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A Practical Approach for Design of PD and PI Like Interval Type-2 TSK Fuzzy Controllers

Mohammad Biglarbegian
Department of Mechanical Engineering
University of Waterloo
Waterloo, ON N2L 3G1
mbiglarb@uwaterloo.ca

William W. Melek
Department of Mechanical Engineering
University of Waterloo
Waterloo, ON N2L 3G1
wmelek@uwaterloo.ca

Jerry M. Mendel
Department of Electrical Engineering
University of Southern California
California 90089-2564
jmendel@sipi.usc.edu

***Abstract*—Interval type-2 fuzzy logic control systems (IT2 FLCSs) have the potential of handling uncertainties better than type-1 FLCSs. However, lack of systematic design methodology of IT2 FLCSs limits their utility. This paper presents systematic methods to design interval IT2 Takagi-Sugeno-Kang (TSK) FLCSs that are PD-type and PI-type fuzzy controllers to satisfy certain desired transient response. We adopt the MacVicar-Whelan rule-base system and present general schemes for the design of IT2 TSK FLCSs, that include the design of the TSK consequent parameters. To validate the performance of the proposed controllers, some nonlinear plants have been considered. Results show that the IT2 TSK FLCSs satisfy the desired performance measures in terms of a set point tracking. Moreover, they reveal remarkable improvements in comparison to their type-1 counterparts for the plants considered in this paper.**

***Index Terms*—Interval type-2 fuzzy logic, Control, TSK.**

I. Introduction

Performance comparisons between Proportional-Integral-Derivative (PID) controllers and type-1 fuzzy logic controllers (T1 FLC) as well as PID and hybrid PID-FLCs have been reported in the literature ([1], [2], [3]). Coleman and Godbole [1] designed three controllers, namely, robust FLC, PID, and sliding mode to control the speed of a third order linear model of a motor. They concluded that the FLC can readily synthesize a robust controller to satisfy the desired performance. Zhao et. al. [2] presented a fuzzy gain scheduling scheme of PID controllers [2], and through simulations demonstrated that a better control performance can be achieved in comparison to PID controllers. Ying [3] compared the performance of T1 FLC with linear PID controllers through several simulations on both linear and nonlinear plants. Results show that for a nonlinear plant, T1 FLCs, when properly tuned, can achieve enhanced performance. The author concludes that when time-delay or nonlinear plants are considered, using T1 FLC is beneficial only when the parameters of T1 FLC are properly designed and fine-tuned. Therefore, it has been shown that either T1 FLCs or hybrid PID-FLCs can achieve significant performance over PID or conventional controllers.

Interval type-2 FLCS (IT2 FLCS) have been shown to outperform their T1 counterparts, i.e., T1 FLCs in several control case studies ([4], [5]). Furthermore, they have successfully been implemented in real-time control applications [4].

Proportional-Integral (PI) and Proportional-Derivative (PD) controllers are well-known for improving transient response and tracking [6]. Hence, these control structures have been often used in PD-type and PI-type fuzzy controller designs [3]. Even though a systematic methodology still lacks for designing T1 FLCSs, several works have reported successful facilitation of the design process [7] and [8]. The methods proposed in the literature for designing T1 FLCSs almost always use the Mamdani inference engine. Lee [9] proposed two types of fuzzy PI controller to reduce rise time as well as overshoot. The control structure of the proposed controllers uses two augmented conventional fuzzy PI-type controllers that employ resetting factors. The first controller determines the resetting rate according to error and error rate, and the second one uses error and control input. Both controllers reduce rise time and overshoot.

Tan and Lai in [10] developed an IT2 fuzzy proportional controller with a variable gain. The gain can be changed depending on the level of uncertainty and performance requirements. Simulation results on an uncertain second-order system showed that the developed controller is more robust and outperforms its T1 counterpart. However, due to its complex structure, whether it can be successfully implemented for real-time control implementation is an open question.

PD and PI are common linear controllers for various industrial applications. However, for many nonlinear systems with nonlinearities and uncertainties, the performance of these controllers may not be satisfactory. Therefore, in this work, we introduce PD and PI like IT2 FLCs that capture the advantage of a linear controller in terms of simplicity and also can handle nonlinearity because of their inference mechanism. To the best of our knowledge, no systematic methodology exists for the design of IT2 TSK FLCs. In this paper, we present two controller structures, PD-type and PI-type IT2 FLCs. Takagi-Sugeno-Kang (TSK) model structure is used for control design. Furthermore, IT2 membership functions are suggested for the design, hence eliminating the extra effort usually needed to define these functions. Moreover, by using the closed-form inference engine proposed in [11], we design a computationally efficient IT2 TSK FLCS that can be easily implemented in real-time control applications.

The organization of this paper is as follows: Section II

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contains background material on IT2 TSK FLSs. Section III presents the control design methodology. Section IV presents the simulation results for the designed IT2 TSK FLCSs and compares their performance with T1 TSK FLCs. Finally, Section V provides the concluding remarks of the paper.

