, and working quickly to optimize with low numbers of trials, but have not yet been evaluated for online robotic gait development. To address the problem of engineering gait development in a biorobotic space, a micro-genetic algorithm (μGA) is evaluated on a biorobotic pectoral fin platform. The μGA effectively optimizes in the gait space with low time costs, discovering new gaits that optimize thrust force production on the swimming fin. The μGA also reveals parameter tuning strategies for changing propulsive forces. Overall, the μGA framework is shown to be effective at online optimization in a large, complex biorobotic gait space.**

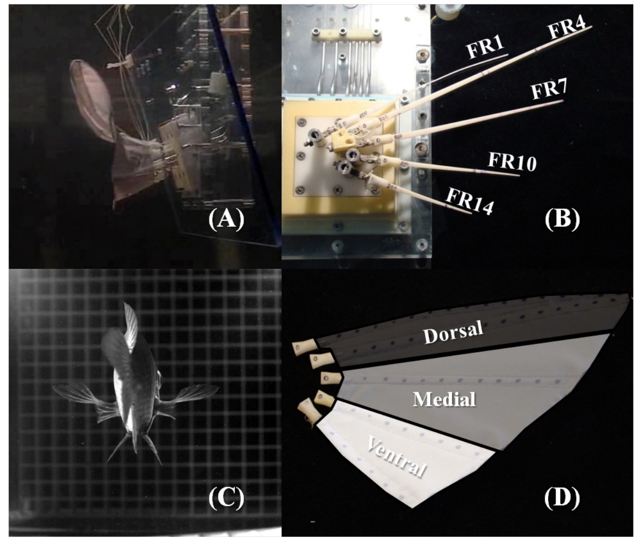

Fig. 1. In this study, a biologically-inspired pectoral fin platform was used to test the effectiveness of a micro-genetic algorithm for developing gaits in large kinematic spaces. The biorobotic fin (A) matches the kinematics, mechanical properties, and hydrodynamics of the steady swimming gait of a bluegill sunfish (C). The fin is composed of 5 fin rays (B) connected by a flexible webbing (D) that is driven by a servo tendon system to produce forces underwater. The kinematics of the first DOF ("cupping") were labeled FR1, FR4, FR7, FR10, and FR14; these indices refer to their biological counterparts. The kinematics of the second DOF ("sweeping") were labeled FR1b, FR10b, and FR14b. The fin was functionally divided into segments. The long, flexible *dorsal leading edge* is formed by the fin rays and webbing of FR1 and FR4; the *ventral leading edge* formed by the shorter length FR10,FR14 and webbing; the *medial area* formed by FR7 and webbing. Sunfish image (C) used with permission of George V. Lauder.

I. INTRODUCTION

Researchers in biologically-inspired locomotion have successfully used robotic platforms to understand and approximate complex animal gaits [1][2][3][4][5]. Biorobotic platforms have also been adapted to meet specific engineering goals [6][7][8], but it is difficult to optimize these platforms for force production over their broad gait spaces (the high dimensional spaces formed by the kinematic parameters). By design, most studies evaluate a small region of the space near the biological behavior of interest. Optimization over the broader gait space could extend the range of behavior possible with bio-inspired platforms.

However, the gait spaces of bio-inspired robots are frequently large and complex due to many actuated degrees of freedom [9], compliant mechanisms [10], and non-linear dynamics, making broad optimization challenging. Optimization can be even more difficult without a numerical system model, making simulation infeasible and local optima hard to identify. Even if a model exists, generalized numerical modeling is often infeasible beyond the gaits and behaviors of interest. These gait spaces are usually too large for empirical evaluation; new gait development strategies must be employed to optimize for engineering goals.

Genetic algorithms, or heuristic approaches that "evolve" a population of solutions based on a fitness function, can successfully optimize in large parameter spaces without a numerical model, but fall short in online implementation. A few studies have evolved behaviors with the use of simulated robot teams [11][12] and in simulated optimization of gait parameters [13][14]. However, traditional genetic algorithms can converge too quickly to local optima [15], exploring small regions of the solution space with a depth-first approach. Genetic algorithms can be time-consuming for online implementation in large spaces, where the evaluation of each solution requires an experimental trial. These factors make the basic genetic algorithm a good choice for simulated robotic gait development, but a poor choice for online biorobotic gait development where spaces are complex and fewer general models exist.

