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Turtle-inspired Localization on Robot

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Abstract— **In nature, some animals exhibit impressive navigation capability using ambient magnetic field. Particularly, certain kinds of sea turtles can associate geomagnetism to spatial representation for positioning in transoceanic migration across the seemingly clueless sea. In robotics, the previous works on magnetic navigation can position a robot using ambient magnetic field, but they focused on an exploitation of extensively-explored magnetic map, i.e. the magnetic field of every inch of the region of interest is recorded for mapping. In this work, we propose an algorithm that is based on our analysis on how sea turtles navigate at sea under magnetic disruption as investigated and reported by biologists [1], [2]. We propose a direct likelihood method that generates pseudo training data to improve the estimation accuracy of the Gaussian mixture models. The experimental evaluation demonstrates that our localization algorithm exhibits stable and accurate positioning results. This work contrasts with the previous works which focused on magnetic localization using extensive data collection. On the contrary, we address whether magnetic localization is still feasible under scarce data samples and how to overcome this challenge.**

I. INTRODUCTION

Animals know their bearings. Over the years, scientists discovered that most, if not all, animals show remarkable navigation and localization skills in the wild using ambient magnetic field [3]. Turtles, birds, lobsters and many more are making use of ambient magnetic field for migration. The nose regions in some species of sharks [4] consist of thousands of *magnetometer*-like sensors for self-localization in deep sea. Even some bacteria are confirmed carrying magnetosome chains to physically orient itself purposely in a magnetic field. Such an invisible yet overwhelming vector field is evidently exploitable in nature. However, ambient magnetic field is not widely adopted in maps for robot's localization.

How exploitable is magnetic field? The general understanding of ambient magnetic field is that it points to the pole. The variation in readings are understood as electromagnetic disturbance due to, for example, an electric wire or an object made of iron in the surrounding. Magnetic localization was often explored in atomic level [5], [6]. Only in recent years, studies [7], [8], [9] revealed that ambient magnetic field can be exploited for localization with meter-grade accuracy. Haverinen and Kemppainen [7] presented a particle filter-based method to position in indoor corridors by using a magnetic map that is constructed by extensive data sampling over every inch of the floor. The significance of their work is that, for the first time the magnetic field is demonstrated to be useful for localization. However, their method can only yield one-dimensional localization in addition to the requirement of extensive data collection and mapping. Later, their method extended to 2-D localization [9] and the data sampling took 2.5 hours in an area of about 35ft^2.

In this paper, we consider the problem of designing a localization method for a robot car in an indoor environment, and address whether magnetic localization is feasible under scarce data samples. From technological perspective, our algorithm can leverage scarce mapping samples – which is composed of a series of position and magnetic field measurement pairs as a training set – for localization. Our key observations are in twofold: i) from the researches on sea turtles reported by biologists [1], some turtles can extract spatial representation solely from geomagnetism and can navigate under magnetic disruption but with degraded effectiveness; ii) the regional magnetic field is closely related due to its electromagnetic characteristics, and hence a series of Gaussian mixtures may characterize magnetic field in a region using sample data that are in close proximity. The first observation gives us evidence on what data would be be necessary for magnetic navigation from a biomimetic view, and is further explained in Section II. The second observation inspires us on the modeling method of the magnetic field and can be best explained by Fig. 1 and 2. In those figures, the measurements of the magnetic field vectors from a tri-axial magnetometer are plotted. The position of each of the magnetic field vector is measured by a millimeter-grade camera tracking system. It shows that the field vectors are closely related to each other in near proximity in terms of the direction and the magnitude.

In our algorithm, the key advantage of modeling the vector field of magnetism using Gaussian mixture models is related to the reduction of the sample complexity. When a robot car steers near a previously sampled position, its measured magnetic field vectors can be found using a series of Gaussian mixture models by regression after training, hence lesser samples are needed as we do not need to cover every inch of the field. On the other hand, to minimize the occasionally large variance in the Gaussian mixture regression, we introduce a direct likelihood method that can match between similar magnetic field vectors in the sampled data when the position estimates from the regression is deemed unreliable. We generate pseudo training data by using the direct likelihood method and add it into the original training set for a new set of model parameters. Therefore, the Gaussian mixture models can evolve to improve its predictability in poorly modeled regions. We evaluate our proposed method on a toy car carrying a tri-axial magnetometer. The ground-truth positions of the car are measured using a camera tracking system for the collection of training

