2013 IEEE/RSJ International Conference on
Intelligent Robots and Systems (IROS)
November 3-7, 2013. Tokyo, Japan

Casimir based fast computation for hydraulic robot optimizations

Satoru Sakai

Abstract— **This paper presents a fast computation method of hydraulic robot dynamics and shows the effectiveness in design optimization that needs a lot of repetitive simulation. First, an exact simplification of the conventional representation of hydraulic robots with external forces is presented based on the Casimir functions. Second, a further exact simplification is given via an integration by parts of the Hamiltonian function as a structural property and a new representation of the hydraulic robots is proposed. The new representation allows us to define a back fluidmotive force (back-FMF) as a hydraulic version of the well-known back electromotive force (back-EMF). Third, the proposed representation and the conventional representation are compared with each other with respect to computational cost and the effectiveness of the proposed representation is confirmed.**

I. INTRODUCTION

In humanoid robotics, hydraulic robots are gaining popularity. It is easy to see that actuators of both BIGDOG's legs [7] and ASIMO's hands are now hydraulic. Also, hydraulic systems are classically important in field robotics, such as construction, agriculture, rescue, demining robotics and so on. These are from the following advantages mainly. First, a hydraulic robot is superior to an electrical robot with respect to the power to weight ratio. Second, a hydraulic robot has a property that the joint displacement can be finite for any external force even if the power source is inactive.

On the other hand, in case of electrical robots, the driving subsystem (the electrical system) is simple and almost static, that is, the driving torque is just proportional to the control input. In this case, fast computation methods have studied well in 1990' [21] assuming that the driving subsystem is negligible.

However, in case of hydraulic robots, the driving subsystem (the hydraulic system) is complex and dynamic due to the existence of compressibility. We can not ignore the driving subsystem any more and it has been more difficult to study fast computation methods for hydraulic (and also pneumatic) robots. Of course, it is needless to say that computer specifications have much better than those in 1990'. But, from the viewpoint of design optimization, fast computation methods are pretty important because a lot of simulations are repeated since actual design optimizations are non-convex problems.

The presented fast computation method in this paper is the state-of-the-arts *modeling* in robotic systems & control. The new results can be applied in other fields such as aircraft industry as well as robotic industry. A new representation in this paper is developed by exact simplifications of the conventional representation of the hydraulic robots. The exact simplifications are from the physical and structural properties. The hydraulic robot dynamics are discussed in port-Hamiltonian framework [18] [12] which would be a generalization of classical mechanics framework and also different from $\dot{x} = f(x) + g(x)u$ framework often used in conventional robotics and the others (e.g. [15], [8], [20], [1]). Although so many related works (e.g. [18], [10]) discuss passivity [17] which is one of the physical and structural properties in port-Hamiltonian framework, this paper repeatedly discusses Casimir function which is another physical and structural property in a class of the port-Hamiltonian framework.

A part of this paper presents an extended version of the previous work [11] which does not focus on (any hydraulic) robot dynamics but on hydraulic actuators only and also does not consider any external forces by which the robots perform tasks and interact with the unknown environment. In this paper, it is observed how the whole body and the external forces are taken into account successfully without destroying the framework in the previous work. Furthermore, the effectiveness of the proposed method is confirmed in design optimization.

This paper is organized as follows. Section II presents a preliminary in this paper. Section III proposes a canonical form of forward dynamics. Section VI gives a main result based on an integration by parts and proposes a new representation of the hydraulic robots. Section V confirms the effectiveness of the new representation with respect to the computational cost in design optimization. This paper is concluded in Section VI.

II. PRELIMINARY

A. Conventional representation

In this section, we review the conventional representation of forward dynamics of a hydraulic robot [6] [13] [4] which is described by

$$\begin{cases} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D\dot{q} = G(q)^{-\mathrm{T}} f + \underbrace{J(q)^{\mathrm{T}} d}_{\tau_d} \\ f = A_+ p_+ - A_- p_- \\ \dot{p}_+ = \ bV_+(q)^{-1} \left[-A_+ \dot{s}(q) + B_+(p_+) u \right] \\ \dot{p}_- = \ bV_-(q)^{-1} \left[+A_- \dot{s}(q) - B_-(p_-) u \right] \end{cases} \tag{1}$$

where $q \in \mathbb{R}^n$ are the joint displacements, $p_+ \in \mathbb{R}^n$ are the cap pressures, $p_- \in \mathbb{R}^n$ are the head pressures, $f \in \mathbb{R}^n$ are the driving forces, $d \in \mathbb{R}^n$ are the external forces and $u \in \mathbb{R}^n$ are the spool valve displacements which are the standard control

Satoru Sakai is with Faculty of Mechanical Engineering, Shinshu University, Japan satorus@shinshu-u.ac.jp

inputs [9]. Figure 1 shows the 1-DOF case of the standard hydraulic arm. Here, the inertia matrix $M(q) = M(q)^{\mathrm{T}} > 0$, the centrifugal and Coriolis matrix $C(q,\dot{q})$and the damping matrix $D(q) = D(q)^{\mathrm{T}} \geq 0$ are defined in the left-hand side of the first equation. The first Jacobian $J(q) \in \mathbb{R}^{n\times n}$, the second Jacobian $G = \nabla_q^{\mathrm{T}} s(q) \in \mathbb{R}^{n\times n}$ with the cylinder displacements $s(q) \in \mathbb{R}^n$ and the cylinder areas A_+, A_- are defined in the right-hand side of the first equation. In the second and third equations, the cylinder volumes $V_+(q) = \mathrm{diag}(A_+(L/2\cdot\mathbf{1} + s(q))), V_-(q) = \mathrm{diag}(A_-(L/2\cdot\mathbf{1} - s(q))) \in \mathbb{R}^n$ with the stroke L, the flows $B_+u, B_-u \in \mathbb{R}^{n\times n}$ through the spool valve are defined with the bulk modulus b. In the paper, the superscript $+$ means the cap-side and the subscript $-$ means the head-side.