Micro-Genetic algorithms (μGAs) present a framework

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[1]J.C. Kahn, Jr. is a Ph.D. Candidate with the Laboratory of Biological Systems Analysis, Mechanical Engineering and Mechanics, Drexel University, Philadelphia, PA, USA. E-mail: kahn at drexel.edu

[2]J.L. Tangorra is an Assistant Professor with the Department of Mechanical Engineering and Mechanics, Drexel University, Philadelphia, PA, USA. E-mail: tangorra at coe.drexel.edu

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to optimize in large parameter spaces by identifying and evolving diverse local optima, but they have yet to be evaluated in online robotic platforms. Recent work by Hedrick et al. developed a micro genetic algorithm μGA for the inverse kinematic problem of hawkmoth flight [16], evolving simulated wing gaits to approximate force trajectories in live moths. Work by Doorly et al. used a general genetic algorithm in online framework to test the evolutionary principle of selection with robots [17]. Theoretical work developing μGAs demonstrates their effectiveness in finding near-optimal solutions in landscapes with multiple local optima [18]. These developments suggest that μGAs could be effective for generating optimized gaits for bio-inspired robots, though to the authors' knowledge μGAs have not been evaluated for this application.

A biologically-inspired robotic model of a bluegill sunfish pectoral fin (Fig. 1) is an excellent candidate for evaluation in the μGA framework. The platform was designed to study the mechanisms of pectoral fin force production during swimming. It approximates the kinematics, mechanical properties, forces, and hydrodynamics of the fish fin and has been used to study the gaits of steady forward swimming [19][20], yaw turn maneuvers [21], and hovering in place [22]. Engineering (non-biological) gaits have been developed by modifying a steady swimming gait [23], though no broad gait optimization has been conducted. Researchers have developed low order numerical models of sunfish steady swimming [24] and yaw turn maneuvers [25] and validated these models against robot performance. But given the variable fin stiffness, non-linear dynamics, and complex vorticity, a general numerical model of kinematics and forces is currently infeasible [26]. The lack of a numerical model, the complexity and size of the gait space, and the empirical nature of the platform make it appropriate for μGA evaluation.

To address the problem of gait optimization in large biorobotic parameter spaces, a μGA is evaluated on the biorobotic pectoral fin platform. The μGA develops swimming gaits that optimize for thrust production. Contributions include the development of methods for implementing a μGA on a robotic platform (Sections II-A,II-B), μGA discovery of engineered gaits for swimming fins, detailed understanding of the parameter space and outputs for fin gaits and propulsive forces, and the comparison of known fish swimming gaits with those found in the μGA framework (Section III).

II. METHODS

To evaluate the effectiveness of a μGA in a large, complex parameter space, the μGA was applied to an existing biologically-inspired robotic (biorobotic) pectoral fin. The μGA was developed based on the methods described in [16] and included the genetic operators of roulette-wheel selection, bit-wise mutation, and crossover of parameters to evolve candidate gaits. Successive generations of candidate swimming gaits were tested with propulsive force measurement on the biorobotic platform. The fitness of a gait was determined experimentally by the average thrust produced through a stroke.

A. Micro-genetic Algorithm

A μGA works by testing a large population of random gaits, sampling quality gaits from the population to form a sub-population, and evolving multiple sub-populations with the use of genetic operators. The μGA first generated a random population (P) of candidate solutions of fixed size (N). This entire random population P was tested with force measurement and fitnesses were computed for each candidate gait. At each major iteration, a fixed number of gaits (i) were sampled from P, forming an sub-population P_i (Fig. 2). The sub-population was then evolved iteratively.

At each loop iteration, genetic operators were used to improve the fitness of gaits in the sub-population P_i. For each generation, elitism was applied to P_i, selecting the first non-dominated vector of the population, P_{elite}. Elitism preserved the genetic information of the best solutions. Next, selection was applied, where $i-1$ candidate solutions were sampled from a fitness-weighted distribution, forming the selected population $P_{i,s}$. The probability of an individual candidate solution's selection $p(X_i = CG_i)$ was given a normalized weight of its fitness as in (1). Following selection, crossover was applied between randomly generated pairs of candidate gaits, in which their genetic information was swapped at a random index, forming two offspring candidate gaits and creating $P_{i,c}$. Crossover shares genetic information of high-fitness gaits, forming offspring of paired gaits. Bit-wise mutation was applied to the members of $P_{i,c}$ with a fixed probability $p(m)$, forming P_i, m. Mutation added randomness to the search by inverting bits of the candidate gait binary representation. The non-dominated solution P_{elite} and the mutated solutions $P_{i,m}$ were merged into a new population P_i, completing one iteration of the μGA. The fitness of the new population P_i was established through force testing. Following testing, when the planned number of generations was reached, the loop terminated.

$$p(X_i = CG_i) = \frac{CG_{i,fitness}}{\sum CG_{fitness}} \quad (1)$$

After loop termination, all elite candidate gaits from the evolved sub-population were saved to the growing portion of the random population. These filtered gaits could be re-sampled in future iterations during the sampling stage. The use of a growing random population is unique to μGAs and typically produces a diverse distribution of solutions along a near-optimal front [27].