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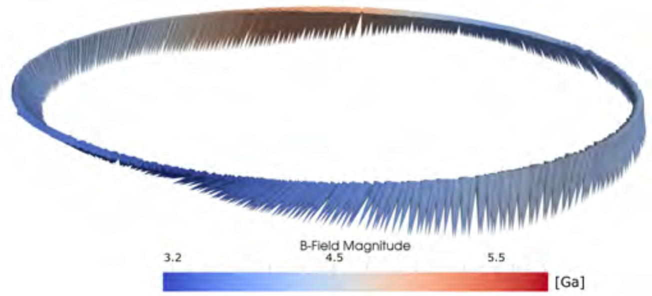

Fig. 1. The field vectors collected along a circular path. This figure shows that the magnetic field in a region is remarkable unique and continuous in terms of the variations in direction and magnitude.

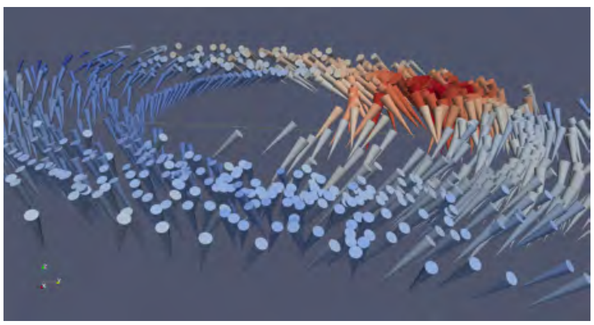

Fig. 2. The magnetic field vectors in a field measured by our setup as shown in Fig. 3.

data and references. The empirical results show that our algorithm can significantly outperform the Gaussian mixture regression, and yields a remarkable accuracy of $\pm$3cm.

The paper is arranged as follows: Section II explains how we exploit biologist's research on sea turtles in geomagnetic navigation. Section III details the design of our algorithm. Section IV gives the experimental setup and results. Section IV-A presents the comparisons between the Gaussian mixture regression and our proposed method. Section VI closes with a conclusion.

II. Decoding Navigation of Turtles

Many kinds of turtles have shown remarkable ability to navigate across hundreds of miles in the seemingly clueless ocean, and get back to the very same spot for laying eggs or perform transoceanic migration [10], [11], [12], [13], [14]. Biologists conducted extensive experiments over last decades trying to unravel turtle's navigation secret. Various theories are being raised, many of them are debatable - but one consensus has been made: turtles make use of the geomagnetic information for navigation [2].

A series of experimental studies on turtles that drew our attention and inspired this work is carried out by Lohmann's group [1], [2], [11]. They found that hatchling turtles does not navigate as good as the older turtles. They captured older turtles and placed them into a pool with devices that generated specific magnetic fields. Then they reproduced magnetic field that was similar to a place hundreds of miles north of the capture site. They found that the turtles swam southward. When the magnetic field mimicked a site hundreds of miles south of the capture site, the turtles swam northward. Similar experiments have been conducted for sites along both latitudinal and longitudinal directions of the capture site, and the turtles could swam towards the direction of the capture site. This finding suggests that the turtles may make use of geomagnetism for navigation, and they may somehow translate a magnetic field to spatial information and eventually know which direction to go, i.e. a possible hypothesis of this ability can be represented in this way,

$$b_k = f(p_k|\theta), \tag{1}$$

where b_k is the magnetic field at place k, and p_k is its spatial representation, namely position, and $f(\cdot)$ is an unknown mapping function parameterized by θ. Older turtles may possess a θ that lead to more accurate mapping between magnetic fields and their spatial representations. Equation (1) means that when given certain knowledge in a form of a function parameter θ, the magnetic field of a place can be retrieved by its spatial representation. During localization, the spatial representation of a magnetic field is retrieved by an inverse of the mapping function, i.e. $f^{-1}(\cdot)$.

