

# A Bilevel Optimization Model and a PSO-based Algorithm in Day-ahead Electricity Markets

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**Abstract**—Strategic bidding problems are becoming key issues in competitive electricity markets. This paper applies bilevel optimization theory to deal with this issue. We first analyze generating company strategic bidding behaviors and build a bilevel optimization model for a day-ahead electricity market. In this bilevel optimization model, each generating company will choose their bids in order to maximize their individual profits. A market operator will determine the output power for each unit and uniform marginal price based on the minimization purchase electricity fare. For solving this competitive strategic bidding problem described by the bilevel optimization model, a particle swarm optimization (PSO)-based algorithm is. Experiment results have demonstrated the validity of the PSO-based algorithm in solving the competitive strategic bidding problems for a day-ahead electricity market.

**Keywords**—bilevel programming, electricity market, optimization, particle swarm algorithm, strategic bidding

## I. INTRODUCTION

Many decision problems have hierarchical structures, for which bilevel programming techniques have been developed. In a bilevel decision problem, a decision maker at the upper level is known as the leader, and at the lower level, the follower [1]. Tremendous research has been done on bilevel problems and a lot of achievements have been obtained [2]-[4]. The investigation of bilevel problems is strongly motivated by real world applications, and bilevel programming techniques have been applied with remarkable success in different domains such as decentralized resource planning [5], electric power market [6], logistics [7], civil engineering [8], and road network management [9][10].

Throughout the world, electric power industries are undergoing enormous restructuring processes from nationalized monopolies to competitive market. Because of the significance and particularity of electricity energy to national economics and the society, electricity market must be operated under the conditions of absolute security and stabilization. The research of electricity market has concerned a lot of researchers, owners and managers from electricity entities and authorities. The competitive mechanism of day-ahead markets is one of very important issues in the electricity market study, which can be described as follows. Each generating company (GC) submits a set of hourly (or half-hourly) generation prices and the available capacities for the following day. According to these

data and an hourly (or half-hourly) load forecast, a market operator (MO) allocates generation output for each unit. Many researches have been done on how to strategically bid prices for those GCs, and how to dispatch generation output for MOs to each of their units. David and Wen [11] gave a literature survey on strategic bidding problems in competitive electricity markets. References [12]-[14] used supply function equilibrium models to describe a day-ahead electricity market to maximize GCs' profits and get Nash equilibrium[17]. References [15]-[17] used game theory to build a strategic bidding model for generating companies, and reach a Nash equilibrium solution. However, these models did not include ramp rate constraints, which are very crucial to guarantee a real optimal solution that considers factors from the real world. In addition, because strategic bidding problems involve two hierarchical optimizations, and are different from a conventional game model, a new Nash equilibrium is needed as a solution. From literature, only Pang and Fukushima [17] gave a general Nash equilibrium concept. Reference [18] used a bilevel optimization method to build a generation output allocation model, but did not consider competitive bidding problem from GCs. Reference [19] built a competitive strategic bidding model using bilevel optimization by means of the Cournot and Bertrand model, but did not include ramp rate constraints in their model. Therefore, this paper addresses the competitive strategic bidding problem in electricity markets using bilevel optimization techniques. It will propose a set of theories including a new Nash equilibrium concept and related bilevel optimization model, and also provide a way, the PSO algorithm, to get solutions.

The rest of this paper is organized as follows: Section II presents a competitive strategic bidding model for GCs and a generation output dispatch model for a MO in a day-ahead electricity market. Based on the analysis of strategic bidding behaviors in Section II, Section III proposes a bilevel optimization model, which includes ramp rate constraints, for describing strategic bidding problems in competitive electricity markets. To solve the problems described by a bilevel optimization model, Section IV develops a particle swarm optimization (PSO) technique-based solution algorithm. A set of experiments have been organized to test the validity of the proposed bilevel optimization model and the PSO-based algorithm. Section V shows these experiment results which can effectively support obtaining a solution for the competitive

strategic bidding problems in an electricity market. Finally, conclusions and further study are highlighted in Section VI.