B. Biorobotic Fin Implementation

The biorobotic fin was developed to approximate the kinematics, mechanical properties, and hydrodynamics of a swimming sunfish pectoral fin (see [20],[22], Fig. 1). The biorobotic fin was composed of multiple fin rays enclosed in a fabric webbing; a servo-tendon system driving up to two degrees of freedom (DOF) on each fin ray to produce gaits (Fig. 1A,B).

To apply a μGA to the biorobotic fin, the components of a gait were parametrized and represented in a genetic algorithm framework. To parametrize kinematic trajectories for each

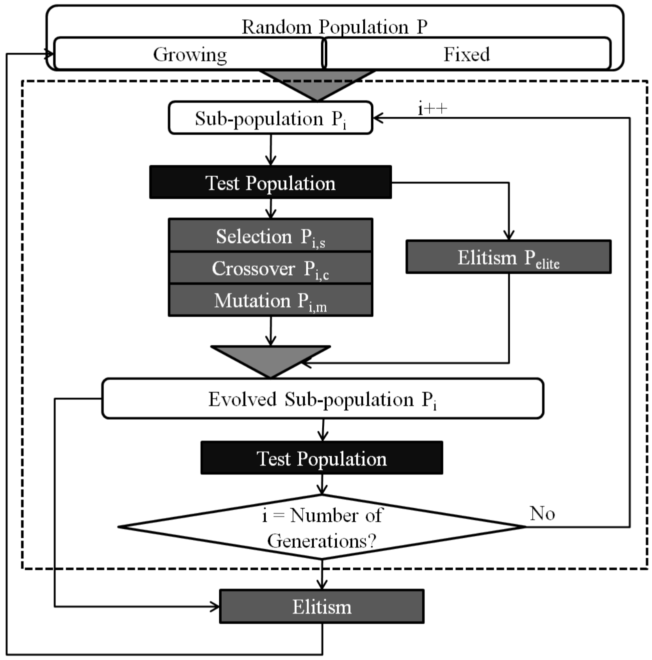

Fig. 2. A block diagram shows the steps of the live testing μGA - a genetic algorithm that tests small populations and allows for reinitialization of the evolving population. The fitness of candidate gaits was determined through testing when the random population was first generated, and at the generation of each new population (shaded blocks). The main program iterated (dotted line) and tested generations of initial populations until the convergence criterion was reached. Convergence was determined by number of generations per iteration. Diagram modified from [28].

actuated fin ray DOF, the underlying kinematic trajectory of steady swimming ([20]) was used and the amplitude (A; degrees), phase offset (P; fraction of period T), and flapping frequency (F; Hz) were varied, forming the range of kinematic patterns (Table I). Changes in fin mechanical properties require time-consuming swaps of fin equipment so fin ray flexural rigidity was fixed at a stiffness with known strong thrust production (EI = 800 times the biological rays;[19]). Parameter values were selected so as to avoid damage to the platform, by restricting phase lags between segments and limiting flapping frequency. The kinematics of an individual fin ray were represented by a binary array of 18 elements, and thus the kinematics of an entire fin were represented using 144 binary elements, forming a “candidate gait” for the genetic algorithm. The solution space specified by the kinematic parameters contains over 2 million possible fin trajectories, so brute force search of the space was not feasible.

TABLE I

PARAMETER SPACE OF FIN RAY KINEMATICS

	Amp. (A;°)	Phase (P;T)	Freq. (F;Hz)
Minimum	0	0.00	0.25
Maximum	63	0.31	1.52
Increment	1	0.01	0.01
$2^{\#bits}$	2^6	2^5	2^7

Each candidate gait was represented by a data cluster containing the kinematic parameters (see Table I), the servo trajectories, the measured propulsive forces in thrust and lateral planes, and the fitness (or quantitative measure of solution quality). The kinematic parameters and servo trajectories were selected by the μGA, while forces and fitness were determined through testing. Average thrust (N) through a fin beat period was used as the fitness criterion to explore basic forward swimming. The fitness landscape was a 25-dimensional space, formed by the {Amplitude, Phase, Frequency} parameters of each of the candidate gaits and the fitness of their forces.

C. Experiments

Candidate gaits were tested in real time on the biorobotic platform with measurement of force and execution of μGA-specified kinematic patterns. All force and kinematic data were sampled at 100 Hz on analog input channels and stored in a single delimited log file. The trials were filmed at 60 frames per second (Exilim FX-1, Casio, JP) to observe gaits and fin bending underwater. Experiments were carried out through use of a custom robot graphical user interface in the LabVIEW programming environment (National Instruments, Austin, TX, U.S.A.) that drove servo kinematics on the robot (described in [20]). The robot was mounted onto a low-friction air bearing carriage (New Way Air Bearings, Aston, PA, USA) and propulsive forces were measured in the thrust and lateral directions (LSB200, Futek Advanced Sensor Technology, LLC., Irvine, CA, USA) in a standing water tank.