Remark The flows $B_\pm u$ are often approximated by Bernoulli law, that is,

$$B_\pm(p_\pm) = \mathrm{diag}\left(k\sqrt{\bar{p} \pm \mathrm{sgn}(u_i)\,(\bar{p} - p_\pm)}\right)$$

where k is the valve coefficient and $\bar{p} := (p_{\mathrm{upper}} + p_{\mathrm{lower}})/2$ is the mean value of the source pressures, that is, the pump pressure p_{upper} and the tank pressure p_{lower}, respectively.

Remark The forward dynamics (1) is a popular and practical model based on several assumptions. For example, the second and third equations in dynamics (1) are discretized from the (compressible) Navier-Stokes equations which are already based on the continuum approximation. Also the control input is the spool displacement and the spool dynamics is neglected.

B. Casimir function

In conventional robotic systems and control, one of the most popular representations is Lagrangian form [2] which is described by

$$\frac{d}{dt}\nabla_{\dot{q}} L(q,\dot{q}) - \nabla_q L(q,\dot{q}) = \tau \tag{2}$$

where $L = T(q,\dot{q}) - U(q) \in \mathbb{R}$ is the Lagrangian with the kinetic energy $T = (1/2)\dot{q}^{\mathrm{T}} M(q)\dot{q}$ and the potential energy $U(q)$ and $\tau \in \mathbb{R}^n$ is the input torque.

It may be observed that the model (1) is a Lagrangian system augmented with two state equations on the pressures. In the sequel we shall use the port Hamiltonian framework which is precisely suited for such systems and allowing for the control inputs not to be generalized forces and the state spaces not being any tangent bundles of the configuration manifold [18] [19].

A Casimir function is one of the properties of port-Hamiltonian systems which describe physical systems including conventional mechanical systems. A standard state space expression of port-Hamiltonian systems is given as

$$\begin{cases} \dot{x} &= (J(x) - R(x))\,\nabla_x H(x) + g(x)u \\ y &= g(x)^{\mathrm{T}} \nabla_x H(x) \end{cases} \tag{3}$$

where $x \in \mathbb{R}^n$ is the state, $u, y \in \mathbb{R}^m$ are the input and the output, respectively. $J(x) = -J(x)^{\mathrm{T}} \in \mathbb{R}^{n\times n}$ is a skew-symmetric matrix and $R(x) \geq 0 \in \mathbb{R}^{n\times n}$ is a semi-positive definite matrix. The function $H(x) \in \mathbb{R}$ is a Hamiltonian

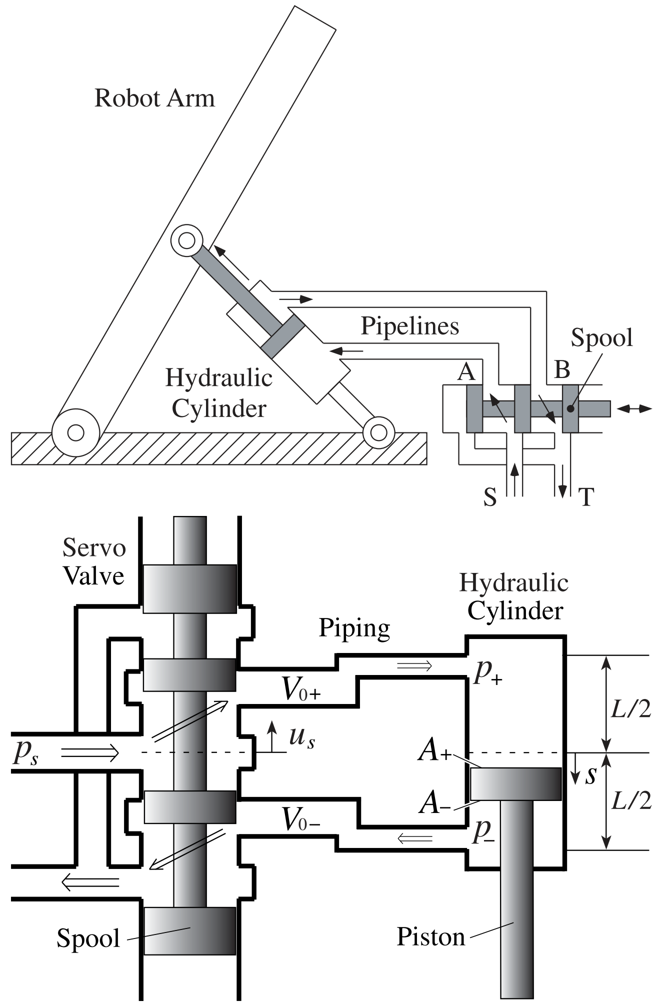

Fig. 1. The standard hydraulic arm (1-DOF case).

function. Here ∇_x denotes the partial derivative with respect x. In case of zero input $u \equiv 0$, if $R(x) = 0$,

$$\frac{dH}{dt} = \nabla_x^{\mathrm{T}} H(x) J(x) \nabla_x H(x) = 0$$

holds and the value of the Hamiltonian function is a conserved quantity. A Casimir function $C(x) \in \mathbb{R}$ is defined as a solution of the following PDE

$$J(x)\nabla_x C(x) = 0. \tag{4}$$

From this definition, in case of zero input $u \equiv 0$, if $R(x) = 0$,

$$\frac{dC}{dt} = \nabla_x^{\mathrm{T}} C(x) J(x) \nabla_x H(x) = 0 \tag{5}$$

holds and the value of the Casimir function is also a conserved quantity. Note that the solution of the PDE (4) does not always exist and if

$$J(x) = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix},\ g(x) = \begin{bmatrix} 0 \\ I \end{bmatrix}, \tag{6}$$

and $u = \tau$, then the state-space equation of system (3) is equivalent to Equation (2).

