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Classes of Optimal Network Topologies under Multiple Efficiency and Robustness Constraints

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***Abstract*—We address the problem of designing optimal network topologies under arbitrary optimality requirements. Using three critical system parameters, efficiency, robustness and cost, we evolve optimal topologies under different environmental conditions. Two prominent classes of topologies emerge as optimal: (1) Star-like topologies, with high efficiency, high resilience to random failures and low cost, and (2) "Circular Skip Lists" (CSL), with high robustness to random failures as well as targeted attacks, and high efficiency at moderate cost. We analyze CSLs to observe that they show several structural motifs that are optimal with respect to a variety of metrics.**

***Index Terms*—Network design, optimal networks, optimal structural motifs**

I. Introduction

Designing optimal network topologies is an important problem across various application domains such as: distributed information systems, supply chain networks, and network-centric warfare (NCW) [1]. The requirements of optimality vary with the purpose for which a network is built. Further, there are conflicting performance requirements within a network that need to be balanced.

Classes of networks have been studied in search of optimal properties especially following the work on complex networks by Strogatz, Barabási and Albert [2]–[4]. Scale-free (SF) networks, with power law degree distributions are shown to have low diameters [5] and high resilience to random failures. Small-world networks, with properties such as low average path lengths (APL) and high clustering, lead to fast propagation of information and high synchronizability [6]. Kleinberg [7] shows that short paths can be found using purely local (node-level) information in a sub-class of small-world networks.

Cohen *et al.* show that while SF networks are robust in the face of random failures [8], they are easily disrupted by targeted attacks [9]. Valente *et al.* [10] report that the most robust structure in the face of random failures or targeted attacks is one with at most three distinct node degrees in the network. Dekker and Colbert [11] study targeted attacks in the context of NCW. They propose node connectivity (the minimum number of node deletions that partitions a network) as the most suitable metric to measure robustness. They report that *vertex-transitive*[1] networks are the most robust networks. Since nodes cannot be distinguished based on their neighbourhood, disruption due to an attack does not depend on the target.

SF networks are also unsuitable for large traffic flows since a small number of nodes handling most of the traffic load can lead to congestion. Unless the load handling capacity of a node is directly proportional to its degree, random-regular graphs and Cayley trees are shown to be better suited for designing traffic flow networks [12]. Guimerà *et al.* [13] study the problem of "searchability", i.e. the ability to find short paths based on local information, in the presence of congestion, and report that only two classes of optimal topologies exist: highly polarized (star-like) networks, when the load on the network is low; and *homogeneous isotropic networks* with symmetric node betweenness, as the load increases. Mondragón [14] reports a sharp transition of network structures from star-like to decentralized as the load on the network increases. Donetti *et al.* [15] study optimal structures for high synchronizability and low first-passage times for random walkers, and propose *entangled networks* with highly homogeneous structural properties.

There are studies on the structural properties of distributed hash tables (DHT) with respect to lookup complexity and resilience to frequent random failures [16], [17]. Cayley graphs are proposed as a unified group-theoretic model for designing optimal DHT topologies [18]. Gummadi *et al.* [19] report that ring structures are most suitable for DHTs. They also argue that the presence of hamiltonian circuits facilitates design of dynamic DHT routing schemes. Further, hamiltonians allow load balancing, also making a network robust. In an earlier work [20], we proposed *circular skip lists* with nearly symmetric degree centrality and multiple Hamiltonian circuits as optimal structures for balancing load in DHTs.

Generally, the approach to addressing the network optimization problem in literature has been either to analyze classes of networks (ex: Cayley graphs or SF networks) for specific optimal properties (ex: low diameter or symmetric betweenness) or to design networks for domain specific performance requirements (ex: DHTs or NCW). Our approach is to address the problem of designing network topologies under a variety of performance trade-offs.

We use genetic algorithm optimization to design both undirected and directed networks for multiple optimality metrics.

