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Flight Control System Design with Hierarchy-Structured Dynamic Inversion and Dynamic Control Allocation

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***Abstract*—A new flight control system design scheme is presented for attitude tracking problem, which integrates hierarchy-structured nonlinear dynamic inversion (NDI) and dynamic control allocation techniques. The hierarchy-structured NDI with two time-scales separation is designed to generate the required aerodynamic moments for given attitude angles command. To avoid the shortcomings in static control allocation approaches, a dynamic control allocation algorithm is adopted to distribute the moments into individual control surfaces. Different from most control allocation methods, the actuator dynamics are considered. The flight control design scheme is evaluated by numerical simulation on a nonlinear six degree-of-freedom aircraft model. Simulation results show the validity and good tracking performance of the flight control system.**

***Index Terms*—flight control, nonlinear dynamic inversion, dynamic control allocation, time-scale separation**

NOMENCLATURE

ϕ, θ, ψ	roll, pitch, yaw attitude angles, rad
p, q, r	roll, pitch, yaw angular rates, rad/s
L, M, N	roll, pitch, yaw moments, N.m
I_x, I_y, I_z	moments of inertia, kg.m^2
I_{xy}, I_{zy}, I_{xz}	products of inertia, kg.m^2
v	virtual control effort, N.m
B	control effectiveness matrix
δ	effector deflection angle, rad
W_u, W_v, W_1, W_2	weighting matrices
ω	bandwidth, rad/s
δ_e	elevator deflection angle, rad
δ_a	aileron deflection angle, rad
δ_r	rudder deflection angle, rad

Superscripts

des	desired
act	actual

Subscripts

l	left effector
r	right effector
c	commanded

I. INTRODUCTION

Conventional flight control design is based on linearized models at a set of operating points (normally including a range of velocity, altitude, angle of attack) and gain scheduling method [1]. Distinct drawbacks of the method lie in: Different control laws need to be designed for each working point. In addition, the number of required gains to be designed and scheduled is very large in the entire flight envelope.

Modern high-performance aircraft operating in a large flight envelope are characterized by significant nonlinearity, especially in attitude control, maneuvering flight, etc. Apparently, flight controllers designed by linearized models are hard to provide stable and satisfying performance due to the aircraft nonlinear behaviors and uncertain aircraft dynamics. Considerable researches have been done for nonlinear flight controllers. Nonlinear dynamic inversion (NDI) is a straightforward and simple method which avoids the gain scheduling problem. Nonlinear dynamics of the aircraft are canceled by feedback linearization. The main difference between the method and conventional linearization methods is that the linearized system is obtained by exact state transformations and feedback rather than linear approximations. NDI approach has been widely used in flight control [2–6].

It is well known that direct dynamic inversion computes total inverse of such a system that the number of control variables is equal to the number of state variables. This fact prevents direct application of dynamic inversion to flight control system. To avoid the problem, two time-scales separation method [7–10] based on singular perturbation theory is introduced, where attitude angles and angular rates of the aircraft are separated into slow state variables and fast state variables, respectively.

Conventional aircraft are configured with independent moment generator in each body-axis, i.e., an elevator for pitch control, ailerons for roll control, and a rudder for yaw control. Due to increased demand on the maneuverability, reliability and survivability, more and unconventional control effectors (such as canards, flaps, elevons, thrust vectorings, etc) are designed for modern tactical aircraft. As a result, the aircraft turn into over-actuated systems. For each body-axis aerodynamic moment, different combinations of actuators can be used. Besides the redundant actuators problem, the coupling between control surfaces and the limits of position and rate on the actuators have to be considered. Therefore, there is a need for control allocation technique to solve reasonable distribution of the control effort among the constrained effectors. Furthermore, optimal control varies with different flight missions, such as minimum power consumption, drag, tear/wear, etc. By using optimization based methods, control allocation technique can exploit the maneuverability of the aircraft to the utmost extent.