μGA trial parameters were tuned to reduce trial time, obtain diverse elite gaits, and evaluate a simple fitness criteria. Each trial had a random population P of 50 candidate gaits, a total of 10 iterations of sub-populations P_i, 5 generations per sub-population, and 5 candidate gaits per generation, leading to a testing of 300 candidate gaits per trial. A total of 5 trials were conducted, each lasting approximately 50 minutes. Fitness was calculated as the average thrust force through the fin stroke. After each generation, elite candidate gait data were streamed to file, including parameters, force, and fitness measures. Each candidate gait took between 4 and 10 seconds to test and save, and genetic operator run-times were negligible.

III. RESULTS & DISCUSSION

The μGA was effective at identifying diverse, locally-optimal gaits for the optimization of thrust in the large biorobotic parameter space. The μGA developed new gaits that extended existing strategies of thrust production on the biorobotic fin. The μGA identified a new non-biological gait for thrust production with comparable force production to the bio-inspired steady swimming gait. This gait, termed “μGA-bimodal” (see Fig. 3), used a high-amplitude, early-deployment, rapid dorsal edge movement ($A_{FR1,4} = [40, 50]^{\circ}$,$P_{FR1,4} \approx 0T$,$F_{FR1,4} = [1.0, 1.3]Hz$) in combination with a high-amplitude, late-deployment, slow ventral edge movement (see Fig. 3). These kinematics caused a thrust-producing instroke followed by a burst of thrust in the later outstroke, something not documented before in fish or the robotic platform. Elite gaits (local optima) of the μGA approximated the kinematics and force production of known bio-inspired gaits of steady swimming and hovering. One elite solution generated matched closely the kinematic parameters of steady swimming (Fig. 5), following the typical pattern of: low or no amplitude along the second degree of freedom fin rays ($A_{FR7,10,14} \rightarrow 0°$), high amplitudes along the dorsal leading edge ($A_{FR1,4} \rightarrow 60°$), and little phase lag between segments ($P_{all} \approx 0T$). μGA solutions typically produced between 80 and 90% of the average thrust of a bio-inspired steady swimming gait.

Another elite solution generated, "μGA-hover," closely matched the kinematics used by the sunfish in hovering, typified by: early deployment of the dorsal leading edge ($P_{FR1,4} \approx 0.0T$), late deployment of the ventral leading edge ($P_{FR7,10,14} \rightarrow 0.3T$), and late, high-amplitude, deployment of the second DOF along the ventral leading edge ($A_{FR10b,14b} \approx 30°$;"lift and drop" pattern detailed in [22]). Typical bio-inspired hover gaits produce nearly balanced lateral and thrust forces (Force Means $\approx 0N$), but when hover was executed at high speeds ($F \approx 1.0Hz$) and with stiff fins ($EI = 800x$), it was a strong thrust producing mode [22].

Local optima were quickly reached in μGA execution. The "μGA-bimodal" gait converged (less than 1% change in solution quality between generations) after 50 total gaits were tested (Fig. 3), "μGA-hover" after 23 gaits, and "μGA-steady" after 10 gaits (each in their respective trials). With trial times ranging on $4 - 10$s, this meant that local optima convergence was obtained on the order of minutes.

The μGA revealed fine-tuning strategies for improving the thrust production of the biorobotic fin. Changes to individual fin ray parameters affected the fitness of candidate solutions (Fig. 4). Fitness was negatively affected by large differences in phase lag between fin rays, except in the case where the ventral rays and dorsal rays were deployed at similar lags respectively (i.e. $P_{FR1} \approx P_{FR4}$ and $P_{FR10} \approx P_{FR14}$), where fitness was positively affected by similar phase lags among segments. Fitness increased as phase lags approached zero ($P_{FR1,4,7,10,14} \rightarrow 0.0T$). Fitness increased as first DOF amplitudes ($A_{FR1,7,10,14}$) increased, excepting fin ray 4, which produced high fitness at lower amplitudes ($A_{FR10} = [10, 20]°$). Increasing the flapping frequency of FR4 tended to increase fitness. Increasing the flapping frequency on other fin rays had no consistent effects on fitness.