In another setting of experiments [1], [14], the turtles were transported to and released in sites hundreds of miles away from the capture site. A group of turtles were attached with strong magnet to disrupt their sense of geomagnetism, while the other group were not. The results showed that the group with magnet can still swim back to the capture site, while the group without magnet can swim back more directly. Similar results are found in both latitudinal and longitudinal cases [15]. Based on our hypothesis that is represented in (1), there are at least two possibilities: 1) the turtles may make use of other senses for navigation, such that there exist a third parameter in the unknown mapping function $f(\cdot)$, hence $b_k = f(p_k, \theta, \gamma_k)$, where γ_k is the measurement associated with p_k from the unknown sense; 2) the turtles can evolve the function parameter θ to improve the mapping accuracy from time to time, such that,

$$p_k = f^{-1}(b_k|\theta^*),\ \theta^* = \underset{\theta}{\text{argmax}}\ J_t, \tag{2}$$

where θ^* is an evolving function parameter to maximize the mapping accuracy. J_t is a time-varying cost function. As no literature can verify what the unknown sense is guiding the turtles amidst disrupted geomagnetism, we therefore consider the most possible case: they may be making an efficient use of all they have in their magnetic map. More specifically, they may make use of the current magnetic field and places where they believe they were at to improve their mapping functions. As a result, the cost function should be a function of the previous position estimates, and their existing knowledge of geomagnetism and its spatial representation, such that,

$$J_t = J_t(\{\hat{p}_k(t-n)\}_{n=0,1,\ldots}, \{p_k, b_k\}_{k=1,2,\ldots}), \tag{3}$$

where $\hat{p}$ is the position estimate at a specific time, $\{p_k, b_k\}$ is a magnetic field measurement and its spatial representation at a place k.

III. DESIGN OF ALGORITHM

In this section, we derive an algorithm that makes use of our hypotheses (1-3) in Section II to mimic the ability of geomagnetic localization as in turtles for a robot car. Particularly, we experiment with our algorithm in an indoor setting that the floor of a room exhibits magnetic field that is similar to geomagnetic isolines for navigation. And we consider that our robot car first briefly explores the room and collects magnetic field measurements with their spatial representation (i.e. 2D positions), and then we employ our algorithm to locate the car using the car's previous mapping knowledge and the current magnetic field measurement, without using other positioning method. The robot car is restricted to locate its position by making use of its magnetic map and the current 3D magnetic readings.

To establish a magnetic map that facilitates retrieval of spatial representation of a magnetic field measurement as hypothesized in (1), we employ a mixture of Gaussian functions to connect the magnetic readings with their spatial representations that are collected during exploration. This model fitting can approximate a map of magnetic field over the explored region. Then we design a method to improve the performance of localization when the accuracy of estimation worsens. We make use of our hypothesis given in (3) to design a method that makes an efficient use of a robot car's previous position estimates and its existing knowledge of the geomagnetism map. Firstly, we make use of the variance of a series of previous position estimates ($\hat{p}(t-n), n = 0, 1, \ldots$) to determine the accuracy of the localization at the moment. Secondly, when this variance is significant, we search for magnetic field readings ($\bar{b}$) that is similar to the current one ($\tilde{b}$) from the existing magnetic map ($\{\bar{b}_i, \bar{p}_i\}_{i=1,2,\ldots}$). Once a match is found, we combine the current magnetic field measurement and the matched spatial representation ($(\tilde{b}, \bar{p})$) as a pseudo training data, and assign it to the existing map via model fitting under the framework of Gaussian mixture model. The details of the algorithm design is given below, and its performance are given in Section IV.

The algorithm is composed of two parts: the Gaussian mixture regression and the direct likelihood method. The Gaussian mixture regression consists of the expectation-maximization (EM) [16] and regression of the Gaussian mixture models. The Gaussian mixture regression is well-studied in literature, we refer readers to Reference [17] for a more comprehensive review.

A. Gaussian mixture method (GMM)

Each entry in our training data is a pair of data that comprises of the position $p \in \mathbb{R}_{N\times 3}$ and the magnetic field vector $b \in \mathbb{R}_{N\times 3}$. The magnetic field vectors measured on board are converted from the sensor's orientation to the inertial frame's orientation. To position using the magnetic field, we need to design a function f as in our hypothesis (1) such that, we can establish a function parameter θ for $b = f(p|\theta)$ via training. Afterwards, when given a magnetic field vector with unknown position, we can inversely find this position, such that $p = f^{-1}(b|\theta)$. To avoid extensive data sampling, we leverage the fact that the regional magnetic field vector are closely related and alike due to the electromagnetic characteristics (as shown in Fig. 1, 2), and hence we employ the Gaussian mixture models to characterize such relation among the magnetic field vectors, such that the probability of a magnetic field vector being in a specific position is probabilistically characterized by,