## II. BIDDING STRATEGY ANALYSIS IN COMPETITIVE ELECTRICITY MARKETS

In an auction-based day-ahead electricity market, each GC will try to maximize its own profit by strategic bidding. Generally speaking, each GC submits a set of hourly generation prices and available capacities for the following day. Based on these data and an hourly-load forecast, a MO will allocate generation output. This is a typical bilevel decision problem. GCs are leaders, and a MO is a follower. In this section, under the analysis of bidding strategy optimization problems, we build a competitive strategic bidding model for GCs and a generation output dispatch model for a MO in a day-ahead electricity market.

### A. Generating Companies' Strategic Pricing Model

In the upper level, each GC (leader) concerns how to choose a bidding strategy, which includes generation price and available capacity. Many bidding functions have been proposed. For a power system, the generation cost function generally adopts a quadratic function of the generation output, i.e. the generation cost function can be represented as

$$C_j(P_j) = a_j P_j^2 + b_j P_j + c_j \quad (1)$$

where  $P_j$  is the generation output of generator  $j$ , and  $a_j, b_j, c_j$  are coefficients of generation cost function of generator  $j$ .

The marginal cost of generator  $j$  is calculated by

$$\lambda_j = 2a_j P_j + b_j \quad (2)$$

It is a linear function of its generation output  $P_j$ . The rule in a goods market may expect each GC to bid according to its own generation cost. Therefore we adopt this linear bid function. Suppose the bidding for  $j$ -th unit at time  $t$  is

$$R_{jt} = \alpha_{jt} + \beta_{jt} P_{jt} \quad (3)$$

where  $t \in T$  is the time interval,  $T$  is time interval number,  $j$  represents the unit number,  $P_{jt}$  is the generation output of unit  $j$  at time  $t$ , and  $\alpha_{jt}$  and  $\beta_{jt}$  are the bidding coefficients of unit  $j$  at time  $t$ .

According to the Justice Principle of "the same quality, the same network, and the same price", we adopt a uniform marginal price (UMP) as the market clearing price. Once the energy market is cleared, each unit will be paid according to its generation output and UMP. The payoff of GC<sub>*i*</sub> is

$$F_i = \sum_{t=1}^T \left( \sum_{j \in G_i} UMP_t P_{jt} - \sum_{j \in G_i} (a_j P_{jt}^2 + b_j P_{jt} + c_j) \right) \quad (4)$$

where  $G_i$  is the suffix set of the units belonged to GC<sub>*i*</sub>. Each GC wishes to maximize its own profit  $F_i$ . In fact,  $F_i$  is the

function of  $P_{jt}$  and  $UMP_t$ , and  $UMP_t$  is the function of all units' bidding  $\alpha_{jt}$ ,  $\beta_{jt}$  and output power  $P_{jt}$ , which will impose impact to each other. Therefore, we can establish a strategic pricing model of these GCs as follows

$$\begin{aligned} \max_{\alpha_{jt}, \beta_{jt}, j \in G_i} F_i &= F_i(\alpha_{t1}, \beta_{t1}, \dots, \alpha_{tN}, \beta_{tN}, P_{t1}, \dots, P_{tN}) \\ &= \sum_{t=1}^T (UMP_t P_{ti} - \sum_{j \in G_i} (a_j P_{jt}^2 + b_j P_{jt} + c_j)) \end{aligned} \quad (5)$$

$$i = 1, 2, \dots, L$$

where  $L$  is GC number,  $P_{ti} = \sum_{j \in G_i} P_{jt}$ ,  $t = 1, 2, \dots, T$ .

The profit calculating for each GC will consider both  $P_{jt}$  and  $UMP_t$ , which can be computed by a MO according to the market clearing model.

