Control of A Formation with One Cyclic Relation in 3-dimensional Space

Yongguang Yu[1,3] Han-Xiong Li[2,3] and Guoguang Wen[1]

1. Department of Mathematics, Beijing Jiaotong University, Beijing 100044, P.R.China.
2. School of Mechanical and Electrical Engineering, Central South University, Changsha 410083, P.R.China.
3. Department of MEEM, City University of Hong Kong, Kowloon, Hong Kong SAR, China

***Abstract*—The paper investigates the flocking behaviors of multi-agent formation in 3-dimensional space which are based on leader following, where the underlying model of a formation is graphical. When the graph is undirected, a class of decentralized control laws for a group of mobile agents are proposed under the conditions that the topology of the control interconnections is fixed. These control laws are a combination of attractive/repulsive and alignments forces which can guarantee the collision avoidance and cohesion of the formation and an aggregate motion along the same heading direction of the leader. And when the graph is directed, a formation of four point agents moving in 3-dimensional space is considered, where one agent is a leader, and the other agents have a cyclic ordering with each one required to maintain a given distance from its neighbor. A control law is obtained, which can be guaranteed the distance preserved for the formation.**

***Index Terms*—Decentralized control; Multi-agent formation; Leader; Cyclic relation.**

I. Introduction

Recent years many researchers have significant interest in formations of mobile autonomous multi-agent, which arises from the broad potential for applications including formation flight, advanced transportation systems, distributed sensors networks, flocking and schooling, search-and-rescue operations, competitive games, and military reconnaissance and surveillance. A formation is defined as a group of mobile agents moving in real 2- or 3-dimensional space. Therefore, decentralized control for the coordination of networks of multiple autonomous agents has attracted a considerable amount of attentions.

Form a control point of view, it is clear that there are tasks at both the level of the whole formation, determining for example waypoints for a path which the center of gravity of the formation should follow as well as control tasks for the individual agents of the formation, such as maintaining their relative positions or shifting from one formation shape to another formation shape.

In 1987, Reynoulds[21] proposed a computer model mimicking animal aggregation. Following his work, several other computer models have appeared in the literature, and led to creation of a new area in computer graphics known as artificial life[27]. Similar problems have become a major thrust in systems and control theory, in the context of cooperative control, distributed control of multi-agent, and formation control, see for example in Refs. [6] [9] [12] [14] [15] [16] [17] [19] [20] [22] [24] [25] [26] [28]. The main goal of above papers is to develop a decentralized control strategy, so that a global objective, such as a tight formation with desired inter-agent distances, is achieved.

The problem of maintaining the shape of moving formation has been studied with graphs depicting the control architecture as follows[7][20]: to each agent corresponds a vertex, and to keep the distance between each agent pair i, j, there is a joint effort of both i and j simultaneously and actively maintained their relative positions. The underlying graph of the formation will have an undirected edge (i, j) between vertices i and j. If enough agent pairs explicitly maintain distances, all inter-agent distances and hence the formation shape will be maintained, *i.e.* the formation will be rigid. However, many applications require the behavior of the multi-agent formations governed by unilateral distance constraint, *i.e.* the responsibility for maintaining a distance is not shared by the two concerned agents but relies on only one of them. So the underlying graph of the model is directed. To each agent, one assigns a set of unilateral distance constraints represents by directed edges: the notation (i, j) for a directed edge connotes that the agent i has to maintain its distance to agent j constant during any continuous move.

In recent control literature, the characterization of a system of above type have attempted under the name of rigidity of a directed graph[5][8][16][19], and appeared to have been first formalized using the notion of persistence for a directed graph[10]. In 2005, Hendrickx *et.al.*[10] generalized the definition of the persistence to $R^d (d \geq 3)$ for seeking to provide a theoretical framework for real world applications, which are often in 3-dimensional space as opposed to the plane.