$$P(x|\theta) = \sum_{j=1}^{M} w_j \cdot \mathcal{N}(x|\mu_j, \Sigma_j) \tag{4}$$

where $x = [\ p\ b\ ]$ is the measurement, w_j is the weights such that $\sum w_j = 1$. μ_j is the mean vector. Σ_j is the covariance matrix. $\mathcal{N}(\cdot)$ refers to the normal density function. θ is the model parameters, such that $\theta = \{\mu_j, \Sigma_j, w_j\}$ for $j = 1, \ldots, M$. M is the number of Gaussian models. To fit the model parameters to the measurement, we write,

$$\theta^* = \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^{N} P(x_i|\theta) \tag{5}$$

where N is the total number of training data. Therefore, the function $f(\cdot)$ is represented by the probability function $P(p_k|b_k)$, meaning the probability of being in p_k when given b_k. We employ the expectation-maximization technique to find θ^*. The idea is to incrementally and iteratively adjust and improve the model parameters so as to achieve a best fit in data. The update equations for the mean vector, the covariance matrix and the weight are as follows,

$$\mu_j = \frac{\sum_{i=1}^{N} P(j|x_i, \theta) x_i}{\sum_{i=1}^{N} P(j|x_i, \theta)} \tag{6}$$

$$\Sigma_j = \frac{\sum_{i=1}^{N} P(j|x_i, \theta)[(x_i - \mu_j)(x_i - \mu_j)^T]}{\sum_{i=1}^{N} P(j|x_i, \theta)} \tag{7}$$

$$w_j = \frac{\sum_{i=1}^{N} P(j|x_i, \theta)}{N} \tag{8}$$

Once all the model parameters are determined, the models can be used to approximate the spatial representation of a magnetic field vector,

$$\hat{p}_i = \mu_i^p + \Sigma_i^{p,b} (\Sigma_i^b)^{-1} (b - \mu_i^b),\ \hat{p} = \sum_{i=1}^{M} h_i \hat{p}_i \tag{9}$$

where h_i is the conditional probability for the magnetic field vector b in a Gaussian model, such that,

$$h_i = \frac{w_i\ \mathcal{N}(b|\mu_i^b, \Sigma_i^b)}{\sum_{i=1}^{M} w_i\ \mathcal{N}(b|\mu_i^b, \Sigma_i^b)} \tag{10}$$

The proof of convergence of this regression method can be found in Reference [18].

B. Direct likelihood method (DLM)

In practice, due to the lack of samples, some query magnetic field vectors would be too ambiguous to be classified by the Gaussian mixture models. Here, we introduce a direct likelihood method to yield position estimates using the collected data set $\{\bar{b}, \bar{p}\}$, and leverage these estimates as pseudo training data to improve the regression of GMM. This method is formulated as a function optimization problem, such that,

$$i^* = \underset{i=1,\ldots,N_{\bar{b}},j=1,\ldots,N_b}{\operatorname{argmin}} E\|\bar{b}(i) - b(j)\| \tag{11}$$

$$\hat{p} = \bar{p}(i^*) \tag{12}$$

where N_b is the total number of queries. To find the position of a magnetic field vector $b(t)$ at time t, the queries needed in the direct likelihood method is $b(t - N_b + 1), b(t - N_b), \ldots, b(t)$. The disadvantages of solely using this method are that, the direct likelihood method yields discrete and *steps*-like responses when plotted against time; moreover, this method is susceptible to a deviated trajectory. Therefore, the results by this method alone may not be suitable for the wheeled robots that require velocity estimates for control and a spatial buffer for motion correction,.

