Adaptive Fuzzy Multivariable Controller Design Based on the Lyapunov Scheme

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***Abstract*— In this paper a new approach to the design of multivariable adaptive-fuzzy controllers is proposed. The approach is based on fuzzy Lyapunov synthesis, that is an extension of the classical Lyapunov synthesis in the domain of computing with words. The closed-loop system is monitored and the parameters of the controller are adapted in order to minimizing the error between the actual and the desired states of the system. Stability of the closed-loop system is shown by the Lyapunov function which is used in the design stage. The performance of the proposed method is evaluated by applying it to a quadruple-tank process as a case study.**

***Keywords*—Fuzzy Control, Lyapunov Scheme, Quadruple-Tank.**

I. INTRODUCTION

There have been challenging issues in design and analyzing fuzzy systems including: How to design and justify an adaptive-fuzzy controller systematically? And how to analyze the closed-loop stability? Transferring human empirical knowledge to the *"if-then rules"* controller is the another difficulty in traditional methods. In many applications of the fuzzy rule-based systems, fuzzy *"if-then"* rules are heuristically obtained from human expert knowledge. The use of neural networks to extract the fuzzy rule-base is investigated in many papers [1], [2], but stability of the closed-loop system can not be shown and guaranteed in this kind of methods. The Model-based fuzzy control approach [3] is another possibility to solve the aforementioned problems. But it usually yields to a non-fuzzy controller that leads to the loss of linguistic interpretability, which is the most important property of fuzzy systems. Recently, a new method for designing the rule-base fuzzy controllers was suggested [4]. Referred to as the *Fuzzy Lyapunov Synthesis* method, it is based on extending classical Lyapunov synthesis to the domain of *computing with words* [5], [6]. The method allows an analytic derivation of the rule-base and furthermore, the same Lyapunov function used in the design phase can be used to prove the closed-loop stability [7].

Classical Lyapunov synthesis suggests a design method for the controller which guarantees $\dot{V}(x) \le 0$ for a Lyapunov function *V(x)* [8]. Fuzzy Lyapunov synthesis follows the same idea but the linguistic description of the plant and control objective will be used by means of computing with words. The basic assumption is that for a Lyapunov function *V(x)* if all the possible linguistic values of the $\dot{V}(x)$ is not positive, then we can conclude $\dot{V}(x) \le 0$, so the stability can be guaranteed. As an example if $\dot{V}(x) =$ *negative* * *negative* + *negative* * *u,* then we can choose *u=positive big* to make $\dot{V}(x) = negative$. An important point addressed here is that $\dot{V}(x)$ might not be negative unless there exists a set of suitable linguistic variables and their arithmetic operations to guarantee this. To guarantee the negativity of $\dot{V}(x)$, a fuzzy Lyapunov synthesis approach, in connection with fuzzy numbers and their arithmetic operations, was investigated in [9].

Design of adaptive-fuzzy controller for SISO (Single Input, Single Output) systems, based on fuzzy Lyapunov synthesis method, was introduced in [10] for the first time, but the proposed method was applicable to only SISO processes. In this paper, design of adaptive-fuzzy controllers based on fuzzy Lyapunov synthesis method is extended to squared MIMO (Multi Input, Multi-Output) systems. The extended version of the arithmetic operations defined in [9] is used in the design procedure to guarantee the stability of the closed-loop system. A quadruple-tank system is considered as a case study to evaluate the performance of the proposed method. The rest of this paper is organized as follows: design of the fuzzy controllers based on modified fuzzy Lyapunov synthesis method for squared MIMO systems is given in section II. Adaptation scheme for the parameters is presented in section III. The proposed method is applied to a Quadruple-Tank system as a case study in section IV. Section V contains the conclusions.