μGA parameters required tuning to determine trial conditions that would produce diverse, high-fitness gaits. Consistent with simulation results in [18], increasing the number of generations per iteration (beyond 5) did not significantly affect the quality of solutions found, and increasing the generation size resulted in a linear increase in testing time. Increasing the size of the starting random population (P) tested was the most effective way to improve the quality of solutions found without significantly adding to testing time. Increasing the number of iterations only improved quality

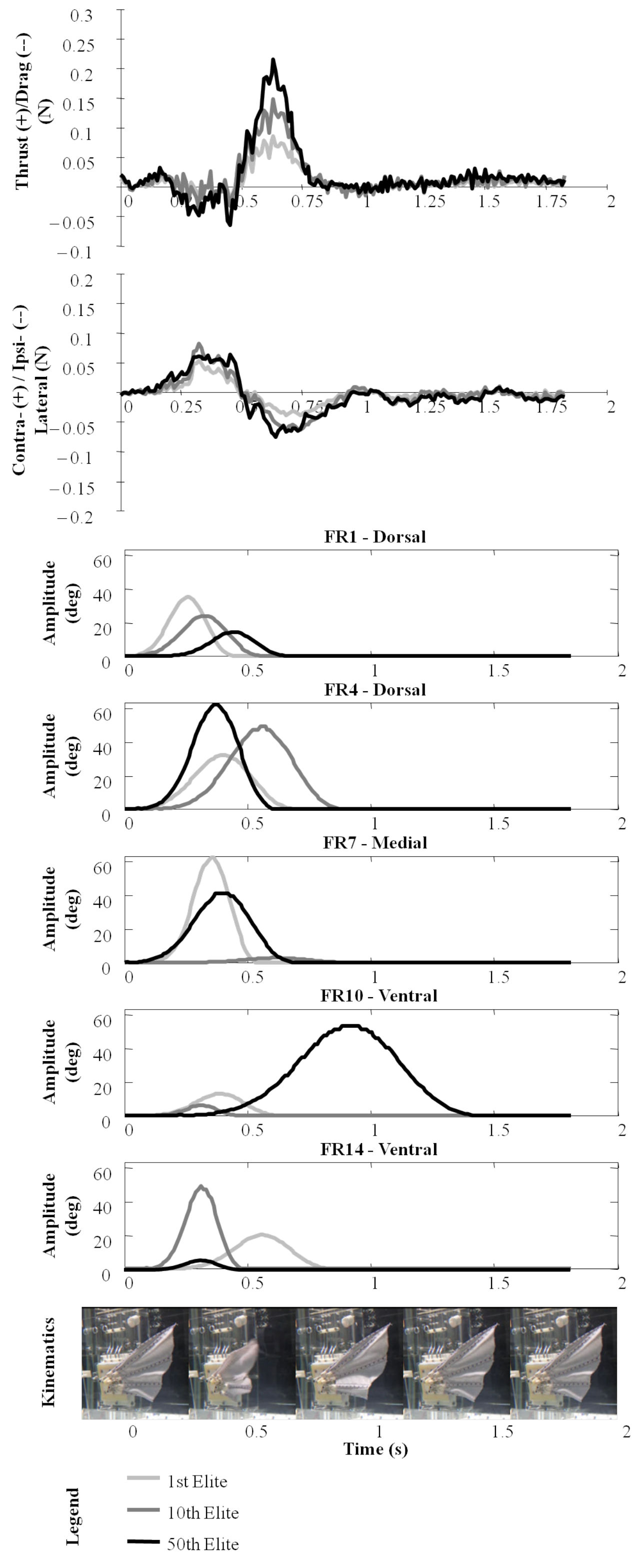

Fig. 3. The μGA evolved a new non-biological swimming gait for thrust production. Evolution of the "bimodal" candidate gait over fifty generations in a local optima region shows the improvement of thrust production (A). The evolution of kinematics (B) show an increase of amplitude on the dorsal leading edge fin rays, causing increase in thrust production through the outstroke (t=$[0, 0.25]$s) and instroke (t=$[0.75, 1.25]$s). "Bimodal" gaits evolved to employ a delayed movement of the ventral kinematics to produce slight thrust in the late instroke (t=$[1.0, 1.5]$s). Data were low pass filtered at 7Hz for clarity.