In view of these disadvantages, when combining with the Gaussian mixture regression, we first determine the variance of the position estimate under a time horizon $\hat{p}(t), \ldots, \hat{p}(t - N_v + 1)$ where N_v is the total number of position estimates in a time horizon. When the variance is higher than a threshold value, we update the Gaussian mixture models by adding the position estimates from the direct likelihood method $\tilde{p}$ and its associated magnetic field vector b to the training dataset. By adding this *pseudo* training data from the direct likelihood method to the GMM, we can ensure that the position estimates are smoothed and the Gaussian distributions can evolve to cover poorly modeled regions. The update procedure is given in Algorithm 1. Although the heuristics of this update is intuitively simple, we find that it can dramatically boost the usability of the position estimates when comparing with the results purely by the Gaussian mixture method. As we have shown in the experiments (Section IV), the results by GMM is not referable at all times. In our proposed method, we introduce the pseudo training data $\{\tilde{p}, b\}$ to improve the Gaussian mixtures and hence improve GMM's predictability in poorly bootstrapped regions. A small random number σ is introduced to the pseudo training data to avoid over-fitting when bootstrapping the new model parameters. The algorithm in Algorithm 1 is written as a post-processing operation for ease of understanding. No future data is required in all operations. In Algorithm 1, **GMM.Fit**($\cdot$) refers to Equation (4) to (8). **GMM.Regression**($\cdot$) refers to Equations (9, 10). **DLM.PseudoData**($\cdot$) refers to Equations (11, 12). All computations are designed to run on a single-threaded processor. The main computation only involves summations and low-dimensional matrix multiplications.

IV. Experiments

To evaluate our method, we implement an IMU that comes with a tri-axial magnetometer (Honeywell HMC5883) on a toy car. The car is tracked by a camera system (from NaturalPoints) for the ground truth data of its position. The accuracy of the ground truth data is up to ± 0.1mm. The reason for the collection of this ground truth data is that we need to collect a set of position and magnetic field vector pairs for the training of the Gaussian mixture models.

A collection of samples is given in Fig. 1 and 2. It is plotted by using Paraview[1]. A color code is used to represent the magnitude of each of the collected magnetic field vector. From the figures, we can observe that the field vectors that are close in proximity are more closely related in terms of their directions and magnitudes, and hence is suitable for the characterization of the Gaussian mixture models. Our car maneuvers in the field to collect the position and the tri-axial magnetic field measurement for training. And then we estimate the model parameters $\theta = \{\mu, \Sigma, w\}$. Afterwards, our car maneuvers in the scene again and positions itself using our algorithm with the measured magnetic field as inputs. For references, the ground truth measurement of the position of the car is collected all the time during the evaluation. Only the orientation information and the 3D magnetic field measurement from the on-board sensor suite are used for the localization in our algorithm.

Algorithm 1 Localization via Ambient Magnetic Field

Input: $\{\bar{b}, \bar{p}\}$, the training set
          $\{b\}$, the query magnetic field vectors
$\{\mu, \Sigma, w\}$ = **GMM.Fit**($\{\bar{b}; \bar{p}\}$)
for all queries
    $\hat{p}(t)$ = **GMM.Regression**($\{\mu, \Sigma, w\}$)
    if $Variance(\{\hat{p}(t), \hat{p}(t-1), \ldots\} >$ threshold
        while always
            $\tilde{p}(t)$=**DLM.PseudoData**($\{b(t), b(t-1), \ldots\}, \{\bar{b}, \bar{p}\}$)
            $\{\mu, \Sigma, w\}$=**GMM.Fit**($\{\bar{b}, b(t); \bar{p}, \tilde{p}(t) + \sigma\}$)
            $\hat{p}(t)$ = **GMM.Regression**($\{\mu, \Sigma, w\}$)
            if $Variance(\{\hat{p}(t), \hat{p}(t-1), \ldots\} <$ threshold
                break
            else if iteration $\geq$ iteration.maximum
                $\hat{p}(t) = \tilde{p}(t)$
                break
            end
        end
    end
end
Output: $\{\hat{p}\}$, the position estimates

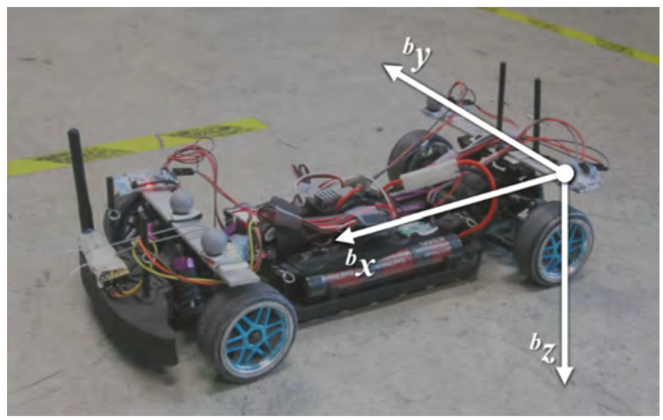

Fig. 3. The setup in the experiments. An off-the-shelf RC car with a tri-axial magnetometer. The coordinate frame of the sensor suite is illustrated in the figure, and is denoted as ^{b}x, ^{b}y, ^{b}z.