### B. A Market Operator's Generation Output Dispatch Model

A MO actually represents the consumer electricity purchase from GCs, under the conditions of security and stabilization. The objective of a MO is to minimize the total purchase fare while encouraging GCs to bid price as low as possible. It is reasonable that the lower the price, the more the output. Thus, the function value of a MO's objective will be calculated according to the bidding price. Most previous bidding strategic models do not include ramp rate constraints, without which, the solution for generating dispatch may not be a truly optimal one. We should consider the ramp rate constraints in the real world when modeling a generating dispatch. However, if a model includes ramp rate as constraints, the number of decision variables involved in the problem will increase dramatically, which imposes stronger request for a more powerful solution algorithm. Based on the analysis above, we build a MO's generation output dispatch model as follows:

$$\begin{cases} \min_{P_{jt}} f = f(\alpha_{t1}, \beta_{t1}, \dots, \alpha_{tN}, \beta_{tN}, P_{t1}, \dots, P_{tN}) = \sum_{t=1}^T \sum_{j=1}^N R_{jt} P_{jt} \\ \sum_{j=1}^N P_{jt} = P_{tD} \\ P_{j\min} \leq P_{jt} \leq P_{j\max} \\ -D_j \leq P_{jt} - P_{t-1,j} \leq U_j, t = 1, 2, \dots, T \end{cases} \quad (6)$$

where  $t \in T$  is the time interval,  $T$  is time interval number,  $j$  represents the unit number,  $P_{jt}$  is the generation output of unit  $j$  at time  $t$ , and  $\alpha_{jt}$  and  $\beta_{jt}$  are the bidding coefficients of unit  $j$  at time  $t$ ,  $P_{tD}$  is the load demand at time  $t$ ,  $P_{j\min}$  is the minimum output power of the  $j$ -th unit,  $P_{j\max}$  is the maximum output power of the  $j$ -th unit,  $D_j$  is the maximum downwards ramp rate of the  $j$ -th unit, and  $U_j$  is the maximum upwards ramp rate of the  $j$ -th unit.

After receiving all GCs' bid data, MO determines the output power of each unit and  $UMP_t$  for all time slot  $t$ .  $UMP_t$  can be calculated according to the following steps:

Step1: calculate output power of each unit  $j$  for all time slot  $t$  using formula (6);

Step2: compute bidding  $R_j$  corresponding to the generation output  $P_j$ ;

Step3: account  $UMP_t = \max_{j=1}^N R_j$ .

### III. A BILEVEL OPTIMIZATION MODEL FOR COMPETITIVE ELECTRICITY MARKETS

From the analysis in Section II, we know that in an auction-based day-ahead electricity market, each GC tries to maximize its own profit by strategic bidding, and each MO tries to minimize its total electricity purchase fare. The decision from either of them will influence the other. This is a typical bilevel decision problem, which has multi-leaders and only one follower, with generating companies as leaders and a market operator as a follower.

By combining the strategic pricing model defined in (5) with the generation output dispatch model defined in (6), we establish a bilevel optimization model for competitive strategic bidding-generation output dispatch in an auction-based day-ahead electricity market as follows:

$$\left\{ \begin{array}{l} \max_{\alpha_{ij}, \beta_{ij}, j \in G_i} F_i = F_i(\alpha_{i1}, \beta_{i1}, \dots, \alpha_{iN}, \beta_{iN}, P_{i1}, \dots, P_{iN}) \\ \quad = \sum_{t=1}^T (UMP_t P_{it} - \sum_{j \in G_i} (a_j P_{ij}^2 + b_j P_{ij} + c_j)), \\ \alpha_{t \min} \leq \alpha_{ij} \leq \alpha_{t \max}, \beta_{t \min} \leq \beta_{ij} \leq \beta_{t \max}, \\ t = 1, 2, \dots, T; j = 1, 2, \dots, N; i = 1, 2, \dots, L \\ \min_{P_{ij}} f = f(\alpha_{i1}, \beta_{i1}, \dots, \alpha_{iN}, \beta_{iN}, P_{i1}, \dots, P_{iN}) \\ \quad = \sum_{t=1}^T \sum_{j=1}^N R_{jt} P_{jt}, \\ \sum_{j=1}^N P_{jt} = P_{tD}, \\ P_{j \min} \leq P_{jt} \leq P_{j \max}, \\ -D_j \leq P_{jt} - P_{t-1, j} \leq U_j, t = 1, 2, \dots, T \end{array} \right. \quad (7)$$