[1]E. W. Weisstein. *Vertex-Transitive Graph*. From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/Vertex-TransitiveGraph.html

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We use diameter, APL and closeness centrality as design metrics for efficiency of communication. For robustness, we use degree centrality, node betweenness and edge connectivity as design metrics. Cost is measured in terms of the number of edges. Different combinations of these metrics are useful in different application domains. We also use two sliders to model application dependent trade-offs between efficiency, robustness and cost. Different types of structures emerge as optimal under different optimality requirements. We observe two prominent optimal topology classes: (1) star-like networks, with low diameters at low costs and low resilience to targeted attacks and (2) circular skip lists (CSL), with low diameters at moderate costs and high resilience to both random failures and targeted attacks.

We observe that the trade-off between efficiency and robustness is pronounced only under severe restrictions on cost. In all other cases, CSLs are optimal topological structures in terms of balancing efficiency, robustness and cost (figure 1). They are characterized by several motifs that are optimal under a variety of requirements: low diameter and low APL; resilience to random failures as well as targeted attacks due to the presence of multiple independent paths; homogeneity with near zero entropy of degree distribution, near zero skew in node and edge betweenness; and hamiltonicity.

Our findings not only corroborate several recent results, but also generalize them. Mondragón [14] reports that the transition from star-like networks to symmetric networks is sharp as the load in a network increases. We observe that this holds under other metrics of robustness as well. Graphs with hamiltonian decompositions are shown to be optimal in DHTs [19]. Valente *et al*. [10] report networks with a nearly symmetric degree centrality as most robust against failures as well as attacks. Guimerà [13] proposes networks with symmetric node betweenness as most optimal to handle congestion in traffic flow networks. Regular graphs with *optimal connectivity* are proposed [11] to be ideal under targeted attacks. We observe that circular skip lists are characterized by all these properties which are optimal under different circumstances.

II. The Network Design Framework

Venkatasubramanian *et al*. [21] propose that a complex network optimizes its structure to maximize its survival fitness (or performance). Performance of a network in turn depends on three critical parameters efficiency, robustness and cost; and an environmental selection pressure variable that decides the trade-off between efficiency and robustness. They define efficiency in terms of APL; robustness in terms of the size of the largest connected component that result in a network upon a single node deletion; and cost in terms of the number of edges. Using these parameters in a fitness function, networks of n nodes with n undirected edges are allowed to evolve under varying emphasis on efficiency and robustness. The star topology emerges as most efficient and least robust, whereas the circle topology emerges as least efficient and most robust.

We extend the above formalism to accommodate multiple constraints and optimality objectives:

- Diameter, APL and closeness centrality are used as design metrics for efficiency. Degree centrality, node betweenness centrality and edge connectivity are used as design metrics for robustness.
- We design topologies with arbitrary number of edges (cost measure). We consider both undirected and directed graphs.
- An environmental variable α decides the trade-off between robustness and efficiency. Variable β is a cost control parameter.

Different combinations of the above design metrics are applicable in different scenarios. In DHTs, minimizing lookup complexity (diameter) while maintaining small and symmetric finger tables across machines (degree regularity) is a design objective. To handle traffic flow, designing networks with low APL while balancing load on the nodes (node betweenness) to avoid congestion is important. In case of NCW and supply-chains, having alternate paths when a communication link fails (edge connectivity) is a design requirement.

A detailed report can be found in [22]. In this paper, we present only a representative subset of our experimental results due to space constraints.

A. *Efficiency*

In this paper, we use efficiency based on diameter in the design process. The worst diameter for a connected graph of n nodes is $n-1$, which is the diameter of a straight line graph in case of undirected graphs, and a circle in case of directed graphs. The best diameter is 1, which is the diameter of a clique (complete graph). In other words, a topology is most efficient if the diameter is 1, and least efficient if it is $n-1$. We map a diameter d that falls in the interval $[1, n-1]$, to a value of efficiency (η_d) in the interval $[0,1]$, as: $\eta_d = 1 - \frac{d-1}{n-2}$

B. *Robustness*

We use two definitions of robustness: ρ_p, based on the skew in degree centrality, to cover the symmetric load perspective (as in DHTs and traffic flow networks); and ρ_λ, based on edge connectivity, to cover the targeted attack perspective (as in NCW).

