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Design Principle of Two Mass Jumping System

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***Abstract*—In our previous research, we developed a jumping legged robot aiming at expansion of mobile robot's activities. In jumping motion, the velocity and mass of robot have large effect on jumping height and motion. In order to analyze the jumping motion, one-mass or two-mass models are proposed. At the moment of jumping, the velocity of the robot in two-mass model changes rapidly because the top mass of robot lifts up the foot mass together. The balance of two masses, upper and lower bodies, is one of the important parameters to have enough jumping height. For designing a real robot, we must estimate/design the robot function, total weight, actuator type, motor power, etc, however, it is difficult to find optimal design under ill-defined constraints. It is known that relative proportion of the body and length in animals follows the elastic similarity law. One reason for the relationship is supposed that animals keep safety and robustness in the strength of body. In this paper, we propose index parameters for jumping robot by taking into account the elastic similarity law in animals, and the mass of the robot and necessary actuator power are estimated.**

***Keywords*—Design princicle, Two-mass system, Jumping robot, Elastic similarity**

I. INTORODUCTION

Locomotion strategy in robotics includes various kinds of methods such as a wheeled mobile robot, a crawler mobile robot, a legged mobile robot and so on. In our previous research, we developed a jumping robot, "Jumping Joe", to extend the activities of mobile robots [1][2][3]. Jumping robots [4] can be categorized into two kinds of systems, which are one-mass system and two-mass system. The robots categorized as the one-mass system [3][5] have a single mass, which includes a main part of jumping motion and others accelerated by the main part. The robots of the two-mass system are modeled with a mass of the jumping part and an accelerated part by the mass. In this paper, the main part of the jumping motion is defined as an actuator which accelerates the other part, and the other part accelerated by actuator is defined as a body. Most of the robots categorized as a three or more mass system [6] can be considered as the two-mass system. The velocity of the one-mass system does not change before and after the jumping motion under an ideal environment condition. On the other hand, the velocity of the two-mass system changes at the moment of jumping (Figure 1). Thus, the mass of the main and other part is necessary to estimate the performance of the actuator, which generates the enough energy for desired jumping motion.

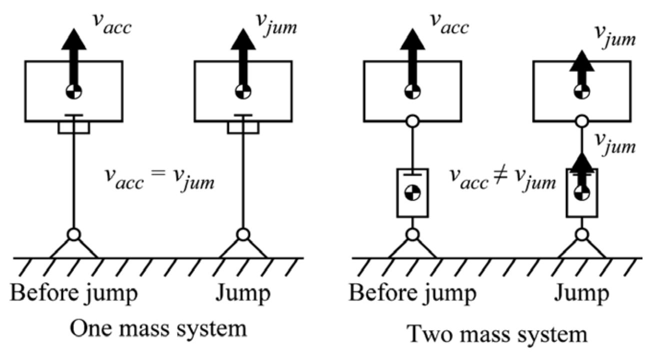

Figure 1. Comparison between one and two mass system

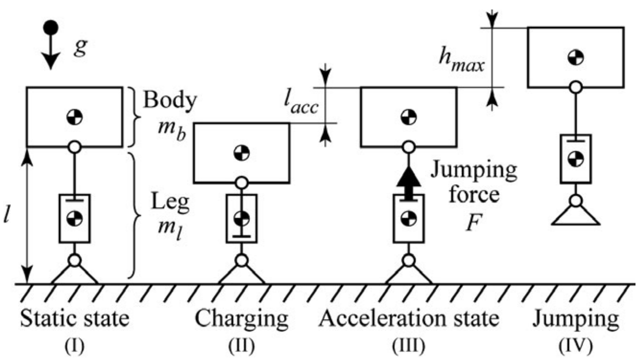

Figure 2. Jumiping robot in two-mass model

In general, it is well known that the mass and the length of have inseparably relationship in the point of similarity. In many cases, we can see that the object follows the similarity law. For example, if the diameter of a column changes, sectional area of the column changes in proportional to the square of the length, and the volume (or mass) changes in proportional to the length cube. However, it has been reported that animals follow the elastic similarity law [7]. In the elastic similarity law, the diameter (e.g., of leg) is proportional to three-second power of the length, and the mass is proportional to forth power of the length. One reason for the relationship is supposed that animals keep safety and robustness in the strength of body. In the animals which follow the elastic similarity law, the mass and length can be estimated by considering the strength of the bones which consists their bodies.