where  $\alpha_{ij}$  and  $\beta_{ij}$  are the bidding coefficients of unit  $j$  at time  $t$ ,  $L$  is the number of generating companies,  $P_{it} = \sum_{j \in G_i} P_{ij}$ ,  $P_{j \min}$  is the minimum output power of the  $j$ -th unit,  $P_{j \max}$  is the maximum output power of the  $j$ -th unit,  $D_j$  is the maximum downwards ramp rate of the  $j$ -th unit, and  $U_j$  is the maximum upwards ramp rate of the  $j$ -th unit.

### IV. A PARTICLE SWARM OPTIMIZATION-BASED ALGORITHM

In this section, we use the strategy adopted in PSO method [20] to develop a PSO-based algorithm to reach solutions for problems defined in (7).

Figure 1 outlines the structure and process of this algorithm. We first sample the leaders-controlled variables to get some candidate choices for leaders. Then, we use the PSO method together with Stretching technology [20] to get the follower's response for every leader's choice. Thus a pool of candidate solutions for both the leaders and the follower is formed. By pushing every solution pair moving towards current best ones, the whole solution pool is updated. Once a solution is reached for the leaders, we use Stretching technology [20] to escape local optimization. We repeat this procedure by a pre-defined count and reach a final solution.

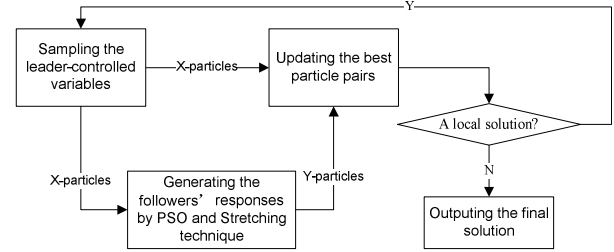


Figure 1. The outline of the PSO based algorithm

The detailed PSO-based algorithm has two parts, Algorithm 1, which is to generate the response from a follower, and Algorithm 2, which is to generate optimal strategies for all leaders. These two algorithms are specified as below.

#### Algorithm 1: Generate the response from a follower

- Step 1: Input the values of  $x_{ij}$  from  $L$  leaders;
- Step 2: Sample  $N_f$  candidates  $y_i$  and the corresponding velocities  $v_{y_i}$ ,  $i = 1, \dots, N_f$ ;
- Step 3: Initiate the follower's loop counter  $k_f = 0$ ;
- Step 4: Record the best particles  $p_{y_i}$  and  $y^*$  from  $p_{y_i}$ ,  $i = 1, \dots, N_f$ ;
- Step 5: Update velocities and positions using
$$v_{y_i}^{k+1} = w_f v_{y_i}^k + c_f r_{1i}^k (p_{y_i}^k - y_i^k) + c_f r_{2i}^k (y_i^{*k} - y_i^k)$$

$$y_i^{k+1} = y_i^k + v_{y_i}^{k+1}$$
- Step 6:  $k_f = k_f + 1$ ;
- Step 7: If  $k_f \geq \text{Max}K_f$  or the solution changes for several consecutive generations are small enough, then we use Stretching technology to obtain the global solution and go to Step 8. Otherwise go to Step 5;
- Step 8: Output  $y^*$  as the response from the follower.