II. MODIFIED FUZZY LYAPUNOV SYNTHESIS

Fuzzy Lyapunov synthesis follows the classical Lyapunov synthesis idea to make $\dot{V}(x) \le 0$ for a Lyapunov function *V(x).* In the modified fuzzy Lyapunov synthesis, the function not only should be negative but also it should behave in a desired manner [7]. In the following, modified fuzzy Lyapunov synthesis approach for a SISO system is extended to a class of MIMO systems. Consider a squared MIMO system described by the nonlinear state space representation:

$$\begin{aligned} \dot{X} &= f(X) + g(X).U \\ Y &= CX \end{aligned} \tag{1}$$

where; $X=(x_1 \cdots x_n)^T$ is the state vector of the system, $f(.)=(f_1(.)\cdots f_n(.))^T$ and $g(.)=(g_1(.)\cdots g_n(.))^T$ are vectors of differentiable functions, U is the input, C is the output matrices and Y is an $n\times 1$ vector containing the output variables. The control objective is to design a tracking controller for the system. Assume that the exact vectors *f(x)* and *g(x)* are *unknown* but we do have some *partial* (fuzzy) knowledge about them given in the form of fuzzy *"if-then rules"*. Based on this linguistic information and by using computing with words idea, we construct two fuzzy approximated functions for *f(x)* and *g(x)* denoted by $\hat{f}(x)$ and $\hat{g}(x)$, respectively. There are some systematic approaches to do this; one of the possible approaches is the use of neural networks [2].

To design the controller rule-base, we choose a Lyapunov function candidate *V(e)* (where *e* is the error vector) and we also consider a desired negative-definite function $\dot{V}_d$, then $\dot{V}$ is calculated as:

$$\dot{V}=\sum_{i=1}^{n}\frac{\partial V}{\partial e_i}.\frac{\partial e_i}{\partial t}=\sum_{i=1}^{n}\frac{\partial V}{\partial e_i}.\frac{\partial e_i}{\partial y_i}.\frac{\partial y_i}{\partial t}$$
$$=\sum_{i=1}^{n}\frac{\partial V}{\partial e_i}.\frac{\partial e_i}{\partial y_i}\sum_{j=1}^{n}\frac{\partial y_i}{\partial x_j}.\frac{\partial x_j}{\partial t}$$
$$=\sum_{i=1}^{n}\frac{\partial V}{\partial e_i}.\frac{\partial e_i}{\partial y_i}\sum_{j=1}^{n}c_{ij}.(f_j(X)+g_j(X).U) \tag{2}$$

Substituting the *f(X), g(X)* with the approximated functions $\hat{f}(x)$, $\hat{g}(x)$, the approximated value of $\dot{V}$ can be obtained as:

$$\dot{V}\approx\sum_{i=1}^{n}\frac{\partial V}{\partial e_i}.\frac{\partial e_i}{\partial y_i}\sum_{j=1}^{n}c_{ij}.(\hat{f}_j(X)+\hat{g}_j(X).U) \tag{3}$$

By requiring $\dot{V}=\dot{V}_d$, we have:

$$\sum_{i=1}^{n}\frac{\partial V}{\partial e_i}.\frac{\partial e_i}{\partial y_i}\sum_{j=1}^{n}c_{ij}.(\hat{f}_j(X)+\hat{g}_j(X).U)=\dot{V}_d \tag{4}$$

Substituting all the possible linguistic values for X, yields a linguistic equation that can be solved by using the arithmetic operations of fuzzy words defined in [9]. The conditions derived from solving (4), yield the rule-base for the fuzzy controller *u*.

III. Adaptation Scheme

To introduce the basic idea of the scheme, we begin with an example. Consider a driver skilled in driving his own passenger car. If he intends to drive a truck and would apply his usual speed-control strategy, he will notice that the performance of the speed control is not so good compared with his own passenger car. Hence he tries to adapt his usual speed-control strategy to fit to the truck under control. Let investigate the example from another point of view: when the skilled driver wants to increase or decrease speed of the truck he actually tries to change it's energy. He likes that the change in energy of the truck be the same with his own passenger car's one. Indeed he uses modified fuzzy Lyapunov synthesis but he has to use some input-output data to adapt the rule-base of the control strategy in his mind. On the other hand the equation described by (4) may not have an explicit solution. So, the error between time derivative of actual and desired Lyapunov functions should be minimized instead of trying to make them exactly the same, so we have:

$$\text{Min } norm(\dot{V},\dot{V}_d) \tag{5}$$

Where *norm* is some norm of error between $\dot{V}$ and $\dot{V}_d$. The other consideration is that since we used the approximate model of the plant in the design procedure, so the actual $\dot{V}$ can be very different from $\dot{V}_d$.