Tanner *et.al.*[24][25][26] developed a class of local control for a group of mobile agents with a fixed and dynamic network topology in R^2, respectively, that results in multi-agent flocking behaviors. In ref.[29] [30], Yu *et.al.* investigated the flocking behaviors of multi-agent formations with a leader agent in 2-dimensional plane. Through constructing a local control laws, they realized that a group of mobile agent could align their velocities with the leader and achieve a constant relative inter-agent distance while avoiding collisions with each others. In Ref. [14], Lee *et.al.* studied the stable flocking of multiple inertial agents on balanced graph in R^2. And Anderson *et. al.* [1] [2] [11] investigated the rigidity and persistence of meta-formations through cooperative control and realized the control of a three-coleader formations in the 2-dimensional plane[3]. In Ref.[31][32], stable flocking motion of multi-agent formation in 3-dimensional space is studied.

Inspired by the results of Refs. [24] [25] [26] [29] [30] [31] [32], in this paper we firstly study the flocking behaviors of multi-agent formation in 3-dimensional space, in which the topology of control interactions between agents is fixed. Each agent regulates its position and orientation based on a fixed set of neighbors. In this case, the control inputs for every agent are smooth, we show that a formation of mobile multi-agent is capable of coordinating itself so that all agents following a leader can achieve flocking behavior if the underlying graph is undirected. Then, we also realize the distance preservation of a formation with a cyclic relation in 3-dimensional space which underlying graph is directed.

The rest of the paper is organized as follows. In section 2, The graph notions and theory in 2- or 3-dimensional space are introduced, and some conditions are also described, which must be fulfilled by architectures that allow maintenance of formation shape during formation movement. In section 3, we presents the control scheme that triggers flocking and analyzes the stability of the closed-loop system when the underlying graph is undirected. In section 4, we mainly investigate the control of four-agent formation with a cyclic relation in 3-dimensional space whose underlying graph is directed. Finally in section 5, the conclusions are given.

II. Graph theory preliminary

In this section, we introduce some notions of formation graph for coordination of multi-agent.

The formation graph of multi-agent is directed graph $G = (V, E)$ that consists of a set of vertices V with element v_i, $i \in I = \{1, 2, \cdots, n\}$ (I is called an index set) and a set of edges E with elements $(v_i, \quad v_j) \in V \times V$ for $i, j \in I$. We say that v_j is a neighbor of v_i if $(v_i, \quad v_j) \in E$. A graph G is called undirected if it is satisfied the property: $\forall \quad i, j \in I, \quad i \neq j, \quad (v_i, \quad v_j) \in E \Longleftrightarrow (v_j, \quad v_i) \in E$.

A graph $G = (V, E)$ is called complete if any two vertices are neighbors. A path of length m from vertex v_i to vertex v_j is a sequence of $m + 1$ distinct vertices starting with v_i and ending with v_j.

Definition 1: A Graph $G = (V, E)$ is said to be connected if there is a path between any two vertices of Graph G.

Rigid graph theory is used to state properties of graph which ensures that the formation being modeled by the graph will be rigid.

Definition 2: A graph is rigid if and only if continuous motion of the vertices of configuration maintaining the distance constraints come from a family motions of all Eucildean space which are desire-preserving. A graph that is not rigid is said to be flexible.

Generally speaking, a rigid formation is one in which the only smooth motions are those corresponding to translation or rotation of the whole formation. See Figs.1 and 2 for some examples of 2-dimensional and 3-dimensional graphs. Obviously, a graph is connected if it is rigid.

Definition 3: The graph $G' = (V', E')$ is called the subgraph of $G = (V, E)$ if the vertex set $V' \subset V$ and E' includes all the edges of E that are incident on a vertex pair in V'.

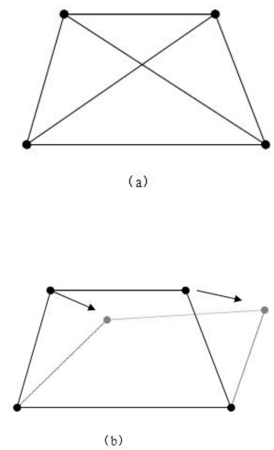

Fig. 1. (a) A rigid formation in R^2, (b) A not-rigid formation in R^2

The following theorem implies that it is possible in 2-dimensional plane to characterize the rigidity of a generic formation corresponding to a given graph.