#### Algorithm 2: Generate optimal strategies for leaders

- Step 1: Sample  $N_l$  particles of  $x_{ij}$ , and the corresponding velocities  $v_{x_{ij}}$ ;
- Step 2: Initiate the leaders' loop counter  $k_l = 0$ ;
- Step 3: For the  $k$ -th particle,  $k = 1, \dots, N_l$ , calculate the optimal response for each leader;

Step 3.1: Sample  $N_l$  particles  $x_{ij}$  within the constraints of  $x_{ij}$ ;

Step 3.2: By calling Algorithm 1, we calculate the rational response from the follower;

Step 3.3: Using PSO technique, we obtain the optimal response for each leader;

Step 4: Calculate the function value of every particle by Formula (7);

Step 5: Record  $p_{x_{ij}}, x_{ij}^*, j = 1, \dots, N_l$  for each  $x_{ij}, j = 1, \dots, N_l$ ;

Step 6: Update velocities and positions using

$$v_{x_{ij}}^{k+1} = w_l v_{x_{ij}}^k + c_1 r_{1l}^k (p_{x_{ij}} - x_{ij}^k) + c_2 r_{2l}^k (x_{ij}^{*k} - x_{ij}^k)$$

$$x_{ij}^{k+1} = x_{ij}^k + v_{x_{ij}}^{k+1}$$

Step 7:  $k_l = k_l + 1$ ;

Step 8: If the sum of the differences between samples and optimal responses is smaller than  $\varepsilon$  or  $k_l \geq \text{Max}K_l$ , then we use Stretching technology to current leaders' solutions to obtain the global solution.

Notations used in the algorithms are detailed in TABLE I.

TABLE I. EXPLANATION OF SOME NOTATIONS USED IN THE PSO-BASED ALGORITHM

|                                  |   |
|----------------------------------|---|
| $N_l$                            | the number of candidate solutions (particles) for leaders   |
| $N_f$                            | the number of candidate solutions (particles) for the follower  |
| $x_{ij}$                         | the $j$ -th candidate solutions for the controlling variables from $i$ -th leader                           |
| $p_{x_{ij}}$                     | the best previously visited position of $x_{ij}$  |
| $x_{ij}^*$                       | current best one for particle $x_{ij}$  |
| $v_{x_{ij}}$                     | the velocity of $x_{ij}$  |
| $k_l$                            | current iteration number for the upper-level problem  |
| $y_i$                            | the $i$ -th candidate solution for the controlling variables from the follower                              |
| $p_{y_i}$                        | the best previously visited position of $y_i$   |
| $y^*$                            | current best one for particle $y$   |
| $v_{y_i}$                        | the velocity of $y_i$   |
| $k_f$                            | current iteration number for the lower-level problem  |
| $\text{Max}K_l$                  | the predefined max iteration number for $k_l$   |
| $\text{Max}K_f$                  | the predefined max iteration number for $k_f$   |
| $w_l, w_f$                       | inertia weights for leaders and follower respectively (coefficients for PSO)                                |
| $c_l, c_f$                       | acceleration constants for leaders and follower respectively (coefficients for PSO)                         |
| $r_{1l}, r_{2l}, r_{1f}, r_{2f}$ | random numbers uniformly distributed in [0, 1] for leaders and follower respectively (coefficients for PSO) |

## V. EXPERIMENTS AND CASE STUDIES

In this section, we employ a real world strategic bidding problem in an electricity market to test the bilevel optimization model and the PSO-based algorithm developed in this study.

### A. Test Data

In order to test the effectiveness of the proposed bilevel optimization model and the algorithm in solving the model defined by (7), a typical competitive strategic bidding case consisting of 3 companies with 6 units and 24 time intervals is chosen. The generation cost function can be calculated by using formula (1), where the cost coefficients  $a_j, b_j, c_j$  of unit  $j$  and other techniques data are given in TABLE II, the load demands for each time interval  $t$  are given in TABLE III.

TABLE II. TECHNICAL DATA OF UNITS

| Unit No. | $a_j$   | $P_{\min}$ (MW) | $P_{\max}$ (MW) |
|----------|---------|-----------------|-----------------|
| 1        | 0.00028 | 50              | 680             |
| 2        | 0.00312 | 30              | 150             |
| 3        | 0.00048 | 50              | 360             |
| 4        | 0.00324 | 60              | 240             |
| 5        | 0.00056 | 60              | 300             |
| 6        | 0.00334 | 40              | 160             |

| Unit No. | $b_j$ | $c_j$ | $D_j$ (MW/h) | $U_j$ (MW/h) |
|----------|-------|-------|--------------|--------------|
| 1        | 8.10  | 550   | 110          | 165          |
| 2        | 8.50  | 180   | 35           | 30           |
| 3        | 8.10  | 309   | 50           | 40           |
| 4        | 7.74  | 250   | 45           | 35           |
| 5        | 7.82  | 290   | 50           | 40           |
| 6        | 7.78  | 200   | 40           | 30           |

In TABLE II, Unit1 and Unit 2 belong to GC1, Unit3 and Unit 4 belong to GC2, Unit 5 and Unit 6 belong to GC3.