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In this research, an estimation method considering the similarity law is applied for two-mass jumping robot, which has the mass of the body m_b [kg] and the mass of the leg m_l [kg]. The jumping robot of two-mass system in this research is shown in Fig.2. m_b [kg] includes the mass of the frame m_{fra} [kg] and the mass of actuator m_{act} [kg]. The frame is used to support the body, and the actuator is used to generate jumping force F [N]. Let the shape of leg be column, where the length is l [m] and the diameter is d [m]. In the jumping motion, the body is considered as the main part to be accelerated and the leg is the other part with actuators. In this paper, the target jumping height and necessary power of the actuator are expressed by the terms named "jumping rate" and "jumping force mass rate". Jumping rate is the height of jumping normalized by the acceleration distance. Jumping force mass rate is the jumping force normalized by the weight of actuator. These two parameters are non-dimensional parameters obtained by the dimensional analysis. The aim in this research is to estimate the minimum required jumping force mass rate to obtain the target-jumping rate.

At first, the non-dimensional parameters are obtained by the dimensional analysis and the physical implication of these parameters is explained. Second, it is shown that the elastic similarity law can be applied for the two-mass system shown in Fig.1. Third, the estimation method to get the relative proportion of the body for the size and total mass are mentioned. And minimum required jumping force mass rate for the robot is estimated from the obtained total mass. Finally, the effectiveness of our proposed estimation method is verified by numerical calculations.

II. ANALYSYS OF TWO-MASS SYSTEM

A. Dimentional analysis

Okubo et. al. [8] show an evaluation method of jumping performance of a robot using non-dimensional parameters, and jumping rate for acceleration distance depends on four non-dimensional parameters had been reported in ref. [9]. In this section, the jumping robot in two-mass system shown in Fig.2 is analyzed by dimensional analysis, and the necessary non-dimensional parameters to estimate the mass and jumping force mass rate are discussed and defined.

Let the body of the robot be applied by constant jumping force F [N] within acceleration distance l_{acc} [m]. The robot jumps to the height h_{max} [m] against gravity g [m/s^2]. This problem relates to seven physical parameters which are m_b, m_l, m_{act}, l_{acc}, h_{max}, g and F. Each physical parameters are expressed by three basic parameters of mass [$\boldsymbol{M}$], length [$\boldsymbol{L}$] and time [$\boldsymbol{T}$]. So, Four (=7-3) independent non-dimensional parameters exist in the problem due to the Buckingham Π theorem. Here, non-dimensional parameters are assumed as followings.

$$m_b^a\, m_l^b\, m_{act}^c\, l_{acc}^d\, h_{\max}^e\, g^f\, F^g \tag{1}$$

Sum of basic parameters of (1) should be equal to zero to be non-dimension. Thus, the relational equation as follows is obtained.

$$\begin{cases} c = -a - b - g \\ d = e \\ f = -g \end{cases} \tag{2}$$

By substituting (2) to (1), four parameters as follows are obtained.

$$\left(\frac{m_b}{m_{act}}\right)^a \left(\frac{m_l}{m_{act}}\right)^b \left(\frac{h_{\max}}{l_{acc}}\right)^e \left(\frac{F}{m_{act}g}\right)^g \tag{3}$$

Here, above non-dimensional parameters are rewritten as follows to clarify physical meaning of them.

$$\hat{J} = \frac{h_{\max}}{l_{acc}} \tag{4}$$

$$\hat{m} = \frac{m_b}{m_l} \tag{5}$$

$$\hat{F} = \frac{F}{m_{act}g} \tag{6}$$

$\hat{J}$, $\hat{m}$ and $\hat{F}$ are defined as jumping rate, mass rate and jumping force mass rate in this paper, respectively. Jumping rate $\hat{J}$ means jumping height per acceleration distance of the robot. $\hat{J}$ doesn't depend on the size of robot assuming that l_{acc} is equal to a certain times of l. Thus, jumping robot can be evaluated without considering the actual size of robot by using $\hat{J}$,

Mass rate $\hat{m}$ is a non-dimensional parameter which means mass distribution between the body and the leg. If total mass of the robot M [kg] is constant, then m_b and m_l are expressed as follows by using $\hat{m}$.

$$m_b = \frac{\hat{m}M}{\hat{m}+1} \tag{7}$$

$$m_l = \frac{M}{\hat{m}+1} \tag{8}$$

The velocity change between before and after the jumping motion depends on $\hat{m}$. Thus $\hat{m}$ is considered to be an index which expresses ease of jumping.