We define skew in degree centrality as the difference in the maximum degree in the graph ($\hat{p}$) and the mean degree of the nodes ($\bar{p}$). (In case of directed graphs, we consider both in and outdegrees.) For a connected graph of n nodes, the worst skew occurs for the star topology. The central node has a degree of $n-1$ and all the nodes surrounding it have a degree of 1. Therefore, the worst skew is $\frac{(n-1)(n-2)}{n}$. The best skew is 0, when all the nodes have the same degree. This occurs when the topologies are *regular* graph topologies as in a circular topology or a clique. This holds for both directed and undirected graphs. Thus, $\rho_p = 1 - \frac{n(\hat{p}-\bar{p})}{(n-1)(n-2)}$

Edge connectivity (λ) is the minimum number of edges whose removal renders a network disconnected. In case of an undirected graph, the tree topologies have the worst connectivity of 1, and the circle has the worst connectivity of 1 in directed graphs. For both cases, the clique has the best connectivity, $n-1$. Thus, robustness, when defined in terms of connectivity is: $\rho_\lambda = \frac{\lambda-1}{n-2}$

C. Cost

We divide cost into two components: (1) infrastructure cost as a function of the number of edges, e, in the network and (2) node level maintenance/"bookkeeping" cost, as a function of the node's degree, p (in case of directed graphs, in and outdegrees, p^{in} and p^{out}). We place upper bounds on both these in our topology design.

Infrastructure Cost: The minimum number of edges (e_{min}) required to have a connected undirected graph is $n-1$ and it is n in case of a directed graph. We associate a cost, $k=0$, to a minimally connected graph. Any "extra" edge has an associated cost. All extra edges cost the same. An undirected clique has the highest cost, with $\hat{e} = \frac{n(n-1)}{2}$ (and $\hat{e} = n(n-1)$, for a directed clique) number of edges. Thus, the cost of a topology is defined as the ratio of the number of extra edges in a topology to the number of extra edges in the clique with the same number of nodes: $k = \frac{e-e_{min}}{\hat{e}-e_{min}}$

Maximum Permissible Degree: The Maximum Permissible Degree, p, is an upper limit on the number of edges that can be incident on a node. In case of a directed graph, p^{in} and p^{out}, are the upper limits on the number of incoming and outgoing edges. Degree is a measure of the local "book-keeping cost". It can also be thought of as the amount of load a node is handling through edges incident on it.

D. Fitness

Fitness of a graph, $\phi(G(V,E))$, is defined in terms of efficiency (η), robustness (ρ) and infrastructure cost (k). The maximum permissible degree is also a design constraint. The general fitness function is as follows:

$$\phi = \alpha\rho + (1-\alpha)\eta - \beta k$$

Here, $0 \le \alpha \le 1$, is an application dependent parameter that acts as a slider between efficiency and robustness. A high value of α indicates that a high emphasis should be placed on the robustness of topologies during topology breeding. The parameter $\beta,\ 0 \le \beta \le 1$ is used for additional cost control (in addition to the upper bound k). When set a high value, β helps the evolutionary process to "squeeze out" the most cost-effective topology that achieves a certain efficiency and robustness (controlled by α) by removing as many superfluous edges as possible.

Thus, the global optimization objective is to find the set of edges to construct the fittest graph:

$$\arg\max_{E} \phi(G(V,E))$$

We use the following two combinations in our "topology breeding" experiments: (1) efficiency in terms of diameter (η_d), robustness in terms of degree centrality (ρ_p), cost, k, and (2) efficiency in terms of diameter (η_d), robustness in terms of edge connectivity (ρ_λ), cost, k. Thus, we have the following fitness functions: (1) $\phi_p = \alpha\rho_p + (1-\alpha)\eta_d - \beta k$, and (2) $\phi_\lambda = \alpha\rho_\lambda + (1-\alpha)\eta_d - \beta k$.