TABLE III. LOAD DEMANDS IN DIFFERENT TIME INTERVALS

| $t$      | 1    | 2    | 3    | 4    | 5    | 6    |
|----------|------|------|------|------|------|------|
| $P_{id}$ | 1033 | 1000 | 1013 | 1027 | 1066 | 1120 |
| $t$      | 7    | 8    | 9    | 10   | 11   | 12   |
| $P_{id}$ | 1186 | 1253 | 1300 | 1340 | 1313 | 1313 |
| $t$      | 13   | 14   | 15   | 16   | 17   | 18   |
| $P_{id}$ | 1273 | 1322 | 1233 | 1253 | 1280 | 1433 |
| $t$      | 19   | 20   | 21   | 22   | 23   | 24   |
| $P_{id}$ | 1273 | 1580 | 1520 | 1420 | 1300 | 1193 |

To simplify computation, the limit of strategic bidding coefficients does not vary by different time slot, we suppose:

$$\alpha_{r_{\min}} = 7, \alpha_{r_{\max}} = 9, \beta_{r_{\min}} = 0.0002, \beta_{r_{\max}} = 0.007, \\ t = 1, 2, \dots, T; j = 1, 2, \dots, N$$

### B. Experiment Results

This example is run by the PSO-based algorithm proposed in this paper, which was implemented by Visual Basic 6.0, and tested on a desktop computer with CPU Pentium 4, 2.8GHz, RAM 1G, Windows XP. running results for variables values are listed from TABLE IV to TABLE VIII.

TABLE IV. RUNNING RESULTS FOR  $\alpha_{ij}$  FROM THE EXAMPLE

| t \ j | 1    | 2    | 3    | 4    | 5    | 6    |
|-------|------|------|------|------|------|------|
| 1     | 8.40 | 8.64 | 8.35 | 8.06 | 8.30 | 8.01 |
| 2     | 7.80 | 8.77 | 7.74 | 8.19 | 7.16 | 8.14 |
| 3     | 8.21 | 7.92 | 7.63 | 7.87 | 7.58 | 7.29 |
| 4     | 8.39 | 7.37 | 8.34 | 8.79 | 7.76 | 8.74 |
| 5     | 8.13 | 7.84 | 7.55 | 7.79 | 7.50 | 7.21 |
| 6     | 7.62 | 8.60 | 7.04 | 8.02 | 8.99 | 7.97 |
| 7     | 8.15 | 8.39 | 8.09 | 7.80 | 8.04 | 7.75 |
| 8     | 8.96 | 7.93 | 8.91 | 7.88 | 8.86 | 7.83 |
| 9     | 7.33 | 7.03 | 8.74 | 8.98 | 8.69 | 8.40 |
| 10    | 7.45 | 8.43 | 8.87 | 7.85 | 8.82 | 7.80 |
| 11    | 8.61 | 8.32 | 8.56 | 8.27 | 7.97 | 8.21 |
| 12    | 8.05 | 7.03 | 8.00 | 8.44 | 7.42 | 8.39 |
| 13    | 8.52 | 8.23 | 8.47 | 8.18 | 7.89 | 8.13 |
| 14    | 7.28 | 8.26 | 8.70 | 7.68 | 8.65 | 7.62 |
| 15    | 7.07 | 8.78 | 8.49 | 8.73 | 8.44 | 8.15 |
| 16    | 8.62 | 7.59 | 8.56 | 7.54 | 8.51 | 8.96 |
| 17    | 7.72 | 7.96 | 7.67 | 7.38 | 7.09 | 7.33 |
| 18    | 7.11 | 8.09 | 8.53 | 7.50 | 8.48 | 7.45 |
| 19    | 9.00 | 7.24 | 8.95 | 8.66 | 8.90 | 8.61 |
| 20    | 7.71 | 8.68 | 7.66 | 8.10 | 7.08 | 8.05 |
| 21    | 8.91 | 7.15 | 8.86 | 8.57 | 8.81 | 8.52 |
| 22    | 8.94 | 7.92 | 8.36 | 7.33 | 8.31 | 7.28 |
| 23    | 7.46 | 7.70 | 7.41 | 7.12 | 8.83 | 7.07 |
| 24    | 8.27 | 7.25 | 8.22 | 7.20 | 8.17 | 8.62 |