Lemma 1[18][14] *Suppose that the PDE (4) has a solution $C(x)$. Then the system (3) is equivalent to the following system*

$$\left\{\begin{array}{lcl} \dot{x} &=& J(x)\nabla_x(H(x)+f(C(x)))+g(x)u \\ y &=& g(x)^{\mathrm{T}}\nabla_x H(x) \end{array}\right. \tag{7}$$

for any function $f(\bullet)\in\mathbb{R}$. That is, the Hamiltonian function in only the state equation of the system (3) is replaced by

$$H \to H + f(C). \tag{8}$$

Proof of Lemma 1. This is immediately proved by Equation (4). ■

Remark Note that the Hamiltonian function in the output equation of the system (3) is not replaced and is still equal to H. In this sense, the following holds

$$y \neq g(x)^{\mathrm{T}}\nabla_x(H(x)+f(C(x))).$$

The definition of Casimir function (4) can be generalized in case of $R\neq 0$.

III. CANONICAL FORM

In this paper, the exact simplification consists of two stages. In the first stage, the forward dynamics is simplified using Casimir functions which is a general property of port-Hamiltonian systems with the solution of the PDE (4). In the second stage, the simplified representation is further simplified using an integration by parts which is a special property of the hydraulic robots. The resulting representation is different from any conventional representations such as in [3] [6] [13]. The first stage corresponds to Section III. The second stage corresponds to Section VI.

Proposition 1 (Canonical form) *Consider the hydraulic robots (1). Then there exists a coordinate transformation which converts the system (1) to the state-space equation of the following system*

$$\left\{\begin{array}{lcl} \begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{C}_+ \\ \dot{C}_- \end{bmatrix} &=& \begin{bmatrix} 0 & +I & 0 & 0 \\ -I & F_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\nabla_{x_c}H + b\underbrace{\begin{bmatrix} 0 \\ \tau_d \\ +V_+^{-1}B_+u \\ -V_-^{-1}B_-u \end{bmatrix}}_{g_C} \\ y &=& g_C^{\mathrm{T}}\nabla_{x_c}H \end{array}\right. \tag{9}$$

with the state $x_c := (q,p,C_+,C_-)$ and the Hamiltonian function

$$\begin{array}{l} H = T \\ \quad +A_+b\mathbf{exp}(C_+/b) - V_+(q)(C_+ + b - b\mathbf{log}(V_+/A_+)) \\ \quad +A_-b\mathbf{exp}(C_-/b) - V_-(q)(C_- + b - b\mathbf{log}(V_-/A_-)). \end{array}$$

and

$$F_{22} := -G(q)\nabla_q(G(q)p) + \nabla_q^{\mathrm{T}}(G(q)p)G(q)^{\mathrm{T}} - D.$$

Proof of Proposition 1. Just for readability, $G(q)=I$ is assumed at first. By taking the moment $p=M(q)\dot{q}$ instead of the velocity $\dot{q}$, the system (1) is equivalent to the state space equation of the following port-Hamiltonian system [3] [6]

$$\left\{\begin{array}{lcl} \dot{x}_p &=& \begin{bmatrix} 0 & +I & 0 & 0 \\ -I & -D & -J_{32}^{\mathrm{T}} & -J_{42}^{\mathrm{T}} \\ 0 & J_{32} & 0 & 0 \\ 0 & J_{42} & 0 & 0 \end{bmatrix}\nabla_{x_p}H + b\underbrace{\begin{bmatrix} 0 \\ \tau_d \\ +V_+^{-1}B_+u \\ -V_-^{-1}B_-u \end{bmatrix}}_{g_p} \\ y &=& g_p^{\mathrm{T}}\nabla_{x_p}H \end{array}\right. \tag{10}$$

with the state $x_p := (q,p,p_+,p_-)$,

$$\begin{array}{lcl} J_{32}(q) &=& -V_+^{-1}A_+b\in\mathbb{R} \\ J_{42}(q) &=& +V_-^{-1}A_-b\in\mathbb{R} \end{array}$$

and the Hamiltonian function

$$\begin{array}{l} H = T \\ \quad +V_+(q)(b(\mathbf{exp}(p_+/b)-1)-p_+) \\ \quad +V_-(q)(b(\mathbf{exp}(p_-/b)-1)-p_-). \end{array}$$

Let us take a coordinate transformation

$$\begin{bmatrix} q \\ p \\ C_+ \\ C_- \end{bmatrix} = \begin{bmatrix} q \\ p \\ -\int J_{32}(q) + p_+ \\ -\int J_{42}(q) + p_- \end{bmatrix} \tag{11}$$

where both C_+ and C_- satisfy Equation (4) and are the Casimir functions. It is a direct calculation to prove that the coordinate transformation (11) converts the system (10) into the system (9) even without the assumption $G(q)=I$. ■

Remark Many other nonlinear friction effects (the Coulomb effect, the Stribeck effects and so on) can be taken into account in the above procedure and the result of Proposition 1 holds as well because mass conservation is free from any energy dissipation. In the following, we still omit such effects to maintain readability.