[1]An open-source scientific data viewer http://www.paraview.org/

TABLE I

A PERFORMANCE OVERVIEW BETWEEN OUR ALGORITHM AND GMM

Method	Accuracy [cm]	Variance [cm]
GMM	5.96	5.83
Ours	3.27	0.68

A. Comparison and analysis

In Fig. 4 to 6, we present the results of localization using the standard Gaussian mixture method (GMM) [19] and our proposed method. The ground truth is in solid black lines. The red dots and lines are the results by our method. The orange dots and lines are from the GMM. To have the same footing for comparison, we use the same model parameters ($\{\mu, \Sigma, w\}$) for both sets of the Gaussian mixture models in both algorithms. Although the GMM can track the position in most of the time, it cannot give a steady and consistent estimation of the position throughout the whole period of time. As shown in the position *against* time graphs on the right-hand sides of Fig. 4 to 6, the GMM gives unsteady estimations (the orange lines). On the contrary, our proposed method can keep track with the positions of the robot with much fewer outliers. Statistically, our proposed exhibits a remarkable increase of $45\%+$ in the mean-absolute accuracy and an increase of $80\%+$ in the variance reduction. The large amount of reduction in the variance of estimates suggests that our method yields a more consistent performance in estimating the positions. The same data sets are used for training in the GMM and our algorithm.

V. DISCUSSION

- *What is difference between the proposed algorithm and the existing algorithms in magnetic localization?*
  The existing algorithms focused on magnetic localization using extensively-explored magnetic map. For example, it was reported [9] that an inch-by-inch magnetic field mapping that took 2.5 hours to complete is needed to cover a region of about 35ft^2. In this work, we address whether magnetic localization is feasible without an extensive exploration of the magnetic field in a region. Particularly, we look into a case that whether a rover, using ambient magnetic field, can position itself on a path that, i) the rover has only visited once before; ii) this path is not exactly the same path it visited, but with a deviation about $\pm 10 \sim 20$cm. We found that our algorithm can still manage to position the car using ambient magnetism and scarce data sample.
- *Is the proposed method a complete solution for robot's localization in indoor or outdoor settings?*
  No, it is not. This work addresses the technical feasibility that whether magnetic localization is still feasible using scarce data samples. From our experimental results, we found that when the rover maneuvers on a place that is far from where it has previously explored during data sampling, our algorithm alone cannot position the car. However, our results sheds a new and important light on robot localization that it may complement with odometry, visual SLAM or laser-based ranging methods, and significantly improve the overall performance, especially during the lost-and-recovery scenario.

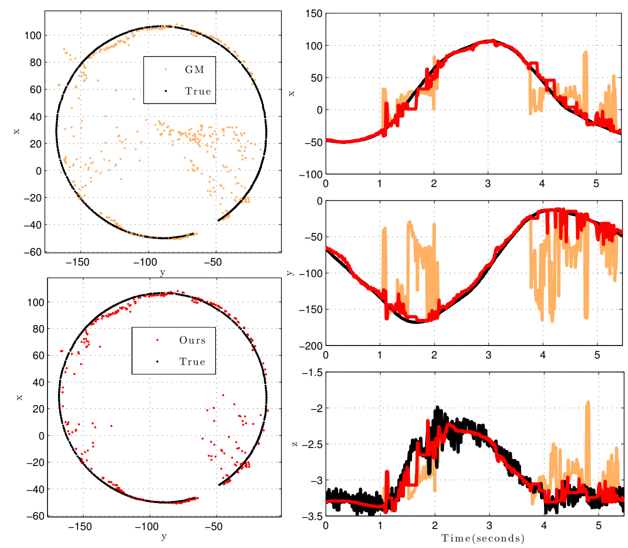

Fig. 4. Experimental results. All plots in this figure represent the results from the same experiment. *Left:* The 2D $x-y$ plots with respect to the inertial frame. *Right:* The positions on each inertial axis against time graphs. This figure presents a comparison between the Gaussian mixture method (GMM) and our proposed method. The same convention of representation is used in Fig. 5 and 6. All units are centimeters. Best view in colors.