TABLE V. RUNNING RESULTS FOR  $\beta_{ij}$  FROM THE EXAMPLE

| t \ j | 1        | 2        | 3        | 4        | 5        | 6        |
|-------|----------|----------|----------|----------|----------|----------|
| 1     | 0.000496 | 0.000247 | 0.000298 | 0.000349 | 0.000401 | 0.000452 |
| 2     | 0.000328 | 0.000489 | 0.000430 | 0.000292 | 0.000233 | 0.000394 |
| 3     | 0.000439 | 0.000490 | 0.000321 | 0.000373 | 0.000424 | 0.000475 |
| 4     | 0.000279 | 0.000220 | 0.000381 | 0.000322 | 0.000484 | 0.000425 |
| 5     | 0.000478 | 0.000308 | 0.000360 | 0.000411 | 0.000462 | 0.000214 |
| 6     | 0.000405 | 0.000266 | 0.000207 | 0.000369 | 0.000310 | 0.000471 |
| 7     | 0.000311 | 0.000362 | 0.000413 | 0.000465 | 0.000296 | 0.000347 |
| 8     | 0.000246 | 0.000407 | 0.000348 | 0.000210 | 0.000451 | 0.000392 |
| 9     | 0.000239 | 0.000290 | 0.000342 | 0.000393 | 0.000444 | 0.000496 |
| 10    | 0.000481 | 0.000343 | 0.000284 | 0.000445 | 0.000387 | 0.000328 |
| 11    | 0.000483 | 0.000234 | 0.000285 | 0.000337 | 0.000388 | 0.000439 |
| 12    | 0.000433 | 0.000374 | 0.000315 | 0.000476 | 0.000417 | 0.000279 |
| 13    | 0.000221 | 0.000272 | 0.000324 | 0.000375 | 0.000426 | 0.000478 |

|    |          |          |          |          |          |          |
|----|----------|----------|----------|----------|----------|----------|
| 14 | 0.000258 | 0.000420 | 0.000361 | 0.000302 | 0.000463 | 0.000405 |
| 15 | 0.000355 | 0.000406 | 0.000457 | 0.000208 | 0.000260 | 0.000311 |
| 16 | 0.000399 | 0.000340 | 0.000202 | 0.000443 | 0.000304 | 0.000246 |
| 17 | 0.000203 | 0.000334 | 0.000385 | 0.000437 | 0.000488 | 0.000239 |
| 18 | 0.000335 | 0.000497 | 0.000438 | 0.000379 | 0.000240 | 0.000481 |
| 19 | 0.000226 | 0.000278 | 0.000329 | 0.000380 | 0.000431 | 0.000483 |
| 20 | 0.000366 | 0.000228 | 0.000469 | 0.000330 | 0.000271 | 0.000433 |
| 21 | 0.000265 | 0.000316 | 0.000367 | 0.000419 | 0.000470 | 0.000221 |
| 22 | 0.000412 | 0.000353 | 0.000215 | 0.000456 | 0.000317 | 0.000258 |
| 23 | 0.000398 | 0.000449 | 0.000201 | 0.000252 | 0.000303 | 0.000355 |
| 24 | 0.000253 | 0.000494 | 0.000356 | 0.000297 | 0.000458 | 0.000399 |