II. Background on IT2 TSK FLSs

A. Interval Type-2 $A2-C0$ TSK Model

IT2 TSK FLSs are TSK models where interval fuzzy sets are used to describe the level of uncertainty in the antecedents and/or consequents. IT2 TSK FLSs have been classified into three different models [12]: first, both antecedent and consequent membership functions are IT2 fuzzy sets ($A2-C2$), second, antecedent and consequent membership functions are IT2 and T1 fuzzy sets, respectively ($A2-C1$), and third, antecedent membership functions are type-2 and consequent are crisp numbers ($A2-C0$).

We only deal with the third case in this paper, so the remainder of this section focuses on review of the $A2-C0$ model since this structure is required as preliminary background for the design of IT2 TSK FLCs.

The general structure of an interval $A2-C0$ TSK model is as follows:

$$\begin{aligned}\text{Rule } i\text{th} \quad &: \quad \text{If } x_1 \text{ is } \widetilde{F}_1^i \text{ and } x_2 \text{ is } \widetilde{F}_2^i \text{ and } \cdots \ x_n \text{ is } \widetilde{F}_n^i \\ \text{Then } y_i \quad &= \quad a_0^i + a_1^i x_1 + a_2^i x_2 + \cdots + a_n^i x_n. \end{aligned} \tag{1}$$

where $i = 1, ..., M$, $\widetilde{F}_j^i$ represents interval type-2 fuzzy set of input state j in rule i, x_1,..., x_n are inputs, x_n are inputs, $a_0^i, \ldots, a_n^i$ are the coefficients of output polynomial for rule i (and hence are crisp numbers), y_i is the output of the ith rule, and M is the number of rules. The above rules allow us to model the uncertainties encountered in the antecedents. If we define the state vector as $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^T$, the output of this model is given as

$$Y_{TSK/A2-C0} = \int_{f^1 \in [\underline{f}^1, \overline{f}^1]} \cdots \int_{f^M \in [\underline{f}^M, \overline{f}^M]} 1 / \frac{\sum_{i=1}^M f^i(\boldsymbol{x}) y_i}{\sum_{i=1}^M f^i(\boldsymbol{x})} \tag{2}$$

where $\underline{f}^i(\boldsymbol{x})$ and $\overline{f}^i(\boldsymbol{x})$ are given by

$$\underline{f}^i(\boldsymbol{x}) = \underline{\mu}_{\widetilde{F}_1^i}(x_1) * \cdots * \underline{\mu}_{\widetilde{F}_n^i}(x_n) \tag{3}$$

$$\overline{f}^i(\boldsymbol{x}) = \overline{\mu}_{\widetilde{F}_1^i}(x_1) * \cdots * \overline{\mu}_{\widetilde{F}_n^i}(x_n) \tag{4}$$

The procedure to compute $Y_{TSK/A2-C0}$ or the bounds y_l and y_r can be performed using iterative Karnik-Mendel (K-M) algorithms [12]. The output of the IT2 TSK model is given by equation (2). In [11], we introduced a new mechanism for IT2 TSK $A2-C0$ systems that has a closed-form. The new inference engine replaces the type-reduction and is given as

$$\begin{aligned} Y_{TSK} = & \, m \frac{\sum_{i=1}^M \underline{f}^i(\boldsymbol{x}) y_i}{\sum_{i=1}^M \underline{f}^i(\mathbf{x})} \\ & + n \frac{\sum_{i=1}^M \overline{f}^i(\boldsymbol{x}) y_i}{\sum_{i=1}^M \overline{f}^i(\boldsymbol{x})} \quad \big(\text{if } M = 1 : m + n = 1\big) \end{aligned} \tag{5}$$

To tune the model variables, the parameters, m and n are introduced in the formulas. Observe that if $\underline{f}^i(\boldsymbol{x}) = \overline{f}^i(\boldsymbol{x})$, (5) reduces to the same T1 TSK. In this paper, we use (5) to design the proposed IT2 TSK FLCSs.

III. Design of IT2 TSK FLCSs

This section presents the design of PD and PI IT2 TSK FLCSs. To identify an IT2 TSK FLCS, rules, membership functions, and inference mechanism must be designed. The next two subsections present the rule structure and membership functions design. The controller structure and design methodology are presented in Subsection III-C

A. Rule bases

Rules are one of the main components of any fuzzy logic system (FLS). Obtaining suitable rules for FLCSs that can capture the behavior of the plant or controller is very crucial and not an easy exercise. Due to the several parameters needed to characterize a FLCS, there is no general and yet systematic method to effectively define the rules. Designers usually exploit expert knowledge or sometimes use their intuition to define the rules for their model. However, it was shown that MacVicar-Whelan rule base can be effectively adopted for design of FLCSs in tracking problems [13].