Furthermore, from Lemma 1, the Hamiltonian function of Equation (9) is simplified as

$$\begin{array}{lcl} H &\to& H - (A_+b\mathbf{exp}(C_+/b) + A_-b\mathbf{exp}(C_-/b)) \\ &=& T \\ && -V_+(q)(C_+ + b - b\mathbf{log}(V_+/A_+)) \\ && -V_-(q)(C_- + b - b\mathbf{log}(V_-/A_-)). \end{array} \tag{12}$$

Note that the above simplification is from the "general" property of port-Hamiltonian systems with Casimir functions. However, in the following Section IV, the further exact simplification will be discussed from the "special" properties of the hydraulic robots.

IV. EXACT SIMPLIFICATION OF CANONICAL FORM

A. Back-fluidmotive force

In this subsection, for hydraulic robots, let us define a generalization of the back-EMF (back electromotive force) of electro-mechanical systems, such as DC motors.

Lemma 2 *Consider the system (9). Then there exists a coordinate transformation which converts the system (9) into the following system*

$$\begin{cases} \begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{f}_A \\ \dot{f}_\perp \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & 0 \\ -I & F_{22} & F_0 & G_0 \\ 0 & -F_0 & 0 & 0 \\ 0 & -G_0 & 0 & 0 \end{bmatrix} \nabla_{x_F} H + \underbrace{\begin{bmatrix} 0 \\ \tau_d \\ g_a u \\ g_b u \end{bmatrix}}_{g_F} \\ y = g_F^{\mathrm{T}} \nabla_{x_F} H \end{cases} \tag{13}$$

with the state $x_F := (q, p, f_A, f_\perp)$,

$$\begin{aligned} F_0(q) &= b(A_+^2 V_+^{-1} + A_-^2 V_-^{-1}) \\ G_0(q) &= bA_+A_-(V_+^{-1} - V_-^{-1}) \\ g_a(q, f_A, f_\perp) &= b(A_+ V_+^{-1} B_+ + A_- V_-^{-1} B_-) \\ g_b(q, f_A, f_\perp) &= b(A_- V_+^{-1} B_+ - A_+ V_-^{-1} B_-) \end{aligned}$$

and the Hamiltonian function

$$\begin{aligned} H = {} & T \\ & + V_+(q)\left(-b - \frac{+A_+ f_A + A_- f_\perp}{\|A\|}\right) \\ & + V_-(q)\left(-b - \frac{-A_- f_A + A_+ f_\perp}{\|A\|}\right). \end{aligned} \tag{14}$$

Furthermore, the following identity holds for all q

$$(\nabla_q - F_0(q)\nabla_{f_A} - G_0(q)\nabla_{f_\perp})(H - T) + \|A\| f_A = 0. \tag{15}$$

Proof of Lemma 2. By a direct calculation, it is proved that the system (9) is converted into the system (13) by the following coordinate transformation

$$\begin{bmatrix} f_A \\ f_\perp \end{bmatrix} = \frac{1}{\|A\|} \underbrace{\begin{bmatrix} A_+ & -A_- \\ A_- & +A_+ \end{bmatrix}}_{A} \begin{bmatrix} C_+ - \mathbf{log}(A_+^{-1} V_+) \\ C_- + \mathbf{log}(A_-^{-1} V_-) \end{bmatrix}$$

where $\|A\|$ denotes the Frobenius norm of the matrix A. The gradient of the function $H - T$ is described by

$$\nabla_{x_F}(H - T) = \begin{bmatrix} -b(A_+ - A_-) - \|A\| f_A \\ 0 \\ -L(A_+^2 - A_-^2)/(2\|A\|) - \|A\| q \\ -(LA_+A_-)/\|A\| \end{bmatrix}$$

and thus it is also confirmed directly that the identify (15) holds for all q. ■

Let us explain the physical meaning of the identify (15). First, $\|A\| f_A$ corresponds to the driving force $A_+p_+ - A_-p_-$ on the mechanical subsystems which is appeared in the equation (1). Second, in the absence of the external forces, the dynamics of $p = M\dot{q}$ (the right-hand side of $\dot{p}$-line) should be equal to forces from the gradients of the function T plus forces from the gradients of the function $H - T$ which should include the driving force as a unique potential force.

Note that the element-wise value of $F_0(q)$ is not constant but always positive as well as that of g_a. In this sense, $F_0(q)$ is refereed as a "back-fluidmotive force variable" as a generalization of the back-electromotive force constant which is well-known in DC motors.

Remark The identity (15)

$$\begin{aligned} & - \quad b(A_+ - A_-) \\ & - \quad F_0(q)\left(\frac{L}{2\|A\|}(A_+^2 - A_-^2) + \|A\| q\right) \\ & - \quad G_0(q)\frac{L}{\|A\|}A_+A_- = 0 \end{aligned}$$

is expressed as an integral form with respect to q as the following

$$\begin{aligned} & - \quad b(A_+ - A_-)q \\ & - \quad \frac{L}{2\|A\|}(A_+^2 - A_-^2)\int F_0(q) \\ & - \quad \frac{L}{\|A\|}A_+A_- \int G_0(q) \\ & = \quad \|A\| \int qF_0(q) \end{aligned} \tag{16}$$

which will be used in the next subsection.