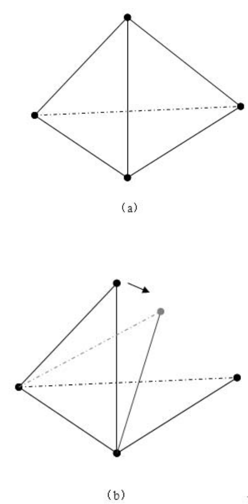

Fig. 2. (a) A rigid formation in R^3, (b) A not-rigid formation in R^3

Theorem 1(Laman's Theorem[13]). A Graph $G = (V, E)$ in R^2 of $|V|$ vertices and $|E|$ edges is rigid if and only if there exists a subgraph $G' = (V, E')$ with $2|V| - 3$ edges such that for any subset V'' of V, the induced subgraph $G'' = (V'', E'')$ of G' obeys $|E''| \leq 2|V''| - 3$. ■

There does not exist any 3-dimensional equivalent results of theorem 1. But a set of such conditions is given in the following theorem.

Theorem 2[28]. A graph $G = (V, E)$ in R^3 of $|V|$ vertices and $|E|$ edges is rigid only if it satisfies the following conditions:

i) there exists a subgraph $G' = (V, E')$ with $3|V| - 6$ edges such that for any subset V'' of V the induced subgraph $G'' = (V'', E'')$ of G' obeys $|E''| \leq 3|V''| - 6$.

ii) if G'' obeys $|E''| = 3|V''| - 6$,

iii) the graph $G' = (V, E')$ is 3-connected, *i.e.* between any two vertices of G'', there are three paths which pairwise are no vertices in common. ■

As mentioned above, the rigidity is an undirected notion, and as noted in Ref.[10], rigidity of a representation implies that if an external observer(or some physical properties) ensures that the distance between the positions of any pair of vertices connected by an edge remains constant, then all the sufficiently close realizations of the induced distance set are congruent to each other. If we consider that the constraints in the formation are not enforced by an external entity, but that each constraint is the responsibility of one agent to enforce, the underlying model of the formation is a directed graph. The persistence of the directed graph means that provided that each agent is trying to satisfy its constraints, the distance between any pair of connected or non-connected agents is maintained constant during any continuous move, and as a consequence the shape of the formation is preserved. The formal definition of the persistence can be found in Refs.[10][28]. A necessary but not sufficient condition for persistence is rigidity. As shown in Fig.3, a persistence graph and a non-persistence graph can have the same underlying undirected graph. In the following we will give some characters and criteria to check the persistence.

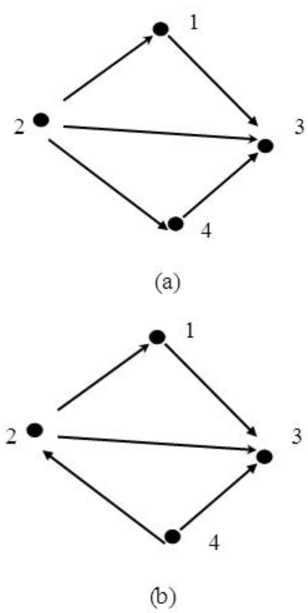

Fig. 3. (a) A rigid but not persistent graph in R^2, (b) A persistent graph in R^2

According to the definition in Refs.[10][28], $d_G^-(i)$ and $d_G^+(i)$ designate respectively the in- and out-degree of the vertex i in the graph G. When no confusion is possible about the graph, we will use $d^-(i)$ and $d^+(i)$, respectively.

Definition 4: In $d-$dimensional space, the number of degrees of freedom(DOF) of a vertex i in a graph $G = (V, E)$ is equal to $\max\{d - d^+(i), 0\}$. And the number of degrees of freedom of a graph is the sum of the numbers of degrees of freedom over all its vertices.

Proposition 1: A persistent graph in $R^d(d \in \{2, 3, \cdots\})$ remains persistent after deletion of any edge(i, j) for which $d^+ \geq d + 1$.

Proposition 2: The number of DOF of a persistent graph in $R^d(d \in \{2, 3, \cdots\})$ can at most be $d(d+1)/2$.