Jumping force mass rate $\hat{F}$ mean jumping force per unit the mass of the actuator. In general, the power mass rate [W/kg] of actuators which are the same kind is known to be constant without depending on the mass of it [10]. If the velocity of these actuators is decelerated with a constant by several gears, then $\hat{F}$ is a constant. In fact, $\hat{F}$ is considered to express performance of the actuator.

B. Mass estimation

Let m_{act} be equal to ρ_{act} (ρ_{act}>0) times m_{fra}, and density of the frame be ρ_{fra}. m_l is expressed as follows from the assumption.

$$m_l = \pi d^2 \rho_{fra}(\rho_{act}+1) \quad (9)$$

And m_b is expressed as follows by using $\hat{m}$.

$$m_b = \hat{m} m_l = \hat{m}\pi d^2 \rho_{fra}(\rho_{act}+1) \quad (10)$$

Here, let the frame be applied by compressive load P [N] witch is shown in Fig.3, when the robot jumps or lands. At this time, minimum compressive load P_{buc} [N] which causes buckling distortion of leg is given by equation of Euler's buckling load as follows.

$$P_{buc} = \frac{n\pi^2 EI}{l^2} = \frac{n\pi^3 Ed^4}{64l^2} \quad (11)$$

n, E and I are coefficient of fixity, Young's modulus and polar moment of inertia of area, respectively. If P is $P>P_{buc}$, the frame causes buckling distortion. And if P is $P<P_{buc}$, the buckling distortion can be prevented. So, d should satisfy as follows.

$$d \geq \sqrt[4]{\frac{64P}{n\pi^3 E} l^2} \quad (12)$$

Here, d is expressed as follows assuming that p is equal to the integral multiple of weight of body $m_b g$.

$$d \cong \sqrt{\frac{64\hat{m}k_{loa}\rho_{fra}g(\rho_{act}+1)}{n\pi^2 E} l^3} \quad (13)$$

k_{loa} (k_{loa}>0) is the weight magnification times the safety factor. M is expressed as follows by using d which is shown in (13).

$$M = m_b + m_l = \frac{64Ck_{loa}\rho_{fra}^2 g(\rho_{act}+1)^2}{n\pi E} l^4 \quad (14)$$

Here,

$$C = \hat{m}(\hat{m}+1) \quad (15)$$

The change of d and M due to change of l is shown in Fig.4. The parameter d is proportional to three-second power of l according to (13), (14) as shown in Fig.4. And M is proportional to fourth power of l. Thus, the jumping robot in the two-mass system follows elastic similarity law.

Here, Let $\hat{m}$ change under the condition that l is constant. It is necessary to increase m_{fra} as m_b increase s depending on $\hat{m}$, and M is constant. The parameter ρ_{act} should change to keep the robustness of robot where M is constant. Therefore, ρ_{act} is requred to satisfy as follows.

$$C(\rho_{act}+1)^2 = const. \quad (16)$$

By substituting (15) to (16), equation as follows is obtained.

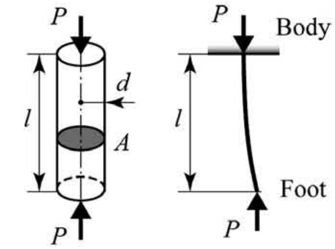

Figure 3. Compressive load to frame

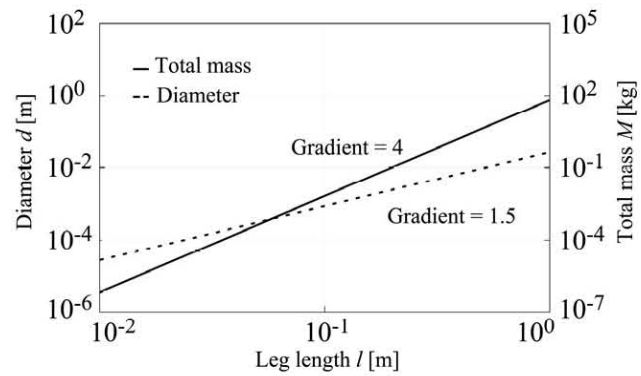

Figure 4. The change of d and M due to change of l

$$\rho_{act} = \sqrt{\frac{k_{act}}{m(m+1)}} - 1 \quad (17)$$

k_{act} (k_{act}>0) is a arbitrary value which is decided by designer.