Many control allocation algorithms have been proposed for flight control of over-actuated aircraft recently, mainly including direct control allocation method [11], pseudo-inverse based methods [12], daisy chaining method [13], linear programming methods [14], quadratic programming based methods [15], [16] etc.

However, the aforementioned methods assume a static mapping from virtual control $v(t)$ to true control $\delta(t)$, which means the resulting control distribution only depends on the present control demand. Dynamic control allocation (DCA) methods [17–19] have become a popular research area recently, where the control allocation result depends on not only present control distribution but also previous distribution. The chances of rate saturation can be reduced by adding a penalizing item on the actuator rates.

A new flight control system design scheme is proposed in this paper, where hierarchy-structured nonlinear dynamic inversion and dynamic control allocation techniques are synthesized. For given attitude angles command, NDI control is used to produce the required aerodynamic moments. A dynamic control allocation algorithm [17] is designed to distribute the moments into individual actuators under position and rate constraints. The control system design scheme is simulated numerically on a nonlinear six degree-of-freedom (DOF) aircraft model.

II. NONLINEAR DYNAMIC INVERSION CONTROLLER

NDI control law and dynamic control allocation are integrated for nonlinear flight control design in this paper. The system structure is shown in Fig. 1. For given attitude angles, an NDI controller with two time-scales separation is first designed to generate the required moments, namely virtual control of control allocation module. The tracking error between the reference trajectory and the corresponding plant states is used to update the NDI control law. Then, optimal control calculated by the control allocation module is applied to the aircraft accounting for the actuator dynamics.

A. *Nonlinear Dynamic Inversion in Flight Control*

This section starts with a brief introduction of nonlinear dynamic inversion. The dynamics of aircraft can be formulated in the form of affine nonlinear system:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where x is the state vector, u is the control vector, and $f(x)$, $g(x)$ are nonlinear functions of x. If $g(x)$ is invertible, the dynamic inversion of the system can be obtained by:

$$u = g^{-1}(x)[\dot{x} - f(x)] \tag{2}$$

In general, the desired response of the system is given in the form of a first-order system:

$$\dot{x} = \omega(x_c - x) \tag{3}$$

where ω is frequency bandwidth, and x_c is the commanded value for x.

B. *Inner Loop and Outer Loop Control Laws Design*

Based on the time-scale separation theory, the NDI attitude controller is composed by two control loops: the inner loop (the fast states loop) and the outer loop (the slow states loop). The outer loop controller is designed according to the kinematic equations of aircraft:

$$\begin{aligned}\dot{\phi} &= p + q\sin\phi\tan\theta + r\cos\phi\tan\theta\\ \dot{\theta} &= q\cos\phi - r\sin\phi\\ \dot{\psi} &= q\frac{\sin\phi}{\cos\theta} + r\frac{\cos\phi}{\cos\theta}\end{aligned} \tag{4}$$

For the commanded attitude angles $[\phi_c, \theta_c, \psi_c]$, the corresponding angular rates command $[p_c, q_c, r_c]$ can be obtained through the outer loop controller.

Taking the input vector from the outer loop controller, the inner loop controller is used to generate the aerodynamic moment vector $[L, M, N]$, which serves as the input of the control allocation module. The inner loop controller is designed

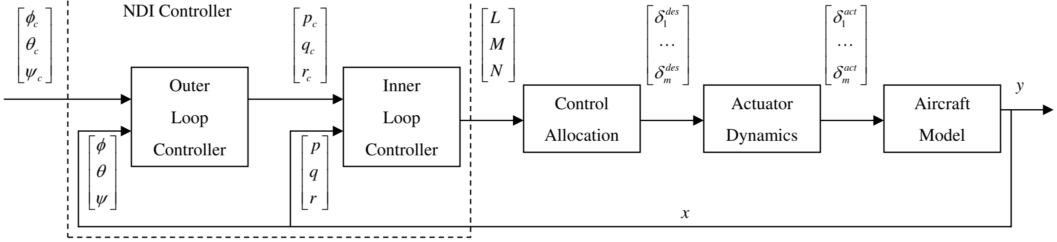

Fig. 1. Flight control system structure

according to the kinetic equations of aircraft:

$$\begin{array}{rcl} \dot{p} & = & c_1 qr + c_2 pq + c_3 L + c_4 N \\ \dot{q} & = & c_5 pr - c_6(p^2 - r^2) + c_7 M \\ \dot{r} & = & c_8 pq - c_2 qr + c_4 L + c_9 N \end{array} \tag{5}$$

where $c_1 = [(I_y - I_z)I_z - I_{xz}^2]/\Sigma$, $c_2 = [(I_x - I_y + I_z)I_{xz}]/\Sigma$, $c_3 = I_z/\Sigma$, $c_4 = I_{xz}/\Sigma$, $c_5 = (I_z - I_x)/I_y$, $c_6 = I_{xz}/I_y$, $c_7 = 1/\Sigma$, $c_8 = [(I_x - I_y)I_x + I_{xz}^2]/\Sigma$, $c_9 = I_x/\Sigma$, $\Sigma = I_x I_z - I_{xz}^2$

The dynamics of attitude angles and angular rates takes the form of (3). According to the singular perturbation theory, the asymptotic stability is guaranteed when the two dynamics are well separated, i.e., the time-scales are not very close to each other [5]. Accordingly, the bandwidths of the inner loop ω_p, ω_q, ω_r, are set as 20 rad/s, and the bandwidths of the outer loop ω_ϕ, ω_θ, ω_ψ are set as 5 rad/s for the aircraft model.

III. DYNAMIC CONTROL ALLOCATION DESIGN

A. *Control Allocation for Flight Control*

The basic control allocation problem can be expressed as a constrained linear mapping problem:

$$v = B\delta \tag{6}$$

subject to

$$\delta_{min} < \delta < \delta_{max}, \quad \dot{\delta}_{min} < \dot{\delta} < \dot{\delta}_{max} \tag{7}$$

where $v \in R^n$ is the desired virtual control vector, $B \in R^{n \times m}$ is the control effectiveness matrix with rank n $(m > n)$, $\delta \in R^m$ is the actual control vector with the position and rate constraints in (7).

Digital control systems are used in modern aircraft, and the rate limits in (7) can be transformed into position limits according to [20]. As a result, the actuator boundary can be rewritten by

$$\underline{\delta}(t) \le \delta \le \overline{\delta}(t) \tag{8a}$$

$$\begin{array}{l} \underline{\delta}(t) = \max[\delta_{min}, \delta(t-T) + \dot{\delta}_{min} T] \\ \overline{\delta}(t) = \min[\delta_{max}, \delta(t-T) + \dot{\delta}_{max} T] \end{array} \tag{8b}$$

where T is the sampling time.

Optimization based control allocation methods aim to find an optimal solution: pick the best one if there are several solutions, or search for a feasible one such that $B\delta$ approximates v as well as possible when there is no exact solution. The optimal control input can be obtained by a two-step optimization, namely sequential quadratic programming.

$$\begin{array}{l} \delta = \underset{\delta \in \Omega}{\text{arg min}} \|W_u(\delta - \delta_p)\|_2 \\ \Omega = \underset{\delta_{min} < \delta < \delta_{max}}{\text{arg min}} \|W_v(B\delta - v)\|_2 \end{array} \tag{9}$$

where δ_p represents some preferred positions of the actuators, Ω is the feasible solution set of control input under the position and rate constraints.