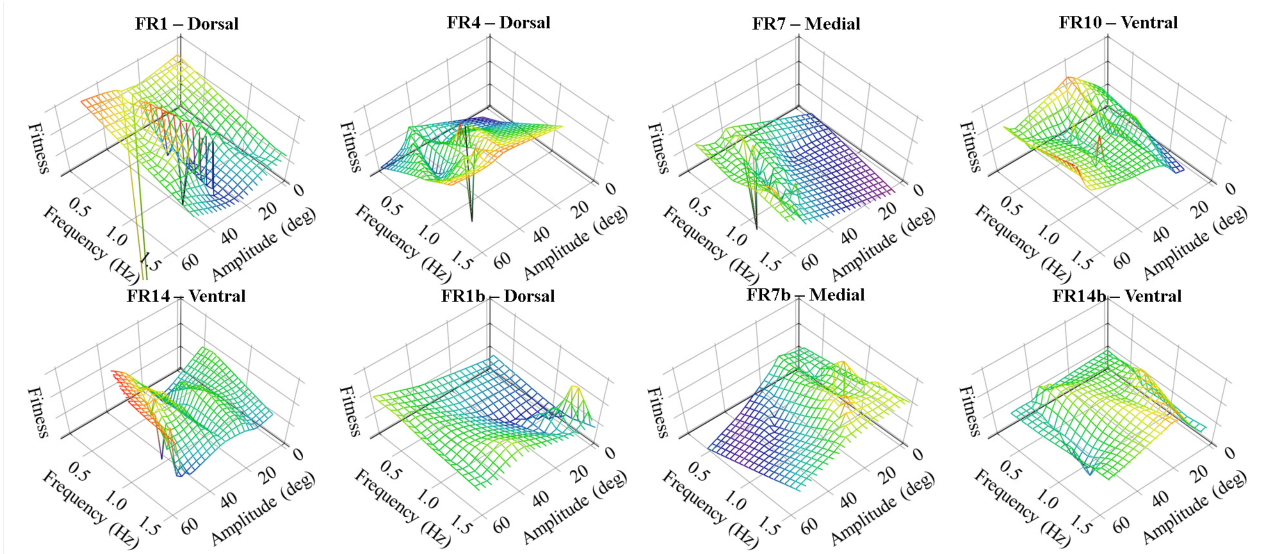

Fig. 4. The μGA revealed fine tuning strategies for fin ray degrees of freedom (DOF) in the biorobotic platform. The kinematic parameters of "Amplitude" and "Frequency" are varied along each of the DOF. "Phase" variations had unclear impacts on fitness and are excluded from these figures. Landscapes were constructed by meshing of 300 candidate solution fitnesses over the broad range explored in one trial of the μGA.

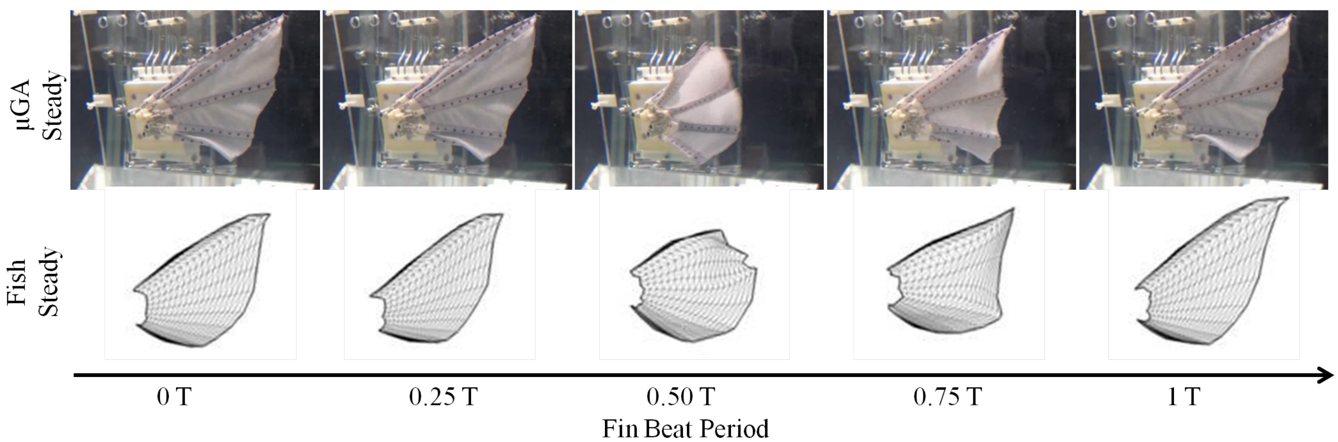

Fig. 5. Elite gaits (local optima) of the μGA approximated the kinematics and force production of known bio-inspired gaits of steady swimming and hovering (not shown). A comparison of an elite (i.e. locally optimal) candidate gait of the μGA (TOP) to a sunfish steady swimming gait (BOTTOM). Small phase differences in the μGA solution led to near-optimal performance of the gait. Steady swimming in both the evolved gait and the biology produces a strong thrust force using the dorsal leading edge segment of the fin with little phase lag between fin segments. μGA solutions typically produced between 80 and 90% of the average thrust of a biologically-inspired steady swimming gait. Steady swimming images modified from [21].

of solutions when the random population was sufficiently large (above 50 solutions), but was a very costly linear operation. Increasing the number of iterations often resulted in exploration of the same solution spaces without adding to diversity. Increasing the bit-wise mutation rate beyond 5% did not have a significant impact on solution quality.