B. Integration by parts of Hamiltonian function

This subsection gives the most important result in this paper. Based on the previous preparations, a further exact simplification of the hydraulic robots is proposed via an integration by parts.

Proposition 2 (Exact simplification of canonical form) *Consider the system (9). Then there exists a coordinate transformation which converts the system (9) into the following system*

$$\begin{cases} \begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{\bar{C}}_+ \\ \dot{\bar{C}}_- \end{bmatrix} = \begin{bmatrix} 0 & +I & 0 & 0 \\ -I & F_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \nabla_{x_{\bar{C}}} H_{\mathrm{simple}} + \begin{bmatrix} 0 \\ \tau_d \\ g_a u \\ g_b u \end{bmatrix} \\ y = g_F^{\mathrm{T}} \nabla_{x_{\bar{C}}} H \end{cases} \tag{17}$$

with the state $x_{\bar{C}} := (q, p, \bar{C}_+, \bar{C}_-)$ and the Hamiltonian function

$$H_{\mathrm{simple}} := T + \|A\| \left(\iint F_0(q) - \bar{C}_+ q\right).$$

Proof of Proposition 2. Let us consider the coordinate transformation

$$\begin{bmatrix} \bar{C}_+ \\ \bar{C}_- \end{bmatrix} = \frac{1}{\|A\|} \begin{bmatrix} A_+ & -A_- \\ A_- & +A_+ \end{bmatrix} \begin{bmatrix} C_+ \\ C_- \end{bmatrix}$$

where the states $\bar{C}_+$ and $\bar{C}_-$ are Casimir functions of the system (13)

$$\begin{aligned} \bar{C}_+ &= -\int F_0(q) + f_A \\ \bar{C}_- &= -\int G_0(q) + f_\perp. \end{aligned}$$

By a substitution of f_A and $f_\perp$ into the function $H-T$,

$$
\begin{aligned}
H-T \quad &= \\
&\quad \tfrac{L}{2}\Big(-\tfrac{1}{\|A\|}\big((A_+^2-A_-^2)(\bar{C}_+-\textstyle\int F_0) \\
&+\quad 2A_+A_-(\bar{C}_- - \textstyle\int G_0)\big) - b(A_+ + A_-)\Big) \\
&+\quad q\Big(-\tfrac{1}{\|A\|}(A_+^2+A_-^2)(\bar{C}_+ - \textstyle\int F_0) - b(A_+ + A_-)\Big) \\
&=\quad (\bar{C}_+ - \textstyle\int F_0)\Big(-\tfrac{L}{2\|A\|}(A_+^2 - A_-^2) - \|A\|q\Big) \\
&+\quad (\bar{C}_- - \textstyle\int G_0)\Big(-\tfrac{L}{\|A\|}A_+A_-\Big) - \tfrac{L}{2}b(A_+ + A_-) \\
&-\quad b(A_+ - A_-)q \\
&=\quad -\|A\|\bar{C}_+q + \tfrac{L}{2\|A\|}(A_+^2 - A_-^2)\textstyle\int F_0 + \|A\|q\textstyle\int F_0 \\
&+\quad \tfrac{L}{\|A\|}A_+A_-\textstyle\int G_0 - b(A_+ - A_-)q + \bar{f}(\bar{C})
\end{aligned}
$$

where

$$
\bar{f}(\bar{C}) := \frac{-L}{2\|A\|}(A_+^2 - A_-^2)\bar{C}_+ - \frac{L}{\|A\|}A_+A_-\bar{C}_- - \frac{L}{2}b(A_+ + A_-)
$$

which depends on Casimirs (and constants) only. From Lemma 1 again, the Hamiltonian function is replaced by

$$
H \to H - \bar{f}(\bar{C}).
$$

Finally, by applying the integral form (16) of the identify,

$$
\begin{aligned}
H-T \quad &\to \quad -\|A\|\bar{C}_+q + \frac{L}{2\|A\|}(A_+^2 - A_-^2)\int F_0 \\
&+ \quad \|A\|q\int F_0 + \frac{L}{\|A\|}A_+A_-\int G_0 - b(A_+ - A_-)q \\
&= \quad -\|A\|\bar{C}_+q + \|A\|q\int F_0 - \|A\|\int qF_0 \\
&= \quad \|A\|\left(-\bar{C}_+q + \iint F_0(q)\right).
\end{aligned}
$$

where the last equation holds via the integration by parts. ■

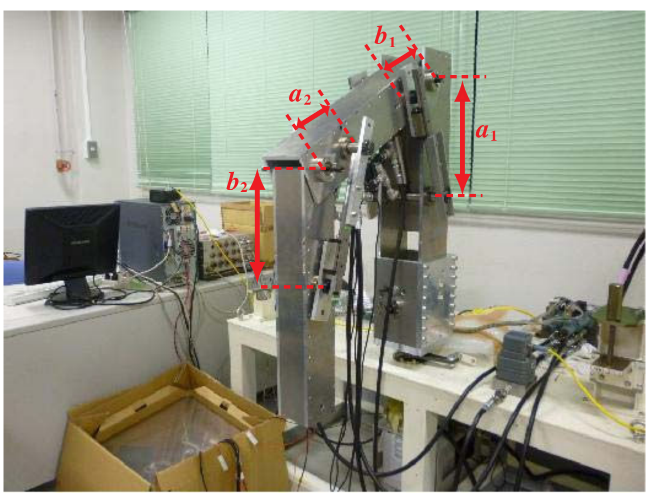

Fig. 2. The 2-DOF hydraulic robot with the design parameters (a,b)

V. Design optimization

In general, an objective function in a robot design optimization consists of analytical terms and non-analytical terms. If the objective function consists of analytical terms only and there are no non-analytical terms, a gradient of the objective function is useful and thus many search methods such as the steepest gradient method and the conjugate gradient method are applicable [16]. However, once the objective function has a non-analytical term, the gradient of the objective function is not useful anymore and the robot forward dynamics should be calculated for each update of the design parameters.