B. *Dynamic Control Allocation Method*

A dynamic control allocation algorithm is presented based on [17], where the control allocation result depends on not only current control distribution but also previous control distribution. The dynamic control allocation approach is an extended quadratic programming method by also penalizing the actuator rates. It takes a form similar to the sequential quadratic programming:

$$\begin{array}{l} J = \underset{\delta(t) \in \Omega}{\min} \|W_1[\delta(t) - \delta_p(t)]\|_2 + \|W_2[\delta(t) - \delta(t-T)]\|_2 \\ \Omega = \underset{\underline{\delta}(t) < \delta < \overline{\delta}(t)}{\text{arg min}} \|W_v[B\delta(t) - v(t)]\|_2 \end{array} \tag{10}$$

The choice of δ_p may correspond to minimum control deflections, drag, radar signature, or wing loading etc. W_1, W_2, W_v are usually chosen as diagonal matrices of proper dimensions.

Compared to (9), not only the position error but also the change in the control input are penalized in the cost function in (10). With the extra term, different weights in W_2 can be chosen, taking the frequency property of different actuators into account. Fast/slow actuators are used to produce the high/low frequency components of the moment command, thus the chances of rate saturation are naturally reduced.

The tradeoff between the two objectives in (10) is governed by the weighting matrices W_1, W_2. A large diagonal matrix W_1 will make a quick convergence to the desired actuator positions, whereas a large W_2 will prevent the actuators moving too fast.

W_1, W_2 allow for actuator prioritization to decide which actuators should be used primarily. Similarly, W_v allows for prioritization among the moments produced in pitch, roll, and yaw.

If no actuators are saturated, the closed form solution of the DCA method can be obtained by:

$$\delta(t) = E\delta_p(t) + F\delta(t-T) + Gv(t) \tag{11}$$

where

$$\begin{array}{l} E = (I - GB)W^{-2}W_1^2 \\ F = (I - GB)W^{-2}W_2^2 \\ G = W^{-1}(BW^{-1})^\dagger \\ W = \sqrt{W_1^2 + W_2^2} \end{array} \tag{12}$$

† is a pesudo-inverse operator. The detailed proof has been given in [17].

C. *Actuator Dynamics*

In most traditional control allocation schemes, the actuator dynamics are neglected, assuming that the actuator response is relatively faster compared with that of the aircraft. However, when the actuator frequencies are comparable with the bandwidth of the aircraft model, the actuator model cannot be ignored. In fact, the presence of actuator dynamics can not only decrease the overall effective bandwidth of the control system but even attenuate the effect of unmodeled nonlinearities [19],

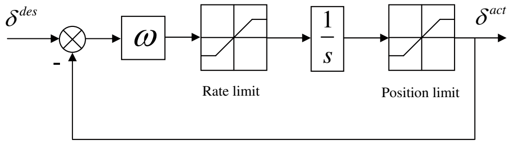

Fig. 2. Actuator dynamics

[21]. The actuator dynamics are considered as a first-order model with the nonlinear effects of saturation and rate limits, which is shown in Fig. 2.

The mathematic model of the actuator dynamics is formulated as follows:

$$H(s) = \frac{\omega}{s+\omega} \tag{13}$$

ω is the bandwidth of actuators, which is taken 20.2 rad/s in this paper.

IV. NUMERICAL EXPERIMENT

The flight control system design scheme is evaluated on a nonlinear six-DOF aircraft model. The aircraft is configured with six actuators. The actuators are left and right elevators (δ_{el}, δ_{er}), left and right ailerons (δ_{al}, δ_{ar}), left and right rudders (δ_{rl}, δ_{rr}), respectively. The position and rate limits of actuator models are summarized in Table I. The hierarchy-structured NDI controller generates the desired moments according to the attitude command. A dynamic control allocation method is applied to distribute the moments into the six actuators of the aircraft.