- *How does the proposed algorithm work?*
  First, our algorithm requires a set of data sample over a region of interest. This set of data sample requires magnetic field measurements and their spatial representations (i.e. 2D positions). Our algorithm will generate a series of Gaussian models to fit the data. Secondly, when a rover is deployed in the region of interest, our algorithm will make use of the current magnetic field measurement and the heading information to compute position estimates. When the variance of the estimates is significant, our direct likelihood method will kick in and generate pseudo training data to improve the regression of the series of Gaussian models.

VI. CONCLUSION

Without any referencing sensors like GPS or GLNOSS, how can sea turtles navigate across hundreds of miles in the seemingly clueless sea for migration and laying eggs? In this paper, we look into the previous experimental studies on sea turtles reported by biologists, and derived hypotheses that guide us to design an algorithm, that can make an efficient use of ambient magnetism for robot's localization.

Leveraging the fact that regional magnetic field vectors are closely related in directions and magnitudes due to electromagnetic characteristics, we demonstrate that Gaussian mixtures can be effectively employed to model and hence extend the spatial coverage of scarce training data. And then these mixture models can be generalized to probabilistically associate the spatial representation with its magnetism.

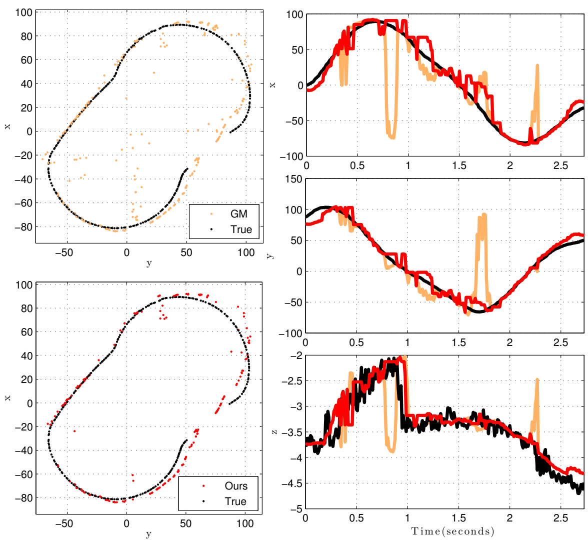

Fig. 5. Experimental results. Note that our algorithm can accurately localize an about 1.5cm vertical displacement as shown in the $z-t$ graph at about $t=1$.

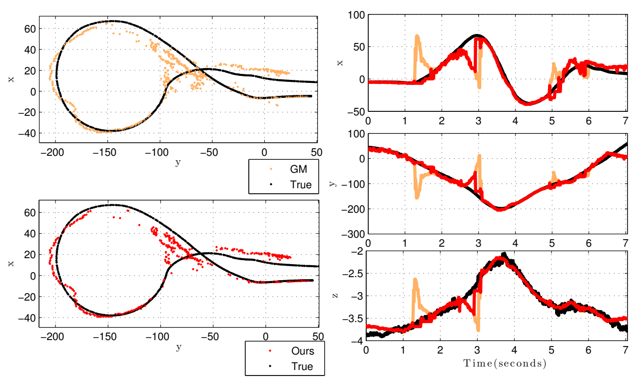

Fig. 6. Experimental results using the platform shown in Fig. 3.

Furthermore, we propose the direct likelihood method that generates pseudo training data to improve the occasionally high variance among position estimates due to insufficient data collection. The combination of these techniques yields a method for a robot car to localize using its ambient magnetic field and heading information. The empirical results demonstrate that, by using an inertial measurement unit that comes with a tri-axial magnetometer, our proposed method enables a robot car to accurately localize its position, and yields a remarkable accuracy of ±3cm. To our best knowledge, this is the first publication that, without extensive mapping of magnetic field, successfully localizes a robot car solely by using its ambient magnetic field and its orientation information. We believe the extent of this work shows promises and sheds a new and important light to complement with existing localization methods like visual SLAM, odometry, infrared laser ranging sensor (i.e. Microsoft Kinect [20]) and laser ranger for long-range localization on robots in both indoor and outdoor settings.