Theorem 3[28]: A graph $G = (V, E)$ is persistent in $R^d(d \in \{2, 3, \cdots\})$ if and only if every subgraph obtained from G by removing edges leaving vertices whose out-degree is greater than d until no such vertex is present anymore in the graph is rigid.

Further details of graph theory can be found in Ref. [4][18][10][28].

III. Control of multi-agent formation whose underlying graph is undirected in R^3

In this section we consider a formation comprising $N + 1$ agents, which can move in a 3-dimensional space. The dynamics can be described by

$$\begin{cases} \dot{b}_i = v_i \\ \dot{v}_i = u_i \end{cases} \quad i = 0, 1, 2, \cdots, N \tag{1}$$

where $b_i = (x_i, y_i, z_i)^T \in R^3$ is the position vector of agent i, $v_i = (\dot{x}_i, \dot{y}_i, \dot{z}_i)^T$ is its velocity and $u_i = (\ddot{x}_i, \ddot{y}_i, \ddot{z}_i)^T$ is the control(acceleration) input. Without loss of generality, suppose that the leader of mobile formation is the agent 0, which is driven at a velocity $v_0(t)$. The position vector of the other agent i relative to the leader agent 0 is denoted by $r_i = b_i - b_0$, and the corresponding velocity of the agent i relative to the leader is $\dot{r}_i = v_i - v_0$. Therefore, the dynamics (1) of the formation can be changed as

$$\begin{cases} \dot{r}_i = v_i - v_0 \\ \ddot{r}_i = u_i - \dot{v}_0. \end{cases} \quad i = 1, 2, \cdots, N. \tag{2}$$

The relative position vector between agent i and j is denoted $r_{ij} = r_i - r_j$.

In order to make the mobile formation achieve flocking behavior by following a leader, we will design a set of local control laws for the followers. The control input u_i for agent i can be divided into two components:

$$u_i = \tilde{u}_i + \hat{u}_i, \qquad i = 1, 2, \cdots, N. \tag{3}$$

The first component $\tilde{u}_i$ is derived from the field produced by an artificial potential function V_i which depends on the relative distance between agent i and its flockmates. This term is responsible for collision avoidance, and cohesion in the group. The second component $\hat{u}_i$ in Eq.(3) regulates the velocity vector of agent i to follow the leader.

To represent the control interconnections between agents, we use a graph with a vertex corresponding to each agent, and edges which can capture the dependence of agent controllers on the state of agents.

For the formation system (2), the corresponding graph $G = (V, E)$ is an undirected graph consisting of a set of vertices $V = \{0, 1, 2, \cdots, N\}$ and a set of vertices $E = \{(i, j)|(i, j) \in V \times V\}$. The set of all neighbors for agent i is called the neighboring set denoted:

$$N_i = \{j|(i, j) \in V \times V\} \subseteq \{0, 1, 2, \cdots, N\} \setminus \{i\}. \quad (4)$$

In order to study the stability of the mobile formation dynamics, we firstly introduce a potential function V_{ij} which can make a pair of neighboring agents i and j keep cohesion and separation for $(i, j) \in E$.

Definition 5: Potential function V_{ij} is a differentiable, nonnegative radially unbounded function of the distance r_{ij} between agents i and j such that

i) $V_{ij}(\|r_{ij}\|) \to \infty$ as $r_{ij} \to 0$,

ii)V_{ij} attains its unique minimum when agents i and j are located at a desired distance.

According to the above definition, we can choose the potential function(see Fig. 4)

$$V_{ij}(\|r_{ij}\|) = \frac{1}{\|r_{ij}\|^2} + \frac{1}{d_{ij}^2} \ln \|r_{ij}\|^2, \quad (5)$$

where $d_{ij} > 0$ is the desired distance of two neighboring agents i and j.