The aircraft model is first trimmed under the given flight condition of 170 m/s airspeed and 2000 m height. The control effectiveness matrix B of the flight condition is computed according to the wind tunnel data.

$$B = 10^6 \cdot \begin{bmatrix} 1.661 & -1.661 & -0.099 & 0.099 & 0.812 & -0.812 \\ -0.427 & -0.427 & 0.007 & 0.007 & -0.422 & -0.422 \\ 0.031 & -0.031 & -0.449 & 0.449 & 0.053 & -0.0.053 \end{bmatrix}$$

The coefficients of the aircraft model are given as follows: $I_x = 39170$ kg.m^2, $I_y = 244660$ kg.m^2, $I_z = 107800$ kg.m^2, and $I_{xy} = I_{zy} = I_{xz} = 0$.

The parameters set in dynamic control allocation are:

$$W_1 = diag([20, 20, 10, 10, 1, 1]), W_2 = I_{6\times 6}, W_v = I_{3\times 3}$$

To illustrate the effectiveness of the flight control system, take the longitudinal motion for an example here. The pitch angle command, i.e., θ_c is given in the form of a slope signal, latitudinal commands of roll and yaw angles are set zeros.

The response results of the attitude angles are given in Fig. 3. It is clearly seen from Fig. 3(a) that the pitch angle command is well tracked. The longitudinal command influences weakly on the latitudinal motion, the outputs of the other two attitude angles are nearly zeros, as is shown in Fig. 3(b).

TABLE I
POSITION AND RATE LIMITS OF CONTROL SURFACES

	Position limit, deg	Rate limit, deg/s
Ailerons	(-30, 30)	(-80, 80)
Rudders	(0, 30)	(-120, 120)
Elevators	(-30, 30)	(-60, 60)

The control allocation results are shown in Fig. 4. In Fig. 4(a), the desired moments produced by the NDI controller are depicted with solid lines, and the actual moments computed by the DCA algorithm are in dashed lines. The latitudinal moments (roll moment L and yaw moment N) are much smaller compared with the longitudinal moment (pitch moment M). The good tracking performance of moments in three body-axis directions demonstrates the good attitude angles response in Fig. 3 on the other hand.

The moment command is distributed into individual control surfaces. The deflection positions and deflection rates of all the control surfaces are shown in Fig. 4(b), Fig. 4(c), respectively. All the control surfaces work within the position and rate constraints, which are listed in Table I.

V. CONCLUSIONS

A new flight control system design scheme is proposed, which synthesizes hierarchy-structured nonlinear dynamic inversion and dynamic control allocation techniques. The actuator dynamics are included in the flight control design. The entire design procedure is given out in detail. The attitude controller is designed using nonlinear dynamic inversion with two time-scales separation to generate the required moments. A dynamic control allocation method is used in the control allocation module, where the deflection rates penalty of control surfaces are also considered compared with conventional static control allocation approaches.

The validity of the scheme is demonstrated by a nonlinear six-DOF aircraft model. Numerical simulation results show the good command-tracking performance.

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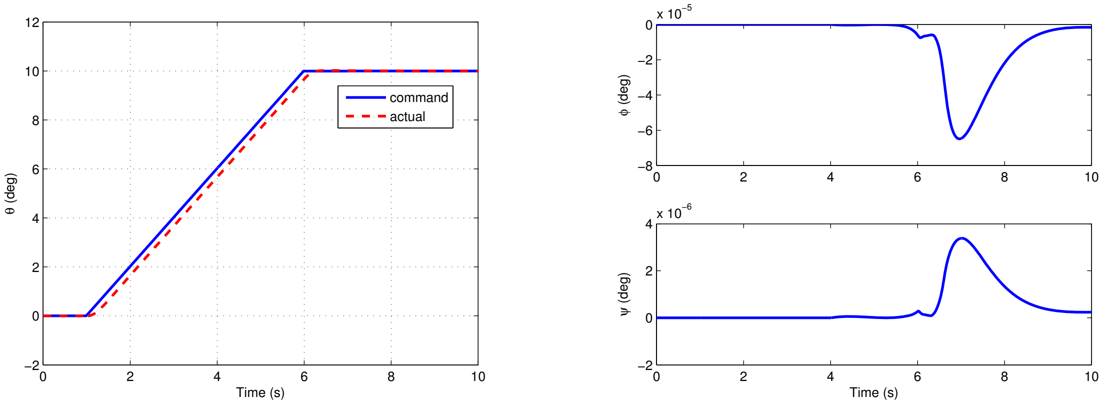
REFERENCES

[1] J. Kawaguchi, Y. Miyazawa and T. Ninomiya, "Flight control law design with hierarchy-structured dynamic inversion approach," *AIAA Guidance, Navigation, and Control Conference*, Honolulu, Hawaii, Aug. 2008.