Since the computational cost of the forward dynamics can be much larger than that of the gradient of the objective function, the proposed representation in the previous sections would be quite important in the various design optimizations.

TABLE I

The hydraulic parameters of the 2-DOF hydraulic robot

Physical parameters	Values
Cap area	$A_+ = 7.0\times 10^{-4}$[m^2]
Head area	$A_- = 5.4\times 10^{-4}$[m^2]
Cylinder stroke	$L = 75\times 10^{-3}$[m]
Pump pressure	$p_{\text{upper}} = 7.0\times 10^{6}$[Pa]
Tank pressure	$p_{\text{lower}} = 0$[Pa]
Bulk modulus	$b = 1.5\times 10^{9}$[Pa]
Flow coefficient	$k = 1.63\times 10^{-7}$[m^2/s/$\sqrt{\text{Pa}}$]

A. Forward dynamics of hydraulic robots

Let us consider a design optimization problem of a 2-DOF hydraulic robot with 3-link in the presence of the gravity as shown in Figure 2. The optimized design parameters $a = (a_1, a_2)$ and $b = (b_1, b_2)$ are the distances between the arm joint and the corresponding cylinder ends.

Figure 3 shows the time responses of the state $q(t), p_+(t), p_-(t)$ by the conventional representation (blue lines) and those by the proposed representation (red lines) in the presence of the non-zero damping matrix $D = 363I$ and the external forces d which is defined as the gravity force ($G(q) = I$). Both responses are sufficiently similar to each other. Note that the joint displacement is finite and the second advantage in Section I is demonstrated.

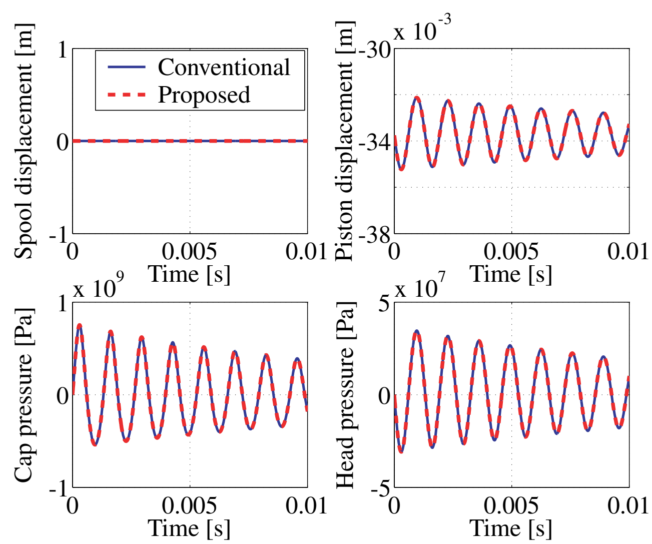

Fig. 3. Disturbance response of $q(t), p_+(t), p_-(t)$

TABLE II

COMPUTATIONAL COST ($u(t) = \sin(2\pi 300t)$)

Trial	Before (conventional) [sec]	After (proposed) [sec]
1	75.00	33.12
2	75.63	30.62
3	69.38	42.81
4	74.22	34.40
5	73.44	39.37

TABLE III

COMPUTATIONAL COST ($u(t) = \sin(2\pi t)$)

Trial	Before (conventional) [sec]	After (proposed) [sec]
1	158.75	53.11
2	159.37	58.42
3	150.13	58.13
4	155.31	57.99
5	158.25	59.13

Table II shows the computational cost of a sine-input response with an input $u = 0.001\sin(2\pi 300t)$ by the conventional representation and that by the proposed representation. In all trials, the computational cost by the proposed representation is much better than that by the conventional representation. Table III shows the computational cost of a sine-input response with an input $u = 0.001\sin(2\pi t)$ by the conventional representation and that by the proposed representation. The responses do not look like any curve due to the high frequency behaviors. Again, in all trials, the computational cost by the proposed representation is much better than that by the conventional representation.

B. Objective function and constraints

The objective function is the sum of the analytic terms and the non-analytic term

$$f(a,b) = w_1 e_1(a,b) + w_2 e_2(a,b) + w_3 e_3(a,b)$$

where e_1 is the manipulability (the volume of the manipulability ellipsoids), e_2 is the workspace (the volume of the workspace) e_3 is the endpoint force (the ∞-norm of the endpoint force in the step response) and $w_\bullet$ is the corresponding weight. The both e_1 and e_2 are the analytical terms which are the explicit functions of the design parameters (a,b), respectively. On the other hand, the third term e_3 is not an analytical term and defined as

$$e_3 := \max_t \|G(q(t))^{-\mathrm{T}} f(t)\|_{\ell 2}$$

whose value is unknown unless the forward dynamics is calculated using the design parameters (a,b). In this sense, the design optimization problem is a non-convex optimization problem. It is needless to say that the computation time of the term e_3 is much larger than that of the term e_1 or e_2.

The constraints of the design parameters (a,b) are

$$\underline{a_i} \le a_i \le \overline{a_i}, \quad \underline{b_i} \le b_i \le \overline{b_i}$$

where $\underline{\bullet_i}$ and $\overline{\bullet_i}$ are from the link length design.