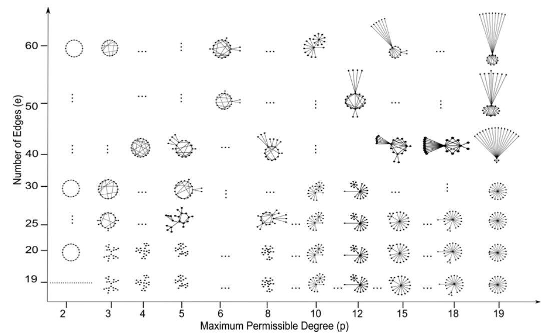

Fig. 1. Undirected Optimal Topology Space with emphasis on robustness, $\alpha = 0$ and cost control parameter, $\beta = 1$, for different no. of edges, e and maximum permissible degree, p. The no. of nodes, $n = 20$.

III. Optimal Topology Families

We conducted topology breeding experiments using different design metrics for networks with up to 200 nodes. Different types of structures emerge at different points in an "optimal topology space" defined by the optimization parameters.

A. Undirected Topologies

Figure 1 shows a sample breed of optimal undirected topologies when the emphasis on robustness, $\alpha = 0$, and the cost control parameter, $\beta = 1$, thus effecting the most efficient (i.e. diameter optimal) as well as the most cost-effective topologies for the given upper bounds on degree, p, and cost, k. We show topologies with a small number of nodes, $n = 20$, so that the structural features can be clearly seen. We make the following observations which hold for any n.

When $e = n-1$ (i.e. cost, $k = 0$) and the maximum permissible degree, $p = 2$, the only possible network is a straight line, with diameter $n-1$, which corresponds to an efficiency, $\eta_d = 0.0$. The degree centrality of a straight line is nearly symmetric, thus it has $\rho_p \approx 1$, for large n. However, a straight line has $\rho_\lambda = 0$, since there is exactly one path between any pair of nodes in the graph.

With $e = n-1$, if we increase p, without regard to robustness (i.e. $\alpha = 0.0$), diameter decreases, and networks with multiple hubs emerge as optimal. This process converges at the star topology. A star is the most efficient topology when $(n-1) \le e < \frac{n(n-1)}{2}$, with a diameter 2. However, it is the least robust topology in terms of degree centrality. All trees are equivalent in terms of robustness when we consider edge connectivity, as all trees have $\lambda = 1$. Failure of any edge in a star partitions a single node from the rest of the network. In case of trees with multiple hubs, the edges in the middle act as "bridges", and their failure causes a greater disruption than the failure of an edge in the periphery. Failure of a node causes maximum disruption in a star: in the worst case, when the central node fails, the network is partitioned into $n-1$ single node components. Thus, star and star-like networks are highly susceptible to targeted attacks.

When $e = n$ and $p = 2$, a circle emerges as optimal, in terms of both efficiency and robustness, regardless of the value of α or β. An undirected circle has a diameter of $\lceil \frac{n}{2} \rceil$. It is 2-regular. Being a hamiltonian circuit, is has an edge connectivity of 2. A circle also has a node connectivity of 2. When $e = n$ and $p > 2$, either trees or circle emerge as optimal structures, depending on the value of α.

When $e > n$, we observe topologies that we call *circular skip lists* (CSL). A CSL is a topology in which each node has edges to one or more other nodes at random distances ("skips") on a logical circle. A circle is the minimal CSL whereas a complete graph is the maximal CSL. When the maximum permissible degree, p is low, CSLs emerge as optimal. As p increases, CSLs start giving way to topologies that are a "hybrid" between CSLs and trees, with a large central loop and small peripheral hubs, as can be seen in figure 1. For higher p, the central loops start growing smaller and hubs larger; the topologies start "shedding" redundant edges (owing to a high value of β); eventually star-like networks emerge.

With increasing e, the onset of hybrid topologies starts for increasingly higher p. We observe that CSLs prevail until the following relation holds, $p <= \frac{2e}{n}$. In fact, the best CSLs occur at the boundary, $p = \frac{2e}{n}$, where topologies utilize nearly all the edges to achieve low diameters. They are also robust with symmetric degree centrality and high edge connectivity, despite not being optimized for robustness. For p beyond this, CSLs start unfolding and shedding edges to form hybrid structures.

With increasing e, we see star-like networks towards the far p end in figure 1, with a small number of hubs forming a highly clustered core. Under the conditions ($\alpha = 0$ and $\beta = 1$), ideally the optimization should converge to the star. However, we see clusters formed as the evolutionary process is unable to throw away all redundant edges, possibly due to local minima.