[2] R. Padhi, S. N. Balakrishnan, "Implementation of pilot commands in aircraft control: a new approach based on dynamic inversion," *AIAA Guidance, Navigation, and Control Conference*, Austin, Texas, Aug. 2003.

[3] B. Bijnens, Q. P. Chu, G. M. Voorsluijs, and J. A. Mulder, "Adaptive feedback linearization flight control for a helicopter UAV,"*AIAA Guidance, Navigation, and Control Conference*, Francisco, CA, Aug. 2005.

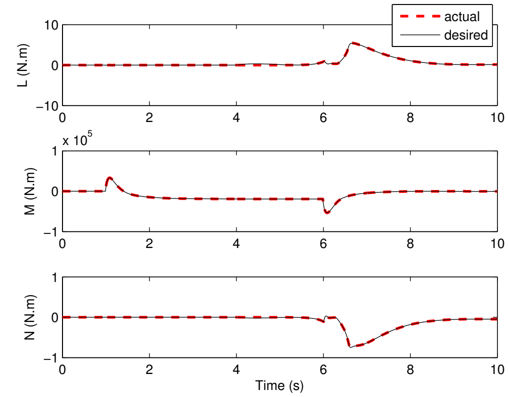
[4] D. Enns, D. Bugajski, R. Hendrick, and G. Stein, "Dynamic inversion: An evolving methodology for flight control," *International Journal of Control*, vol. 59, no.1, pp. 71-91, 1994.



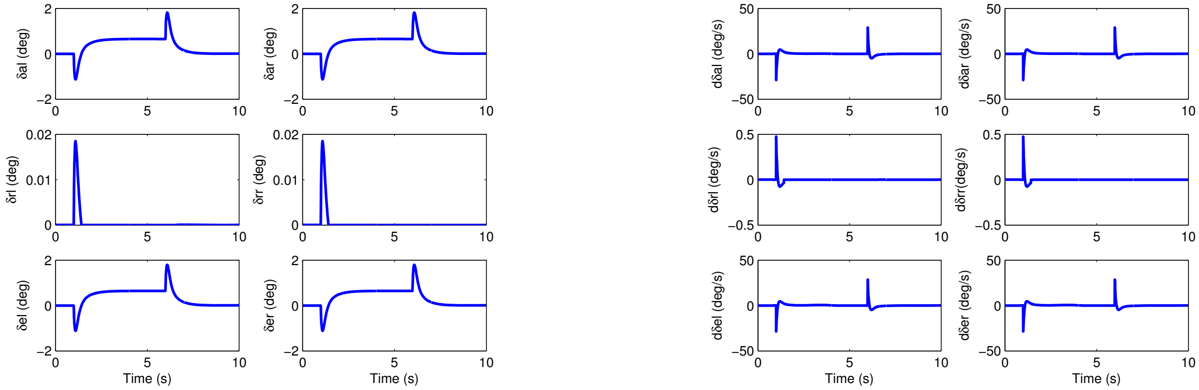
(a) Tracking result of pitch angle command

(b) Outputs of roll angle and yaw angle

Fig. 3. Tracking results of attitude angles



(a) Desired moments generated from NDI controller (solid line) and actual moments produced by control allocation (dashed line)



(b) Control surface deflection positions

(c) Control surface deflection rates

Fig. 4. Control allocation results

[5] R. P. Costa, Q. P. Chu and J. A. Mulder, ”Reentry flight controller design using nonlinear dynamic inversion,”*Journal of Guidance, Control, and Dynamics*, vol. 40, no.1, pp. 64-71, 2003.