To solve the above problems, parameters of the designed controller are adapted in order to minimize the error between the actual $\dot{V}$ and the desired one ($\dot{V}_d$). We define the squared error between the actual and desired functions as $E=0.5(\dot{V}-\dot{V}_d)^2$ and let $a=(a_1\cdots a_l)$ be the parameters of the fuzzy controller, that are the parameters of the fuzzy sets in the premise and consequent parts of the rules defining u_m. The MIT rule [11] is used to obtain the adaptation law:

$$a_l(t_{k+1})=a_l(t_k)-\eta\frac{\partial E}{\partial a_l} \tag{6}$$

Where, $\eta\rangle 0$ is a constant and t_k, t_{k+1} are two consecutive time steps. Calculating $\partial E/\partial a_l$ yields:

$$\frac{\partial E}{\partial a_l}=(\dot{V}-\dot{V}_d)\frac{\partial \dot{V}}{\partial a_l}$$
$$=(\dot{V}-\dot{V}_d)\frac{\partial}{\partial a_l}\sum_{i=1}^{n}\frac{\partial V}{\partial e_i}.\frac{\partial e_i}{\partial y_i}\sum_{j=1}^{n}c_{ij}.(f_j(X)+g_j(X).U)$$
$$=(\dot{V}-\dot{V}_d)\sum_{i=1}^{n}\frac{\partial V}{\partial e_i}.\frac{\partial e_i}{\partial y_i}\sum_{j=1}^{n}c_{ji}g_{jm}(X).\frac{\partial u_m}{\partial a_l} \tag{7}$$

Having substituted (7) in (6), we have:

$$a_l(t_{k+1})=a_l(t_k)-\eta(\dot{V}-\dot{V}_d)\sum_{i=1}^{n}\frac{\partial V}{\partial e_i}.\frac{\partial e_i}{\partial y_i}\sum_{j=1}^{n}c_{ji}g_{jm}(X).\frac{\partial u_m}{\partial a_l} \tag{8}$$

However since $\dot{V}$ and $g(X)$ are *unknown*, we replace (8) with:

$$a_l(t_{k+1})=a_l(t_k)-\eta\left(\frac{V(t_{k+1})-V(t_k)}{t_{k+1}-t_k}-\dot{V}_d\right)\times\sum_{i=1}^{n}\frac{\partial V}{\partial e_i}.\frac{\partial e_i}{\partial y_i}\sum_{j=1}^{n}c_{ji}\hat{g}_{jm}(X).\frac{\partial u_m}{\partial a_l} \tag{9}$$

Note that all the quantities in the right-hand side of (9) are now *known* online and $a_l(t_{k+1})$ can be calculated.

IV. Case Study

Performance of the proposed method is validated by applying it to a Quadruple-Tank process. The goal of this case study is to show the performance of the proposed adaptive method step by step.

A. Plant Description

The quadruple-tank process introduced and developed by Johansson at Lund Institute of Technology in 1996 [12]. The system has some special features such as nonlinear dynamics, multi-input multi-output, and an adjustable zero location that makes it ideal for illustrating many concepts in multivariable control, particularly performance limitations due to right-half plane zeros. Effective control of this system has been one of the major challenges for the researchers in the field of control systems. Several control methods have been introduced and tested on this system, among which we find: Decentralized Proportional Integral (PI) Control [13-15], Feedback Relay Auto-Tuning PID Control [16], Linear Quadratic Optimal Control [17], Nonlinear Model Predictive Control [18], Model Predictive Control Based on Quadratic Cost Function [19], Neural Model Predictive Control [20], Quantitative Feedback Method [21].

Quadruple-Tank process consists of four interconnected tanks. The target is to control the level in the lower two tanks using two pumps. The schematic diagram of the process is shown in Fig. 1.