IV. CONCLUSIONS

Overall, this study demonstrated that a μGA framework is effective for optimizing in biorobotic gait spaces. Several diverse gaits were developed for thrust production that were comparable in quality to previous bio-inspired gaits. The μGA discovered new gaits that extended the capabilities of the biorobotic platform in short numbers of experiments. The μGA identified gaits approximating the biological gaits of steady swimming and hovering, and both were local optima in the gait space. The μGA gait space also provided insight into the effects on fitness of tuning individual parameters in the robot degrees of freedom. μGA parameter tuning was straightforward.

Future work can be done to improve the quality and diversity of gaits developed in the μGA framework. While regions of local optima were explored, precise local optima were not determined in this study. For future work, a simplex algorithm could be used to better explore the space of local optima with hill climbing, using methods from [29]. The μGA could be modified to produce better solution diversity without increasing trial time with the technique of "niching," with methods from [30].

The μGA framework will be used in future study with the biorobotic fin platform to develop new gaits that optimize for other useful engineering goals. Simple changes could optimize for balanced forces through the fin stroke, strong lateral forces to produce maneuver behaviors, or the inverse kinematics problem. For instance, the μGA framework could be used to search for gaits that minimize the mean square error between a desired force trajectory and the observed, developing inverse kinematics for force trajectories. In similar ways, the μGA can extend the effectiveness of biorobotic platforms.

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REFERENCES

[1] S. Floyd, T. Keegan, J. Palmisano, and M. Sitti, "A novel water running robot inspired by basilisk lizards," in *2006 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2006, pp. 5430–5436.

[2] A. L. Desbiens and M. R. Cutkosky, "Landing and perching on vertical surfaces with microspines for small unmanned air vehicles," in *2nd International Symposium on UAVs, Reno, Nevada, USA June 8–10, 2009*. Springer, 2010, pp. 313–327.

[3] S. Sefati, I. Neveln, M. A. MacIver, E. S. Fortune, and N. J. Cowan, "Counter-propagating waves enhance maneuverability and stability: a bio-inspired strategy for robotic ribbon-fin propulsion," in *2012 IEEE RAS & EMBS International Conference on Biomedical Robotics and Biomechatronics (BioRob)*. IEEE, 2012, pp. 1620–1625.

[4] S. H. Suhr, Y. S. Song, S. J. Lee, and M. Sitti, "Biologically inspired miniature water strider robot," in *Proceedings of the Robotics: Science and Systems I*. Boston, USA, 2005, pp. 319–325.

[5] K. Karakasiliotis and A. J. Ijspeert, "Analysis of the terrestrial locomotion of a salamander robot," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2009*. IEEE, 2009, pp. 5015–5020.

[6] E. Moore, D. Campbell, F. Grimminger, and M. Buehler, "Reliable stair climbing in the simple hexapod'rhex'," in *IEEE International Conference on Robotics and Automation (ICRA), 2002*, vol. 3. IEEE, 2002, pp. 2222–2227.

[7] U. Saranli and D. E. Koditschek, "Back flips with a hexapedal robot," in *IEEE International Conference on Robotics and Automation (ICRA), 2002*, vol. 3. IEEE, 2002, pp. 2209–2215.

[8] A. T. Asbeck, S. Kim, M. Cutkosky, W. R. Provancher, and M. Lanzetta, "Scaling hard vertical surfaces with compliant microspine arrays," *The International Journal of Robotics Research*, vol. 25, no. 12, pp. 1165–1179, 2006.

[9] A. Crespi and A. J. Ijspeert, "Online optimization of swimming and crawling in an amphibious snake robot," *Robotics, IEEE Transactions on*, vol. 24, no. 1, pp. 75–87, 2008.

[10] R. Altendorfer, N. Moore, H. Komsuoglu, M. Buehler, H. Brown, D. McMordie, U. Saranli, R. Full, and D. Koditschek, "Rhex: A biologically inspired hexapod runner," *Autonomous Robots*, vol. 11, no. 3, pp. 207–213, 2001.

[11] J. Pollack, H. Lipson, S. Ficici, P. Funes, G. Hornby, and R. Watson, "Evolutionary techniques in physical robotics," *Evolvable Systems: from biology to hardware*, pp. 175–186, 2000.

[12] H. Lipson, J. Bongard, V. Zykov, and E. Malone, "Evolutionary robotics for legged machines: from simulation to physical reality," in *Proceedings of the 9th Int. Conference on Intelligent Autonomous Systems*, 2006, pp. 11–18.