[6] D. M. Acosta, S. S. Joshi, ”Adaptive nonlinear dynamic inversion control of an autonomous airship for the exploration of Titan,”*AIAA Guidance, Navigation, and Control Conference*, Hilton Head, CA, Aug., 2007.

[7] A. Chakrabortty, M. Arcak, ”Time-scale separation redesigns for stabilization and performance recovery of uncertain nonlinear systems,”*Automatica*, vol. 45, pp. 34-44, 2009.

[8] Y. Shtessel, J. Buffington, and S. Banda, ”Mutiple timescale flight control using reconfigurable sliding modes,” *Journal of Guidance, Control, and Dynamics*, vol. 22, no.6, pp. 873-883, 1999.

[9] N. Hoyakimyan, E. Lavretsky, and A. J. Sasane, ”Approximate dynamic inversion for nonaffine in control systems via time-scale separation,” *Journal of Dynamical Systems and Control*, vol. 13, no.4, pp. 451-465, 2007.

[10] Y. Shtessel, J. Buffington, and S. Banda, ”Tailless aircraft flight control using multiple time scale reconfigurable sliding modes,” *IEEE Trans. on Control Systems Technology*, vol. 10, no.2, pp. 288-296, 2002.

[11] W. C. Durham, ”Constrained control allocation,” *Journal of Guidance, Control, and Dynamics*, Vol. 16, no. 4, pp. 717-725, 1993.

[12] B. M. Min, E. T. Kim, and M. J. Tahk, ”Application of control allocation methods to SAT-II UAV,” *AIAA Guidance, Navigation, and Control Conference*, San Francisco, California, Aug., 2005.

[13] J. M. Buffington, D. F. Enns, ”Lyapunov stability analysis of daisy chain control allocation,” *Journal of Guidance, Control, and Dynamics*, Vol. 19, no. 6, pp. 1226-1230, 1996.

[14] M. Bodson, ”Evaluation of optimization methods for control allocation,” *Journal of Guidance, Control, and Dynamics*, vol. 25, no.4, pp. 703-711, 2002.

[15] J. A. M. Petersen, M. Bodson, ”Constrained quadratic programming techniques for control allocation,” *IEEE Trans. Control Systems Technology*, Vol. 14, no. 1, pp. 91-98, 2006.

[16] J. A. M. Petersen, M. Bodson, ”Interior-point algorithms for control allocation,” Journal of Guidance, Control, and Dynamics, Vol. 28, no. 3, pp. 471-480, 2005.

[17] O. Härkegård, ”Dynamic control allocation using constrained quadratic programming,” *Journal of Guidance, Control, and Dynamics*, vol. 27, no.6, pp. 1028-1034, 2004.

[18] W. C. Arun Kishore, S. Sen, G. Ray, and T. K. Ghoshal, ”Dynamic control allocation for tracking time-varying control demand,” *Journal of Guidance, Control, and Dynamics*, vol. 31, no.4, pp. 1150-1157, 2008.

[19] Y. Luo, A. Swrrani, S. Yurkovich et al, ”Model-Predictive dynamic control allocation scheme for reentry vehicles,” *Journal of Guidance, Control, and Dynamics*, Vol. 30, no. 1, pp. 100-113, 2007.

[20] W. C. Durham, K. A. Bordingnon, ”Multiple control effector rate limiting,” *Journal of Guidance, Control, and Dynamics*, Vol. 19, no. 1, pp. 30-37, 1996.

[21] H. Ahmad, T. Young, ”Control allocation with actuator dynamics for aircraft flight controls”, *AIAA Aviation Technology Integration and Operation Conference*, Belfast, Northern Ireland, Sep. 2007.