C. Jumping force mass rate estimation

Let the velocity of body in immediately after and before jumping in Fig.1 be v_{acc} and v_{jum}, respectively. Relation between v_{acc} and v_{jum} is given as follows by the law of conservation of momentum.

$$v_{jum} = e_{los} v_{acc} \quad (18)$$

Here,

$$e_{los} = \frac{\hat{m}}{\hat{m}+1} \quad (19)$$

e_{los} expresses transfer coefficient which is the rate of velocity transferred in the vicinity of jumping. If $\hat{m}$ is infinity, the robot is considered the same as one-mass model because of $e \cong 1$. Here, let the body be accelerated by F within l_{acc}. At this time, v_{acc} is given as follows.

$$v_{acc} = \sqrt{\frac{2l_{acc}(F\hat{m} + F - \hat{m}Mg)}{\hat{m}M}} \quad (20)$$

An equation as follows is obtained by energy balance of the robot starts to jump and height of the robot is h_{max}.

TABLE I. SPECIFICATION OF A HUMAN

l_{acc} [m]	M [kg]	h_{max} [m]	$\hat{h}$ [-]	$\hat{m}$ [-]
0.60	65	0.55	0.92	1.9

TABLE II. DESIGN PARAMETERS

k_{loa}	K_{act}	n	ρ_{fra}	E	$\hat{m}$
[-]	[-]	[-]	[kg/m³]	[N/m²]	[-]
20	1000	0.25	27x10²	69x10¹⁰	1.9

$$h_{\max} = \frac{v_{jum}^2}{2g} \tag{21}$$

By substituting (18)-(20) to (21) and sorting out it using non-dimensional parameters, equation as follows is obtained.

$$\hat{J} = \frac{1}{Mg}\{\frac{F(\hat{m}^2 + m) - Mg\hat{m}^2}{(m+1)^2}\} \tag{22}$$

$\hat{F}$ required to obtain j as above is given by solving (22) for F and substituting it to (6).

$$\hat{F} = \frac{M\{\hat{J}(\hat{m}+1)^2 + \hat{m}^2\}}{m_{act}(\hat{m}^2 + \hat{m})} \tag{23}$$

III. DESIGN OF JUMPING ROBOT

A. Design example

In this section, the design of the jumping robot which has the same $\hat{J}$ as a human is discussed. The effectiveness of the estimation method in this research was verified by using dynamic numerical simulation software. In this research, the jumping robot which has leg length from 10^{-2} [m] to 10^{0}[m] was designed. Then, the data of a genetic adult male who is shown in Table 1 was used. Here, $\hat{m}$ used while the design is equal to a human, and l_{acc} is equal to l. Design principle in this research is shown as follows.

1. Target $\hat{J}$ is equal to that of the human.
2. Design parameters k_{loa} and k_{act} take arbitrarily values. In addition, n is decided by the end condition between frame and P. And E and I by the material of the frame.
3. d and each masses are estimated using (9)-(10), (13) and (14).
4. $\hat{F}$ is estimated using (23).

Design parameters used in the estimation are shown in Table I. h_{max}, which is obtained by the simulation, and F are shown in Fig. 5. $\hat{F}$ and $\hat{J}$ in all leg length are 6.44 and 0.92, respectively.

Robot which has the target $\hat{J}$ can be designed by using this design principle in any leg length. $\hat{F}$ which is needed to obtain target $\hat{J}$ depended on l. This means that $\hat{F}$ can be estimated without depending on the size. And needed F in the