Topologies shown in figure 1 occur when $\alpha = 0$. As we increase α, (while keeping $\beta = 1$), we see hubs expanding into smaller hubs leading to a straight line, when $e = n - 1$. For higher e, CSLs prevail regardless of p. And star-like networks start transforming into circles trying to achieve a symmetric degree centrality (or a higher edge connectivity). As the cost control parameter β is relaxed, CSLs emerge everywhere except at low values of e, regardless of α. We also see the clusters becoming stronger when p and e increase, eventually forming a clique (complete graph), which is the most efficient as well as the most robust topology, when cost is not a constraint.

Figure 2 shows the sharp transition from star-like networks to CSLs as an effect of α on the breeding process. Here, $n = 20$, $p = n - 1$ and $\beta = 1.0$. The robustness metric used is that of degree centrality, ρ_p. When $\alpha = 0$, we observe highly efficient star-like networks. As α increases, star-like networks give way to lesser fit hybrid networks. The point where the fitness curves start rising is the value of α at which CSLs start emerging. As soon as $e > n$, we observe the emergence of CSLs even for a small emphasis on robustness, despite p being $n - 1$. A similar trend occurs when edge connectivity is used as the robustness metric.

We define the *value* of an optimal network as, $v_p = \eta_d + \rho_p - \beta k$. (When edge connectivity is used as the robustness measure in the breeding process, the corresponding definition of value is, $v_\lambda = \eta_d + \rho_\lambda - \beta k$.) Figure 3 plots v_p of optimal networks that emerge at different α. Again, as e increases, We can see a sharp rise in v_p due to the emergence of CSLs. Also, the "flattening" of the v_p curves indicate that the trade-off between efficiency and robustness is pronounced only under a severe restriction on cost. That is where star-like networks with high efficiency and low robustness to targeted attacks occur.

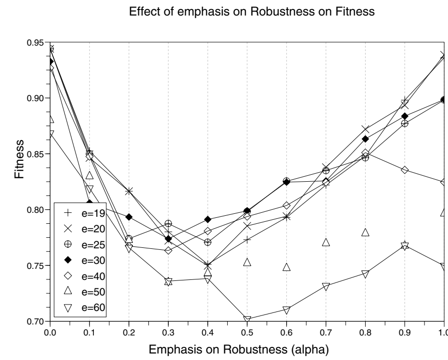

Fig. 2. *Fitness vs* α. Except for very small no. of edges (e), star-like networks start "folding" into CSLs even at a low emphasis on robustness (α), despite the maximum permissible degree, $p = n - 1$. No. of nodes, $n = 20$ and cost control parameter, $\beta = 1.0$.

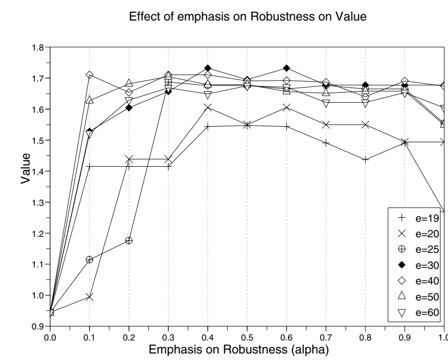

Fig. 3. *Value vs* α. The *value* of a network, $v_p = \eta + \rho_p - \beta k$ rises sharply. Despite $p = n - 1$ and $\beta = 1$, we observe that CSLs set in even for small values of α. Hence, the "flattening" of the value curves.

Figure 4 shows undirected optimal topologies when the emphasis on robustness is maximum ($\alpha = 1$) and the cost control parameter is relaxed ($\beta = 0$). Edge connectivity is used as the metric of robustness. Thus, the breeding process evolves topologies with as high an edge connectivity, λ, as possible, constrained only by the maximum permissible degree, p, and the number of edges, e. When $e >= \frac{np}{2}$, we observe that CSLs emerge with λ of $p - 1$. Further, only a few node pairs have $p - 1$ edge independent paths between them, with most node pairs having p edge independent paths, which is the theoretical upper bound. (The maximal possible connectivity of a graph is equal to the minimal node degree in the graph.) Again, trees