[13] D. Marbach and A. J. Ijspeert, "Online optimization of modular robot locomotion," in *Mechatronics and Automation, 2005 IEEE International Conference*, vol. 1. IEEE, 2005, pp. 248–253.

[14] L. Yang, C. Chew, A. Poo, and T. Zielinska, "Adjustable bipedal gait generation using genetic algorithm optimized fourier series formulation," in *Intelligent Robots and Systems, 2006 IEEE/RSJ International Conference on*. IEEE, 2006, pp. 4435–4440.

[15] R. Salomon, "Evolutionary algorithms and gradient search: similarities and differences," *IEEE Transactions on Evolutionary Computation*, vol. 2, no. 2, pp. 45–55, 1998.

[16] T. Hedrick and T. Daniel, "Flight control in the hawkmoth manduca sexta: the inverse problem of hovering," *The Journal of Experimental Biology*, vol. 209, no. 16, pp. 3114–3130, 2006.

[17] N. Doorly, K. Irving, G. McArthur, K. Combie, V. Engel, H. Sakhtah, E. Stickles, H. Rosenblum, A. Gutierrez, R. Root *et al.*, "Biomimetic evolutionary analysis: robotically-simulated vertebrates in a predator-prey ecology," in *IEEE Symposium on Artificial Life, 2009*. IEEE, 2009, pp. 147–154.

[18] K. Krishnakumar, "Micro-genetic algorithms for stationary and non-stationary function optimization," in *SPIE: Intelligent control and adaptive systems*, vol. 1196, no. 8. Philadelphia, Pennsylvania: Society of Photo-Optical Instrumentation Engineers, 1989, pp. 289–296.

[19] J. Tangorra, G. Lauder, I. Hunter, R. Mittal, P. Madden, and M. Bozkurttas, "The effect of fin ray flexural rigidity on the propulsive forces generated by a biorobotic fish pectoral fin," *The Journal of Experimental Biology*, vol. 213, no. 23, pp. 4043–4054, 2010.

[20] C. Phelan, J. Tangorra, G. Lauder, and M. Hale, "A biorobotic model of the sunfish pectoral fin for investigations of fin sensorimotor control," *Bioinspiration & Biomimetics*, vol. 5, no. 3, p. 035003, 2010.

[21] J. R. Gottlieb, J. L. Tangorra, C. J. Esposito, and G. V. Lauder, "A biologically derived pectoral fin for yaw turn manoeuvres," *Applied Bionics and Biomechanics*, vol. 7, no. 1, pp. 41–55, 2010.

[22] J. Kahn, B. Flammang, and J. Tangorra, "Hover kinematics and distributed pressure sensing for force control of biorobotic fins," in *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2012, pp. 1460–1466.

[23] J. Tangorra, G. Lauder, P. Madden, R. Mittal, M. Bozkurttas, and I. Hunter, "A biorobotic flapping fin for propulsion and maneuvering," in *IEEE International Conference on Robotics and Automation, 2008*. IEEE, 2008, pp. 700–705.

[24] M. Bozkurttas, R. Mittal, H. Dong, G. Lauder, and P. Madden, "Low-dimensional models and performance scaling of a highly deformable fish pectoral fin," *Journal of Fluid Mechanics*, vol. 631, p. 311, 2009.

[25] S. Ramakrishnan, M. Bozkurttas, R. Mittal, and G. V. Lauder, "Thrust production in highly flexible pectoral fins: A computational dissection," *Marine Technology Society Journal*, vol. 45, no. 4, pp. 56–64, 2011.

[26] G. Lauder, P. Madden, R. Mittal, H. Dong, and M. Bozkurttas, "Locomotion with flexible propulsors: I. experimental analysis of pectoral fin swimming in sunfish," *Bioinspiration & Biomimetics*, vol. 1, no. 4, p. S25, 2006.

[27] E. Zitzler and L. Thiele, "Multiobjective evolutionary algorithms: A comparative case study and the strength pareto approach," *IEEE Transactions on Evolutionary Computation*, vol. 3, no. 4, pp. 257–271, 1999.

[28] C. Coello Coello Coello and G. Toscano Pulido, "A micro-genetic algorithm for multiobjective optimization," in *Evolutionary Multi-Criterion Optimization*. Springer, 2001, pp. 126–140.

[29] J. Yen, J. Liao, B. Lee, and D. Randolph, "A hybrid approach to modeling metabolic systems using a genetic algorithm and simplex method," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 28, no. 2, pp. 173 –191, apr 1998.

[30] S. W. Mahfoud, "Niching methods for genetic algorithms," 1996.