C. Optimization method

Figure 4 shows a standard design optimization procedure which is an iteration of the forward dynamics calculation and a search method in this paper which is a modified particle swarm optimization (PSO) [5]. The PSO is an efficient search method for non-convex optimization problems. The design parameters are the position of N-particles and are updated based on a stochastic linear combination of the best particle posit on x_{group} in all particles and the self-best position $x_{self,i}$ of the individual i $(i = 1, \cdots, N)$

$$\begin{aligned} x_i &\leftarrow x_i + v_i \\ v_i &\leftarrow w v_i + c_1 r_1 (x_{self,i} - x_i) + c_2 r_2 (x_{group} - x_i), \end{aligned}$$

where w is the inertia, c_1, c_2 are the weights and r_1, r_2 are the random variables. The constraints are taken into account as the regularity condition [16].

In this section, the workspace is focused and two cases, that is, Case I ($w_1 = 0.4, w_2 = 0.2, w_3 = 0.4$) and Case II ($w_1 = 0.2, w_2 = 0.6, w_3 = 0.2$) are investigated. Ten particles are used in the modified PSO with $w = 1, c_1 = c_2 = 0.4$ under the constraints $\underline{a} = 0.10, \overline{a} = 0.25, \underline{b} = 0.15, \overline{b} = 0.30$. The standard computer (CPU 2.0[GHz], Adams-Bushforth-Moulton method) is used.

D. Optimization results and discussion

Figure 5 and Figure 6 show the optimization results of Case I and Case II, respectively. Since the piston displacements are the minimum in both cases, it is clearly shown that the robots are designed differently. Figures 7-10 show that all ten particles converge to a fixed position numerically in both cases. Note that ten hydraulic arms (ten particles) are simulated simultaneously in each parameter update based on Figure 4.

The computational cost by the proposed forward calculation is 101 sec and that by the conventional forward calculation is 223 sec even though the optimized results are the same in both calculations. By a comparison with the computational cost of the parameters update, it is clear that the proposed method contributes to reduce the total computational cost in design optimization.

VI. CONCLUSIONS

This paper presents a fast computation method of hydraulic robot optimizations. First, an exact simplification is proposed based on Casimir function. Second, a further exact simplification is given via a structural property. The new representation allows us to define a back fluidmotive force (back-EMF) which is a hydraulic version of back-EMF. Third, the proposed representation and the conventional representation are compared with respect to the computational cost and the effectiveness of the proposed representation is confirmed by numerical experiment. Almost 50 % of the computational cost is cut and the result will contribute more in more complex cases such as high-degree of freedom hydraulic arms and legs.

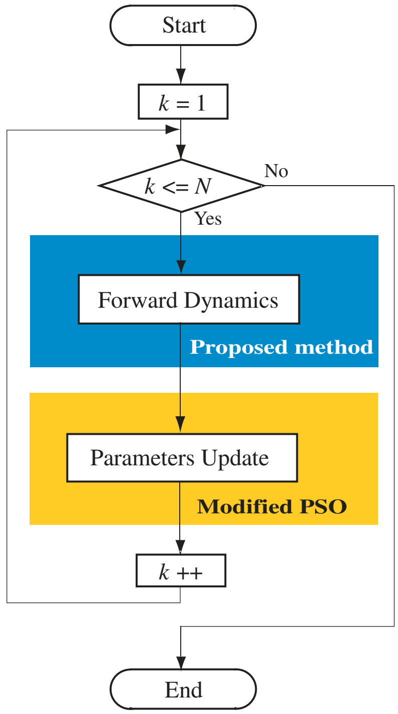

Fig. 4. The flowchart of design optimization

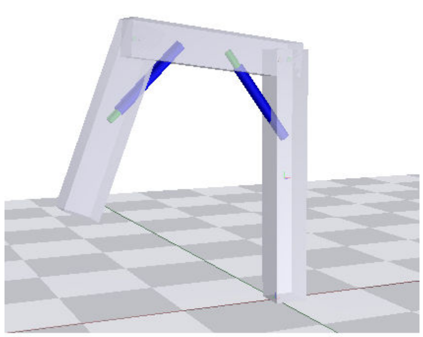

Fig. 5. Optimized design (Case I)

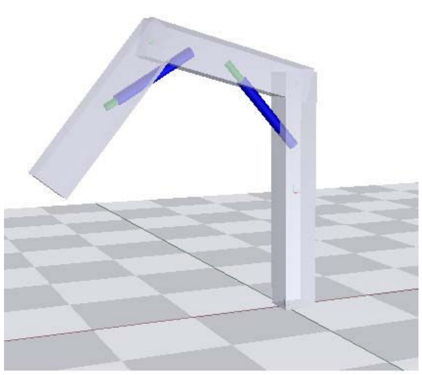

Fig. 6. Optimized design (Case II)

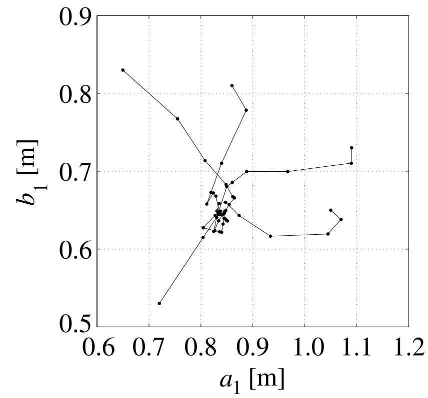

Fig. 7. Parameter updates a_1, b_1 (Case I)