REFERENCES

[1] K. J. Lohmann, C. M. Lohmann, L. M. Ehrhart, D. A. Bagley, and T. Swing, "Animal behaviour: geomagnetic map used in sea-turtle navigation," *Nature*, vol. 428, no. 6986, pp. 909–910, 2004.

[2] D. K. J. Lohmann and C. M. Lohmann, "Sea turtles, lobsters, and oceanic magnetic maps," *Marine and Freshwater Behaviour and Physiology*, vol. 39, no. 1, pp. 49–64, 2006.

[3] T. Maugh, "Magnetic navigation an attractive possibility," *Science*, vol. 215, no. 4539, p. 1492, 1982.

[4] C. Meyer, K. Holland, and Y. Papastamatiou, "Sharks can detect changes in the geomagnetic field," *Journal of the Royal Society Interface*, vol. 2, no. 2, pp. 129–130, 2005.

[5] T. Sugiyama and N. Nagaosa, "Localization in a random magnetic field in 2d," *Physical review letters*, vol. 70, no. 13, pp. 1980–1983, 1993.

[6] Y. Avishai, Y. Hatsugai, and M. Kohmoto, "Localization problem of a two-dimensional lattice in a random magnetic field," *Physical Review B*, vol. 47, no. 15, p. 9561, 1993.

[7] J. Haverinen and A. Kemppainen, "Global indoor self-localization based on the ambient magnetic field," *Robotics and Autonomous Systems*, vol. 57, no. 10, pp. 1028–1035, 2009.

[8] A. Kemppainen, J. Haverinen, I. Vallivaara, and J. Röning, "Near-optimal slam exploration in gaussian processes," in *IEEE 2010 International Conference on Multisensor Fusion and Integration for Intelligent Systems (IEEE MFI 2010)*, 2010.

[9] I. Vallivaara, J. Haverinen, A. Kemppainen, and J. Roning, "Simultaneous localization and mapping using ambient magnetic field," in *Multisensor Fusion and Integration for Intelligent Systems (MFI), 2010 IEEE Conference on*. IEEE, 2010, pp. 14–19.

[10] L. C. Boles, K. J. Lohmann, *et al.*, "True navigation and magnetic maps in spiny lobsters," *Nature*, vol. 421, no. 6918, pp. 60–63, 2003.

[11] K. J. Lohmann, "Sea turtles: navigating with magnetism," *Current Biology*, vol. 17, no. 3, pp. R102–R104, 2007.

[12] S. Johnsen and K. J. Lohmann, "Magnetoreception in animals," *Physics today*, vol. 61, p. 29, 2008.

[13] L. Avens, J. Braun-McNeill, S. Epperly, and K. J. Lohmann, "Site fidelity and homing behavior in juvenile loggerhead sea turtles (caretta caretta)," *Marine Biology*, vol. 143, no. 2, pp. 211–220, 2003.

[14] L. Avens and K. J. Lohmann, "Navigation and seasonal migratory orientation in juvenile sea turtles," *Journal of Experimental Biology*, vol. 207, no. 11, pp. 1771–1778, 2004.

[15] N. F. Putman, C. S. Endres, C. M. Lohmann, and K. J. Lohmann, "Longitude perception and bicoordinate magnetic maps in sea turtles," *Current Biology*, vol. 21, no. 6, pp. 463–466, 2011.

[16] A. Dempster, N. Laird, and D. Rubin, "Maximum likelihood from incomplete data via the em algorithm," *Journal of the Royal Statistical Society. Series B (Methodological)*, pp. 1–38, 1977.

[17] S. Calinon, *Robot programming by demonstration: A probabilistic approach*. CRC Press, 2009.

[18] C. Bishop, *Pattern recognition and machine learning (information, science and statistics)*. Springer, 2006.

[19] J. Bilmes, "A gentle tutorial of the em algorithm and its application to parameter estimation for gaussian mixture and hidden markov models," *International Computer Science Institute*, vol. 4, p. 126, 1998.

[20] E. Suma, B. Lange, A. Rizzo, D. Krum, and M. Bolas, "Faast: The flexible action and articulated skeleton toolkit," in *Virtual Reality Conference (VR), 2011 IEEE*. IEEE, 2011, pp. 247–248.