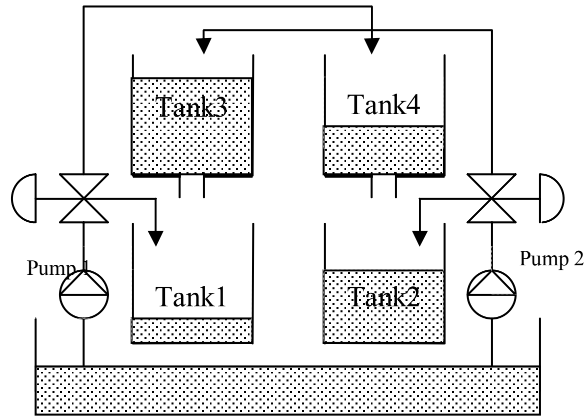

Fig. 1 The Quadruple-Tank process

The process inputs are v1,v2 (input voltages to the pumps) and the outputs are y1,y2 (voltages from level measurement devices). Mass balance's and Bernoulli's law yield [12]:

$$\begin{aligned}
\dot{h}_1 &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} u_1 \\
\dot{h}_2 &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} u_2 \\
\dot{h}_3 &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3} u_2 \\
\dot{h}_4 &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4} u_1
\end{aligned} \tag{10}$$

Where A_i is cross-section of tank i, a_i is cross-section of the outlet hole, h_i is water level of tank i and the parameters $\gamma_1, \gamma_2 \in (0,1)$ are ratios of the control valves connected to the pumps 1 and 2, respectively.

B. *Modelling Stage*

The model of the plant can be given by the standard form of the state space equations.

$$\dot{X} = \underbrace{\begin{bmatrix} -\frac{a_1}{A_1}\sqrt{2gx_1} + \frac{a_3}{A_1}\sqrt{2gx_3} \\ -\frac{a_2}{A_2}\sqrt{2gx_2} + \frac{a_4}{A_2}\sqrt{2gx_4} \\ -\frac{a_3}{A_3}\sqrt{2gx_3} \\ -\frac{a_4}{A_4}\sqrt{2gx_4} \end{bmatrix}}_{f(x)} + \underbrace{\begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix}}_{G(x)} u$$

Suppose that the exact vector *f(x)* is not known but we have some partial information about it. The *G(x)* is assumed to be known (because of simplicity of G this assumption is not impractical). Suppose the available general information about *f(x)* is:

1. The higher the level in the upper tanks the more the outgoing flow.
2. The higher the level in tanks 3 and 4, the faster increase in the level of water in tanks 1 and 2, respectively.
3. The more the applied input voltages to the pumps 1 and 2, the faster increasing in height of water in tanks 1 and 2, respectively.

Based on the above linguistic information and some experimental data, a fuzzy approximated function of f(x) denoted by $\hat{f}(x)$ is obtained. Tables I, II and III show the approximated function $\hat{f}(x)$.

Four Gaussian membership functions are considered for the input variables: Very Low, Low, High and Very High denoted by VL, L, H and VH, respectively. Membership functions are defined as:

TABLE I. PARAMETERS OF $\hat{f}_1(h_1, h_3)$

	VL	L	H	VH
VL	0	-0.0032	-0.0046	-0.0056
L	0.0032	0	-0.0015	-0.0024
H	0.0046	0.0015	0	-0.00096
VH	0.0056	0.0024	0.00096	0

TABLE II. PARAMETERS OF $\hat{f}_2(h_2, h_4)$

	VL	L	H	VH
VL	0	-0.0022	-0.0033	-0.0039
L	0.0022	0	-0.001	-0.0017
H	0.0033	0.001	0	-0.00069
VH	0.0039	0.0017	0.00069	0

TABLE III. PARAMETERS OF $\hat{f}_3(h_3)$ AND $\hat{f}_4(h_4)$

	$\hat{f}_3(x)$	$\hat{f}_4(x)$
VL	0	0
L	-0.0032	-0.0022
H	-0.0046	-0.0033
VH	-0.0056	-0.0039

$$
\begin{aligned}
\mu_{VL}(x) &= \exp\left(\left(\frac{x}{0.05}\right)^2\right) \\
\mu_{L}(x) &= \exp\left(\left(\frac{x-0.08}{0.05}\right)^2\right) \\
\mu_{H}(x) &= \exp\left(\left(\frac{x-0.17}{0.05}\right)^2\right) \\
\mu_{VH}(x) &= \exp\left(\left(\frac{x-0.25}{0.05}\right)^2\right)
\end{aligned}
\tag{11}
$$