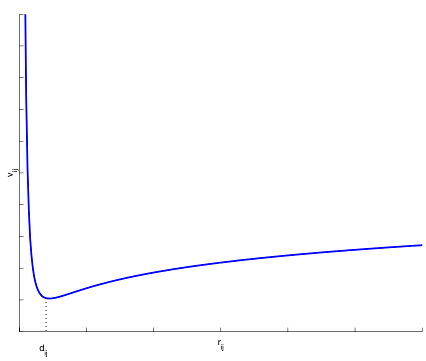

Fig. 4. An inter-agent potential function

Hence the total potential of agent i can be expressed as

$$\begin{aligned} V_i = & \sum_{j \in N_i} V_{ij}(\|r_{ij}\|) \\ = & X_{N_0}(i) V_{i0}(\|r_{i0}\|) + \sum_{j \in N_i \setminus \{0\}} V_{ij}(\|r_{ij}\|), \end{aligned} \quad (6)$$

where

$$X_{N_0}(i) = \begin{cases} 1 & i \in N_0 \\ 0 & i \notin N_0. \end{cases}$$

Theorem 4. Consider a mobile formation of $N+1$ agents with dynamics (2), in which the agent 0 is the leader. It is rigid and the corresponding control laws satisfy

$$u_i = \tilde{u}_i + \hat{u}_i \qquad i = 1, 2, \cdots, N, \quad (7)$$

where $\tilde{u}_i = -\sum_{j \in N_i} \nabla_{r_i} V_{ij}$, $\hat{u}_i = -\dot{r}_i + \dot{v}_0$. Then, all agent velocity vectors asymptotically become the same as the velocity $v_0(t)$ of leader, relative distance that agents maintain between them become constant, collisions between interconnected agents are avoided and the system approaches a configuration that minimize all agent potentials.

Proof: Consider the following positive semi-definite function

$$W(t) = \frac{1}{2} \sum_{i=1}^{n} (2X_{N_0}(i) V_{i0}(\|r_{i0}\|) + \sum_{j \in N_i \setminus \{0\}} V_{ij}(\|r_{ij}\|) + \dot{r}_i^T \dot{r}_i). \quad (8)$$

Using the Lyapunov theory and Lasalle's principle[23], we easily conclude the results of Theorem 4. ■

IV. Control of a formation with a cyclic relation in R^3

In this section we consider a formation in R^3 which consists of four agents, where one agent is an leader, and the other three agents have a cyclic relation. The specific underlying directed graph can be shown in Fig.5. It is obvious that the graph is persistent.

Suppose that the three agents are initially at incorrect distance from one another. The dynamics can be described by

$$\dot{b}_i = v_i, \qquad i = 0, 1, 2, 3 \quad (9)$$

where $b_i = (x_i, y_i, z_i)^T \in R^3$ is the position vector of agent i, $v_i = (\dot{x}_i, \dot{y}_i, \dot{z}_i)^T$ is its velocity. Without loss of generality, suppose that the leader of mobile formation is the agent 0, which is driven at a velocity $v_0(t)$. The position vector of the other agent i relative to the leader agent 0 is denoted by $r_{i0} = b_i - b_0$, and the corresponding velocity of the agent i relative to the leader is $\dot{r}_{i0} = v_i - v_0, \quad i = 1, 2, 3$.

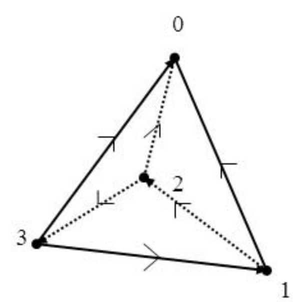

Fig. 5. A formation with a leader and a cyclic relation

Obviously, directed formation control is straightforward if the underlying directed graph depicting the control structure is acyclic[5][14][19]. Challenging problems therefore arise when the graph has cycle. Anderson[3] *et.al* investigated the control of a three-coleader formation in the plane. We will set up the equations with a control law for restoring the correct distances in R^3. Notation is defined in reference to Fig.5, r_{i0} is the the current distance from agent i to the leader agent 0 $\quad (i =$

1, 2, 3); r_{12}, r_{23} and r_{31} are the current distances of from agent 1 to agent 2, from agent 2 to agent 3 and from agent 3 to agent 1, respectively. And the corresponding d_{ij} is the distance which ought to be maintained between agent i to agent j.

We can know that the speed of the agent i which moves in the direction of leader agent 0 can be defined as follows:

$$\dot{r}_{i0} = d_{i0} - r_{i0}, \qquad i = 1,\, 2,\, 3. \tag{10}$$

According to the results of last section, the distance between agent i to the leader agent 0 is easily preserved. In the following,we mainly investigate how control based on the distance preservation can be achieved among the other three followers with a cyclic relation. Under the conditions, the problem can be simplified as the control of three coleader formation in the plane as in Ref.[3] whose underlying directed graph as shown in Fig.6.