TABLE VI. RUNNING RESULTS FOR  $UMP_t$  FROM THE EXAMPLE

|        |        |        |        |        |        |
|--------|--------|--------|--------|--------|--------|
| t = 1  | t = 2  | t = 3  | t = 4  | t = 5  | t = 6  |
| 1.67   | 6.16   | 12.70  | 18.39  | 1.33   | 1.89   |
| t = 7  | t = 8  | t = 9  | t = 10 | t = 11 | t = 12 |
| 5.52   | 14.62  | 12.62  | 3.94   | 4.66   | 17.19  |
| t = 13 | t = 14 | t = 15 | t = 16 | t = 17 | t = 18 |
| 19.62  | 7.01   | 10.14  | 6.08   | 18.25  | 16.41  |
| t = 19 | t = 20 | t = 21 | t = 22 | t = 23 | t = 24 |
| 16.63  | 15.98  | 17.91  | 12.14  | 14.75  | 17.53  |

TABLE VII. RUNNING RESULTS FOR  $P_{ij}$  FROM THE EXAMPLE

| t \ j | 1   | 2   | 3   | 4   | 5   | 6   |
|-------|-----|-----|-----|-----|-----|-----|
| 1     | 138 | 35  | 349 | 61  | 290 | 160 |
| 2     | 155 | 40  | 336 | 64  | 250 | 155 |
| 3     | 213 | 45  | 323 | 72  | 210 | 150 |
| 4     | 272 | 50  | 310 | 80  | 170 | 145 |
| 5     | 379 | 55  | 297 | 90  | 130 | 115 |
| 6     | 455 | 60  | 284 | 96  | 90  | 135 |
| 7     | 556 | 65  | 271 | 104 | 60  | 130 |
| 8     | 603 | 70  | 258 | 112 | 85  | 125 |
| 9     | 620 | 75  | 245 | 120 | 120 | 120 |
| 10    | 630 | 80  | 232 | 128 | 155 | 115 |
| 11    | 573 | 85  | 219 | 136 | 190 | 110 |
| 12    | 543 | 90  | 206 | 144 | 225 | 105 |
| 13    | 483 | 85  | 193 | 152 | 260 | 100 |
| 14    | 517 | 80  | 180 | 160 | 290 | 95  |
| 15    | 448 | 75  | 167 | 168 | 285 | 90  |
| 16    | 488 | 70  | 154 | 176 | 265 | 100 |
| 17    | 515 | 65  | 141 | 184 | 245 | 130 |
| 18    | 626 | 60  | 155 | 210 | 225 | 157 |
| 19    | 517 | 30  | 130 | 196 | 250 | 150 |
| 20    | 680 | 60  | 160 | 230 | 290 | 160 |
| 21    | 642 | 90  | 180 | 238 | 240 | 130 |
| 22    | 561 | 120 | 215 | 224 | 200 | 100 |
| 23    | 458 | 130 | 250 | 232 | 160 | 70  |
| 24    | 414 | 140 | 240 | 239 | 120 | 40  |

Under these solutions, the objective values for both the leaders and the follower are listed in TABLE VIII

TABLE VIII. OBJECTIVE VALUES FOR THE DECISION MAKERS

| The 1st GC | The 2nd GC | The 3rd GC | MO     |
|------------|------------|------------|--------|
| 29952      | 15605      | 15217      | 203866 |

## VI. CONCLUSIONS

Based on the analysis on the competitive strategic model for day-ahead electricity market, this paper presents a bilevel optimization model for bidding problems in electricity markets. The proposed PSO-based algorithm can obtain solutions for the competitive strategic bidding problems by providing GCs a competitive strategic bidding within network security constraints. The algorithm has been implemented in a bilevel decision support system software.

Considering the cost with uncertainty, building fuzzy optimal strategic bidding model for day-ahead electricity market will be our next research issue. We will also develop more applications to test and improved the developed bilevel decision support system software.

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