The *product* inference engine and the *center of gravity* defuzzifier is used to obtain the fuzzy approximated functions. Hence,

$$
\hat{f}_1(h_1,h_3) = \frac{\sum_{i,j=1}^{4} d_{ij}(h_1,h_3) T_{ij}^1}{\sum_{i,j=1}^{4} d_{ij}(h_1,h_3)} \tag{12}
$$

Where $d_{ii}(h_1,h_3)$ is the firing strength of rule (i,j) and T_{ii}^1 is the value of entry (i,j) of table I. For example consider the rule represented by entry (2,3) in table I, then we have:

if h_1 *is High and* h_3 *is Low then* $\hat{f}_1$ *is near -0.0015*

For the above rule we have:

$d_{2,3}(h_1,h_3) = \mu_H(h_1) \times \mu_L(h_3)$ and $T_{23}^1 = -0.0015$.

The values of $\hat{f}_2(h_2,h_4)$, $\hat{f}_3(h_3)$ and $\hat{f}_4(h_4)$ can be obtained similarly. Therefore, the approximated function is obtained. The next step is to generate the control rule-base using the Lyapunov scheme.

C. Generating the Control Rule-Base

The proposed method in section II is now applied to the Quadruple-Tank system. The first step is to choose a positive-definite Lyapunov function candidate V and a desired negative-definite function $\dot{V}_d$. We choose:

$$
V = \frac{1}{2}((h_1 - r_1)^2 + (h_2 - r_2)^2) \tag{13}
$$

$$
\dot{V}_d = -c((h_1 - r_1)^2 + (h_2 - r_2)^2) \tag{14}
$$

Where r_i, i=1,2 is the reference input signals and h1, h2 are the water level in tanks 1 and 2, respectively and c is a positive constant. Indeed we would like $\dot{V}$ to behave as $\dot{V}_d$. Note that if we succeed in equating $\dot{V} = \dot{V}_d$, then the system will obviously be stable because $\dot{V}_d$ is a negative-definite function. The next step is to differentiate V and use the requirement $\dot{V} = \dot{V}_d$ to derive the rule-base. Having used the chain rule, we have:

$$
\begin{aligned}
\dot{V} &= (h_1 - r_1)\frac{\partial h_1}{\partial t} + (h_2 - r_2)\frac{\partial h_2}{\partial t} \\
&= (h_1 - r_1).(f_1(h_1,h_3) + \frac{\gamma_1 k_1}{A_1} u_1) \\
&\quad +(h_2 - r_2).(f_2(h_2,h_4) + \frac{\gamma_2 k_2}{A_2} u_2)
\end{aligned}
\tag{15}
$$

The requirement $\dot{V} = \dot{V}_d$ results in:

$$
\begin{cases}
f_1(h_1,h_3) + \dfrac{\gamma_1 k_1}{A_1} u_1 = -c(h_1 - r_1) \\
f_2(h_2,h_4) + \dfrac{\gamma_2 k_2}{A_2} u_2 = -c(h_2 - r_2)
\end{cases}
\tag{16}
$$

Solving equation (16) for u_1 and u_2 results in:

$$
\begin{cases}
u_1 = \dfrac{A_1}{\gamma_1 k_1}(-f_1(h_1,h_3) - c(h_1 - r_1)) \\
u_2 = \dfrac{A_2}{\gamma_2 k_2}(-f_2(h_2,h_4) - c(h_2 - r_2))
\end{cases}
\tag{17}
$$

But we have only fuzzy approximated function of *f(X)*, so we substitute the fuzzy approximated function of *f(X)* in (17):

$$
\begin{cases}
u_1 = \dfrac{A_1}{\gamma_1 k_1}(-\hat{f}_1(h_1,h_3) - c(h_1 - r_1)) \\
u_2 = \dfrac{A_2}{\gamma_2 k_2}(-\hat{f}_2(h_2,h_4) - c(h_2 - r_2))
\end{cases}
\tag{18}
$$