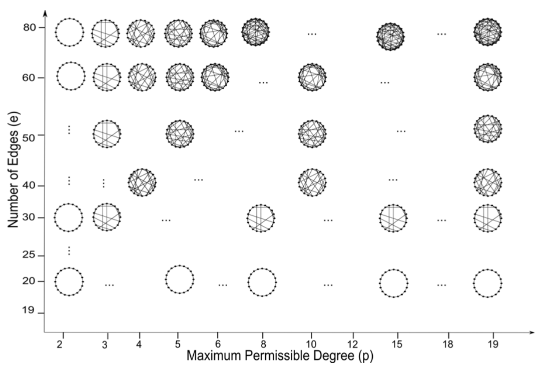

Fig. 4. Undirected Optimal Topology Space with emphasis on robustness, $\alpha = 1$ and cost control parameter, $\beta = 0$, for different no. of edges, e and maximum permissible degree, p. The no. of nodes, $n = 20$. Robustness metric used is edge connectivity, ρ_λ.

can be observed only when α is reduced drastically and β increased.

B. Directed Topologies

We conducted similar experiments for directed graphs. Figure 5 shows the optimal directed graph topologies for $n = 20$, $\alpha = 0.0$ and $\beta = 1.0$.

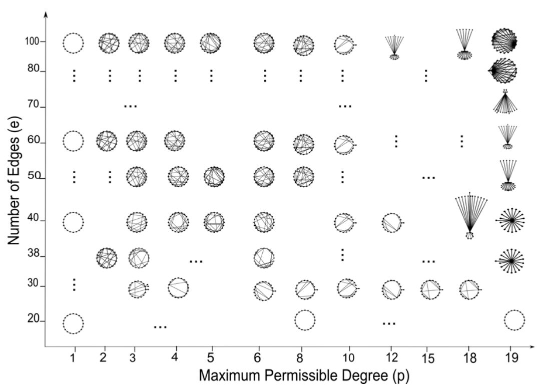

Fig. 5. Directed Optimal Topology Space with emphasis on robustness, $\alpha = 0$ and cost control parameter, $\beta = 1$, for different no. of edges, e and maximum permissible degree, p. The no. of nodes, $n = 20$.

Circular skip lists occur more naturally in directed graphs. For small number of edges, CSLs help achieve strong connectivity. The first directed hubs occur around $e = 2(n-1)$, when p is very high. Similar to the case of undirected graphs, large hubs with highly clustered cores can be seen for high values of e and p. However, CSLs are prevalent everywhere else. This effect is even more complete as α increases.

Figure 6 shows directed optimal topologies when edge connectivity, λ, is used as the robustness metric. When the emphasis on robustness is maximum ($\alpha = 1$) and the cost control parameter is relaxed ($\beta = 1$), we see topologies very similar to the corresponding case of undirected graphs (figure 4). CSLs with high edge connectivity prevail. Similarly, when $e > np$, CSLs with λ of $p - 1$ emerge.

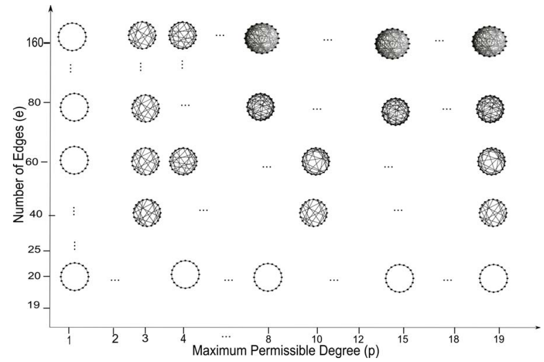

Fig. 6. Directed Optimal Topology Space with emphasis on robustness, $\alpha = 1$ and cost control parameter, $\beta = 0$, for different no. of edges, e and maximum permissible degree, p. The no. of nodes, $n = 20$. Robustness metric used is edge connectivity, ρ_λ.

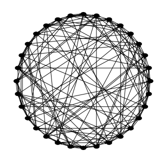

Fig. 7. A directed circular skip list with $n = 32$, $p = 5$ and $e = 160$. CSLs are highly symmetric and possess several optimal properties.