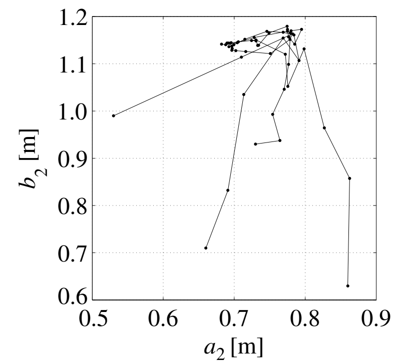

Fig. 8. Parameter updates a_2, b_2 (Case I)

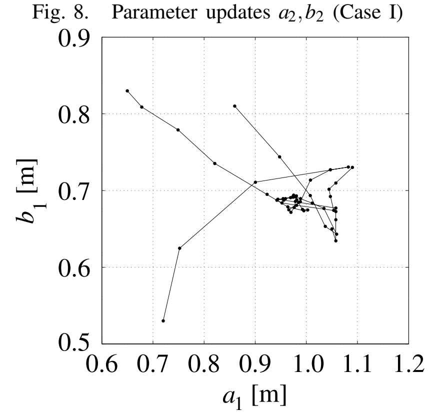

Fig. 9. Parameter updates a_1, b_1 (Case II)

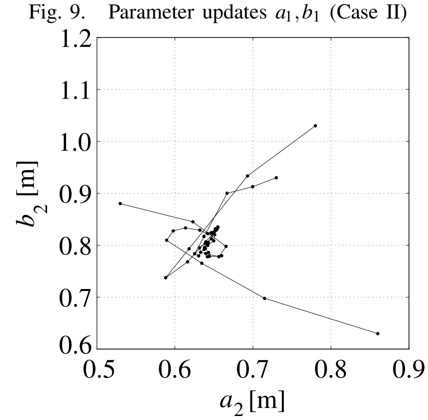

Fig. 10. Parameter updates a_2, b_2 (Case II)

ACKNOWLEDGEMENT

I would like to express my gratitude to Mr. Yutaro Ichinose and thank Mr. Akihiro Fujisawa.

REFERENCES

[1] A. Bonchis, P.I. Corke, D.C.Rye, and Q.P. Ha. Variable structure methods in hydraulic servo systems control. *Automatica*, 37(1):589–595, 2001.

[2] F. Bullo and A. D. Lewis. *Geometric control of mechanical systems*. Springer-Verlag, 2005.

[3] G. Gernot and K Schlacher. Energy-based nonlinear control of hydraulically actuated mechanical systems. In *Proc. of the 44th Conference on Decision and Control*, pages 7520–7525, 2005.

[4] W. Kemmetmuller and A. Kugi. Immersion and invariance-based impedance control for electrohydraulic systems. *International Journal of Robust and Nonlinear Control*, 20:725–744, 2010.

[5] J. Kennedy and R. C. Eberhart. Particle swarm optimization. In *IEEE Int. Conf. Neural Networks*, pages 1942–1948, 1995.

[6] A. Kugi and W. Kemmetmuller. New enegy-based nonliner controller for hydraulic piston actuators. *European Journal of Control*, 10(5):432–450, 2003.

[7] Raibert Marc. Bigdog, the rough-terrain quadruped robot. In *Proc. IFAC World Congress 2008*, pages 10822–10825, 2008.

[8] F. Mazenc and E. Richard. Stabilization of hydraulic systems using a passive property. *Systems and Control Letters*, 44.

[9] H. Merrit. *Hydraulic Control Systems*. John Willey & Sons, 1967.

[10] R. Ortega and Eloisa Garcia-Canseco. Interconection and damping assignment passivity-based control: A survey. *European Journal of Control*, 10(5):432–450, 2004.

[11] S. Sakai. Fast computation by simplification of a class of hydro-mechanical system. In *Proc. of IFAC Lagrangian and Hamiltonian method for method for nonlinear control*, pages 7–12, 2012.

[12] S. Sakai and S. Stramigioli. Port-Hamiltonian approaches to motion generations for mechanical systems. In *Proc. of IEEE Conference on Robotics and Automation*, pages 69–74, 2007.

[13] S. Sakai and S. Stramigioli. Passivity force control of hydraulic robots. In *Proc. of IFAC Symposium on Robot Control*, 2009.

[14] S. Sakai and S. Stramigioli. Casimir based impedance control. In *Proc. of IEEE ICRA*, pages 1384–1391, 2012.

[15] M.R. Sirouspour, BC Vancouver, and S.E. Salcudean. Nonlinear control of hydraulic robots. *IEEE Transactions on Robotics and Automation*, 17(2):173–182, 2001.

[16] Gilbert Strang. *Computational Science and Engineering*. 2007.

[17] M. Takegaki and S. Arimoto. A new feedback method for dynamic control of manipulators. *Trans. ASME, J. Dyn. Syst., Meas., Control*, 103:119–125, 1981.

[18] A. J. van der Schaft. *L_2-Gain and Passivity Techniques in Nonlinear Control*. Springer-Verlag, London, 2000.

[19] A. J. van der Schaft and B. M. J. Maschke. The Hamiltonian formulation of energy conservation physical systems and external ports. In *Archiv fur Electronic und Ubertrangungstechnik*, pages 362–371, 1995.

[20] B. Yao, F. Bu, J. Reedy, and G. T.-C. Chiu. Adaptive robust motion control of single rod hydraulic actuators: Thorey and experiment. *IEEE/ASME Transactions on Mechatoronics*, 5(1):79–91, 2000.

[21] T. Yoshikawa. *Foundation of Robotics*. MIT Press, 1990.