Linguistic interpretability is a favorable property of fuzzy controllers but obviously (18) is *not* a fuzzy controller and can not be represented conveniently by a set of linguistic rules. On the other hand fuzzy Lyapunov synthesis follows a different route. We substitute all the possible combination of the fuzzy terms describing h_1 and h_2 in (18) and using arithmetic operations of fuzzy words we can obtain the rule-base of *U*. For example when h_1 is Low and h_3 is High, substituting the values for h_1 and h_3 and $\hat{f}_1$ in (18) we get:

$$
u_1 = \frac{A_1}{\gamma_1 k_1}(-\text{"}near\ \ 0.0015\text{"} - c(Low - r_1)) \tag{19}
$$

For calculation u from (19) the extension of four arithmetic operations to fuzzy words is needed. Here we use the arithmetic operations derived in [9]. Therefore, every fuzzy word is replaced by its center of gravity, thus we have:

$$
u_1 = \frac{A_1}{\gamma_1 k_1}(-0.0015 - c(0.08 - r_1)) \tag{20}
$$

Note that all values of the right-hand side of (20) are now known online and u_1 can be obtained. Thus u_1 is *near* the obtained number when h_1 is Low and h_3 is High. Continuing in this manner, we get the rule-base for u_1. The rule-base for u_2 can be obtained in the same way.

Now using the *product* inference engine and *center of gravity* defuzzifier, we can calculate the value of u as follows:

$$
u_{1,2}(h_1,h_3) = \frac{\sum_{i,j=1}^{4} d^{1,2}{}_{ij}(h_1,h_3) T_{ij}^{1,2}}{\sum_{i,j=1}^{4} d^{1,2}{}_{ij}(h_1,h_3)} \tag{21}
$$

Where $d^{1,2}{}_{ij}(h_1,h_3)$ is the firing strength of rule (i,j) and $T_{ij}^{1,2}$ are the values obtained from (18).

D. Controller Parameters Adaptation

In this section we use the approach described in section III to derive adaptation laws for the parameters a_j and b_j of u_1 and

u_2, respectively. It should be emphasized that the same approach can be easily applied to adapt all the parameters of u_1 and u_2, including the parameters of the fuzzy sets in the premise and consequent parts of the rules, defining u_1 and u_2.

To derive the adaptation scheme for the parameters of the controller (a_j and b_j), we define the squared error between the actual and desired behavior of $\dot{V}$ as:

$$E = \frac{1}{2}(\dot{V} - \dot{V}_d) \tag{22}$$

By differentiating E with respect to a_j and b_j using (7):

$$\begin{aligned}\frac{\partial E}{\partial a_l} &= (\dot{V} - \dot{V}_d)\frac{\partial \dot{V}}{\partial a_l} \\ &= (\dot{V} - \dot{V}_d)(h_1 - r_1)\frac{\gamma_1 k_1}{A_1}\frac{\partial u_1}{\partial a_l}\end{aligned} \tag{23}$$

Using (23) and (6) the adaptation law for u_1 is obtained as:

$$a_l(t_{k+1}) = a_l(t_k) - \eta(\dot{V} - \dot{V}_d)(y_1 - r_1)\frac{\gamma_1 k_1}{A_1}\frac{\partial u_1}{\partial a_l} \tag{24}$$

The adaptation law for u_2 can be obtained in the same way:

$$b_l(t_{k+1}) = b_l(t_k) - \eta(\dot{V} - \dot{V}_d)(y_2 - r_2)\frac{\gamma_2 k_2}{A_2}\frac{\partial u_2}{\partial b_l} \tag{25}$$

E. Simulation Results

Two types of controllers are simulated and the results are presented in this section. The first one is the fuzzy Lyapunov based controller described by (20) without any adaptation law for adjusting the parameters. The second one is the adaptive version of the controller. Equations (23) and (24) are used to adapt the firing strength (d_{ij}) of the rules with η=0.01. The Parameters of the quadruple-tank used in the simulation are listed in table IV.