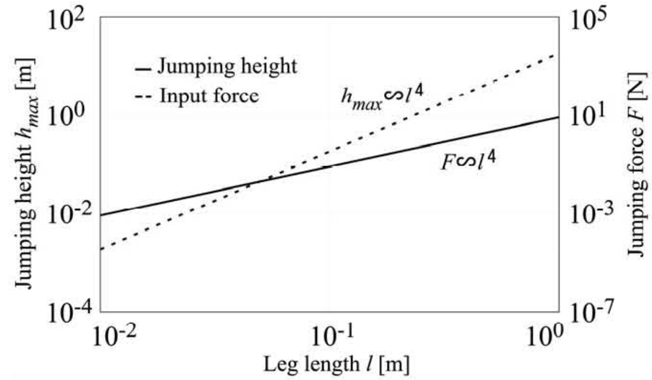

Figure 5. The change of h_{max} and F due to the change l

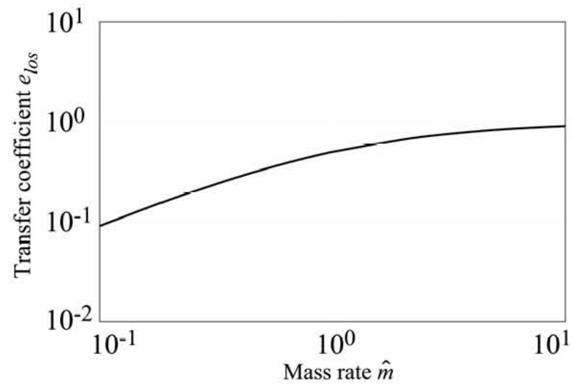

Figure 6. Relation between $\hat{m}$ and e_{los}

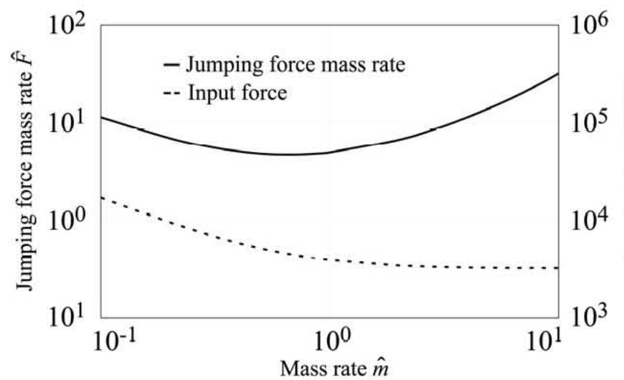

Figure 7. The effect of $\hat{m}$ on $\hat{F}$ and F under $\hat{J}$ =constant

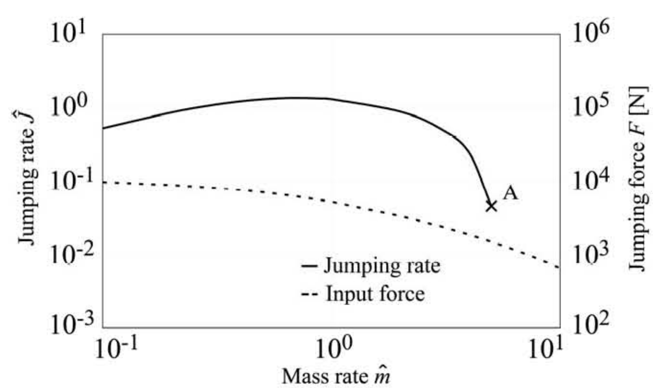

Figure 8. The effect of $\hat{m}$ on $\hat{J}$ and F under $\hat{F}$ =constant

robot was proportional to fourth power of l. In two-mass model, M is proportional to fourth power of l as indicated chapter 2 (refer Fig.4). Thus, it is considered that F depends on M.

B. The Effect of the change of mass rate

The effect of the change of $\hat{m}$ which is considered in previous chapter is discussed in this paragraph. Relation between $\hat{m}$ and e is shown in Fig.6. If m increases, e approaches asymptotically to 1, as shown in Fig.6. Thus, a large $\hat{m}$ is better than small $\hat{m}$ if only considering e.

Next, the effect of $\hat{m}$ on $\hat{J}$ and $\hat{F}$ is verified. Here, design parameters used in the verification is equal to value used in previous paragraph. l, $\hat{F}$ and $\hat{J}$ are 1.0[m], 6.44 and 0.92, respectively. The effect of $\hat{m}$ on needed $\hat{F}$ and F under $\hat{J}$ =constant is shown in Fig.7. Needed $\hat{F}$ is minimum when $\hat{m} \cong 0.7$, as is shown in Fig.7. And F downs asymptotically to 3.2×10^3 [N] due to the change of $\hat{m}$. The effect of $\hat{m}$ on $\hat{J}$ and F under $\hat{F}$ =constant is shown in Fig.8. Obtained $\hat{J}$ is a maximum when $\hat{m} \cong 0.7$, as is shown in Fig.8. And F downs due to the change of $\hat{m}$. The robot is obtained force to jump because F is too small, form $\hat{m}$ of point A in Fig.8. Optimal $\hat{m}$ which $\hat{F}$ is a minimum and $\hat{J}$ is a maximum exist as is understood in above verification.

IV. CONCLUSION

In this research, it is shown that jumping robots follow the elastic similarity law through the analysis of two-mass system. Jumping force mass rate can be estimated considering the effect of the change of leg length and the strength of frame. And the optimal mass rate where jumping rate is a maximum or jumping force mass rate is a minimum exits.

As the future work, optimal mass rate to obtain maximum jumping rate and minimum jumping force mass rate will be studied. And effectiveness of this estimation will be verified by experiment using real machine.

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