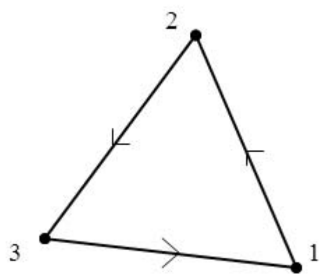

Fig. 6. A formation of three-coleader in R^2

Similar to the assumptions in Ref.[3],with respect to the four-agent formation in R^3, we have the following assumptions.

Assumption 1: Agent i knows the current and correct distances $r_{i,i+1}$, $d_{i,i+1}$ from the agent $i+1$(agent 1 being identified with agent 4). So the internal angle α_i of the triangle formed by the three agents at agent i is known($i = 1, 2, 3$).

Assumption 2: The distances d_{12}, d_{23} and d_{31} are satisfied the triangle inequalities, *i.e.*, the steady state of the three agents sub-formation is supposed to tend a well-defined triangle.

Assumption 3: During the motion of the formation, any of the r_{12}, r_{23} and r_{31} do not become zero, and do not tend to zero as time tends to infinity.

According to the results in Ref.[3], we have the following control equations:

$$\begin{cases} \dot{r}_{12} = (d_{12} - r_{12}) + (d_{23} - r_{23})\cos\alpha_2 \\ \dot{r}_{23} = (d_{23} - r_{23}) + (d_{31} - r_{31})\cos\alpha_3 \\ \dot{r}_{31} = (d_{31} - r_{31}) + (d_{12} - r_{12})\cos\alpha_1\,. \end{cases} \tag{11}$$

Defining the error variables $e_{i0} = r_{i0} - d_{i0}$, $i = 1, 2, 3$; $e_{12} = r_{12} - d_{12}$, $e_{23} = r_{23} - d_{23}$ and $e_{31} = r_{31} - d_{31}$, and combining the equations (10) and (11), we have

$$\begin{bmatrix} \dot{E}_0 \\ \dot{E}_1 \end{bmatrix} = \begin{bmatrix} -I & O \\ O & A_{22} \end{bmatrix} \begin{bmatrix} E_0 \\ E_1 \end{bmatrix} \tag{12}$$

where $E_0 = (e_{10},\, e_{20},\, e_{30})^T$, $E_1 = (e_{12},\, e_{23},\, e_{31})^T$, I and O are a 3×3 identity matrix and a 3×3 zero matrix, respectively;
$$A_{22} = \begin{pmatrix} -1 & -\cos\alpha_2 & 0 \\ 0 & -1 & -\cos\alpha_3 \\ -\cos\alpha_1 & 0 & 1 \end{pmatrix}.$$

Due to the α_i are the functions of r_{12}, r_{23} and r_{31}, the equation (12) is not a linear differential equation. However, we have the following results.

Theorem 5. Under the assumptions 1,2,3, and if there exists some positive constant ϵ which can be guaranteed that $\alpha_i \in [\epsilon,\, \pi - \epsilon]$ for $i = 1,\, 2,\, 3$, the solution of equation (12) is globally exponentially convergent,*i.e.* The distance preservatin of the four-agent formation with a cyclic relation is realized.

Using the Lyapunov theory, we can easily prove the results.

V. Conclusions

In this paper we demonstrate how a mobile multi-agent formation following a leader can cooperate to exhibit a flocking behavior. If the topology of control interconnections is fixed, we model mobile multi-agent formation in 3-dimensional space and introduce a class local control laws which can make the coordinated flocking motion stabilize asymptotically. The control policy ensures that all agents of formation eventually align with each other and have the same heading direction of the leader while avoid collisions and keep a rigid formation at the same time. As the underlying graph of the formation is directed, the control of a formation with a cyclic relation in R^3 is investigated when the distances between the agents are initially incorrect. And a globally exponentially convergent result is obtained. There are still many problem about directed formation deserved to study.

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