Figures 2 and 3 show the simulation results of the system for the first and second controllers, respectively. In the first case the input is sum of a sinusoidal and a step function applied at t=10 sec. For the second case a sinusoidal input is also applied at t=10 sec. Note that only one of the inputs is excited and the second input is considered as a constant signal, in both cases. Adapting the parameters of the second controller is started after applying the input (after t=10 sec). The error signal between the process output and the reference signal is shown in figure 4. As shown in the figure, adjustment of the parameters of the controller results in reduction of the error amplitude. Note that the scale of figure 4 is much smaller than the scale of figures 2 or 3.

TABLE IV. PARAMETERS USED IN THE SIMULATION

h^0_1, h^0_2	0.125, 0.125	Cm
h^0_3, h^0_4	0.15, 0.15	Cm
k_1,k_2	3.33, 3.35	Cm^3/volt
γ_1, γ_2	0.7, 0.6	-
A_1,A_2	28	Cm^2
A_3,A_4	32	Cm^2
a_1,a_2	0.071	Cm^2
a_3,a_4	0.057	Cm^2
k_c	0.5	Volt/Cm
g	981	Cm/s

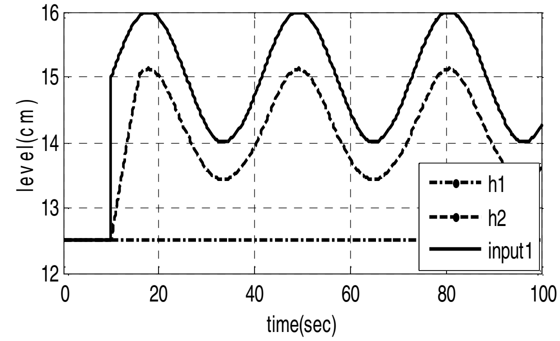

Fig 2. Response of the closed-loop system with the first controller

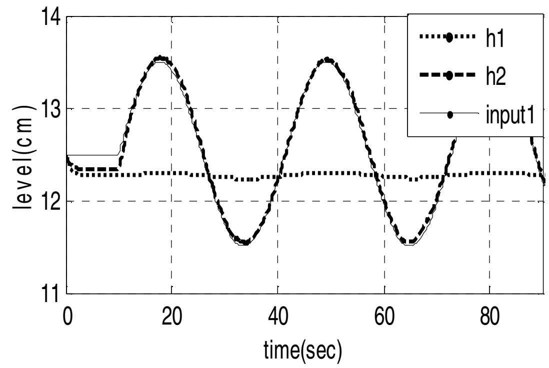

Fig 3. Response of the closed-loop system with the second controller

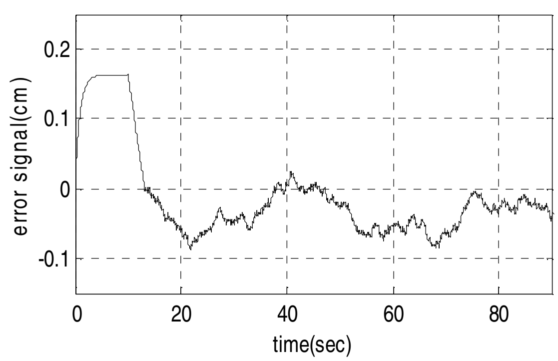

Fig 4. The error signal in the second (adaptive) case

As shown in figure 4, the error amplitude is always less than 0.1 cm. Therefore we can conclude that the performance of the designed adaptive fuzzy controller is satisfactory in tracking the reference signal.

The proposed method has been validated by using the Quadruple-Tank system. The method can also be applied to any squared MIMO system by using the proposed procedure, consequently a stable adaptive-fuzzy controller can be designed without need to the specific human expert knowledge.

V. CONCLUSION

In this paper a systematic approach for designing stable adaptive fuzzy controllers for MIMO systems has been proposed. The approach is based on modified fuzzy Lyapunov synthesis method that follows the classical Lyapunov synthesis method in computing with words idea. The proposed method can be applied to different systems for both tracking and regulation problems. Stability of the method can be proved by a Lyapunov function. The approach is validated by applying it to a quadruple-tank system as a case study. Simulation results show the performance of the designed adaptive fuzzy controller.

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