C. Two Classes of Optimal Topologies

Thus, two prominent classes of topologies emerge as optimal under different optimality requirements: (1) star-like networks and (2) circular skip lists. We can observe that star-like networks are optimal only when both the following design requirements are satisfied: (1) very low emphasis on robustness (or very high emphasis on efficiency), and (2) severe restrictions on cost. In all other cases CSL are optimal topologies in terms of balancing efficiency, robustness and cost.

IV. Optimal Structural Motifs of CSLs

Both directed and undirected circular skip lists show several structural features that are potentially optimal under varied applications. We discuss some of them using an example directed CSL of 32 nodes, shown in figure 7.

Hamiltonian Circuits: CSLs contain multiple hamiltonian circuits. Presence of hamiltonian circuits is an optimal feature.

TABLE I
Structural Properties of an example directed CSL, $n = 32$, $p = 5$, $e = 160$

Property	Value
Diameter and APL	3 and 2.53
Eccentricity (min, max) and Radius	(3, 3) and 3
Skew in Closeness	0.0016
Entropy of Indegree and Outdegree	0.0 and 0.0
Skew in Betweenness: Node and Edge	0.005 and 0.0025
Edge Connectivity (min, max)	(4, 5)

Hamiltonians are at least biconnected (both in terms of nodes and edges). They are also considered important in applications such as DHTs [19] as they facilitate design of dynamic routing schemes and load balancing.

Symmetric Centrality Measures: Centralities measure the importance of nodes and edges in a graph. If a small number of nodes/edges have a significantly higher centrality value than the rest, a network is susceptible to congestion. Also, since such nodes/edges are presumably more important than the rest of the network, they are susceptible to targeted attacks.

We measure the skew in node (or edge) betweenness as the difference between the maximum node (or edge) betweenness in the network and the average betweenness. A (normalized) value of 1.0 for the skew indicate that there is only one node (or edge) in the network through which all the traffic flows; whereas a value of 0.0 indicate a uniform load distribution. Table I shows typical values of betweenness skews for CSLs, which are close to 0. Networks with such homogeneity are not easily congested under heavy traffic. Also, such networks are resilient to both targeted attacks and random failures.

Entropy of Degree Distribution: Entropy of degree distribution is indicative of the heterogeneity of a network. It is measured as, $H = -\sum_p P(p) log P(p)$, where $P(p)$ is the fraction of nodes with a degree p. Typically, CSLs are regular or nearly regular graphs. Thus, have an entropy of nearly 0.

Symmetric Distances: CSLs have low diameters and APLs. The distances between node pairs are also homogeneous, leading to a very low skew in closeness centrality (which measures the per node APL). Node eccentricities (which measures the greatest separation a node suffers from another node in the network) are also symmetric. Radius (which is the smallest eccentricity) is typically very close to the diameter, thus there are very few "peripheral" nodes which incur more communication cost than the rest of the nodes.

Multiple Independent Paths: CSLs are nearly *optimally connected* [11]. Given a CSL with minimum degree, p_{min}, its edge connectivity, λ is at least, $p_{min} - 1$, when $k >= \frac{np}{2}$ (and $k >= np$, for directed graphs).

Recent studies have reported optimal networks with features such as: homogeneous degree [10], [12], betweenness [13], [14] and optimal connectivity [11]. We observe that CSLs are characterized by all these optimal features.

V. Future Directions

We study the design of optimal network topologies under different optimality requirements. Two prominent classes of topologies emerge as optimal: (1) star-like networks and (2) circular skip lists. We analyze CSLs to show that they have structural properties that are optimal under a variety of requirements. Thus, CSLs potentially form the underpinnings of optimal network design.

This work is part of a larger vision, which is to develop a deeper theoretical understanding of network design in terms of general design principles. In that light, this work has lead to at least three future directions: (1) identifying structural signatures from the optimal networks that emerge, which can be used by autonomous agents to *snap* or quickly (re)construct networks in the face of perturbations (2) developing efficient deterministic techniques to construct topologies that are equivalent to the optimal circular skip lists (3) developing theoretical results that can provide guarantees about CSLs in the phase of perturbations [23].

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