The rule structure for an IT2 TSK PD-FLC is as follows:

$$\text{If } e \text{ is } \widetilde{F}_1^i \text{ and } \Delta e \text{ is } \widetilde{F}_2^i, \text{ Then } u_i = a_1^i e + a_2^i \Delta e \tag{6}$$

Similarly, IT2 TSK PI-FLC has the rule structure of the form

$$\text{If } e \text{ is } \widetilde{F}_1^i \text{ and } \Delta e \text{ is } \widetilde{F}_2^i, \text{ Then } \Delta u_i = a_1^i e + a_2^i \Delta e \tag{7}$$

where $i = 1, ..., M$, $\widetilde{F}_j^i$ represents the IT2 fuzzy set of input state j in rule i, a_1^i and a_2^i are the coefficients of the output function for rule i (and hence are crisp numbers, i.e., type-0 fuzzy sets), u_i and Δu_i are the outputs of the ith rule for the two controllers, respectively, and M is the number of rules. Error, e, and its rate of change, Δe, are defined as

$$e \equiv r - y \tag{8}$$

$$\Delta e \equiv e(kT) - e((k-1)T) \tag{9}$$

where r is the set point, y is the output of the closed-loop system, T is the sampling period of the discrete system, and k is an integer. The above rules allow us to model the uncertainties encountered in the antecedents. Lower and upper firing strengths of the ith rule, $\underline{f}^i$ and $\overline{f}^i$, are given by

$$\underline{f}^i(\boldsymbol{e}) = \underline{\mu}_{\widetilde{F}_1^i}(e_1) * \underline{\mu}_{\widetilde{F}_n^i}(e_n) \tag{10}$$

$$\overline{f}^i(\boldsymbol{e}) = \overline{\mu}_{\widetilde{F}_1^i}(e_1) * \overline{\mu}_{\widetilde{F}_n^i}(e_n) \tag{11}$$

where $\underline{\mu}_{\widetilde{F}_j^i}$ and $\overline{\mu}_{\widetilde{F}_j^i}$ represent the jth ($j = 1, 2$) lower and upper membership functions of rule i, respectively, and "$*$ " is a t-norm operator. Error vector, $\boldsymbol{e}$, is defined as

$$\boldsymbol{e} = [e, \Delta e]^T \tag{12}$$

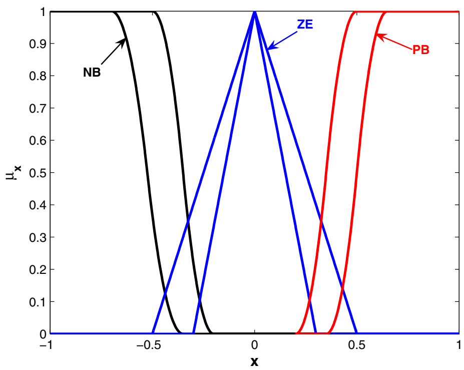

Fig. 1. Membership functions for e and Δe of the proposed IT2 TSK FLCS for a system with 9 rules.

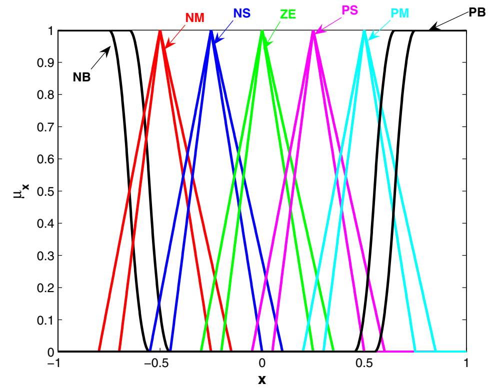

Fig. 2. Membership functions for e and Δe of the proposed IT2 TSK FLCS for a system with 49 rules.

A general MacVicar-Whelan rule-base [7] uses error, e, and change in error, Δe, to determine u or Δu, for IT2 TSK-PD or IT2 TSK PI-FLCs, respectively. We introduce two rule-bases for our control design. The first one, which has 9 rules, is defined by Table I where three linguistic variables, "NB," "ZE," and "PB," represent "negative big," "zero," and "positive big," respectively. To examine the effect of rule numbers on the performance of the closed-loop system, we introduce the second rule-base. As shown in Table II, this rule-base has 49 rules and the linguistic terms "NM," "NS," "PS," and "PB" represent "negative medium," "negative small," "positive small," and "positive big," respectively.

TABLE I
FUZZY RULE-BASE FOR A SYSTEM WITH 9 RULES

$\Delta e/e$	NB	ZE	PB
NB	NB	NB	ZE
ZE	NB	ZE	PB
PB	ZE	PB	PB

TABLE II
FUZZY RULE-BASE FOR A SYSTEM WITH 49 RULES

$\Delta e/e$	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NM	NS	NS	ZE
NM	NB	NM	NM	NM	NS	ZE	PS
NS	NB	NM	NS	NS	ZE	PS	PM
ZE	NB	NM	NS	ZE	PS	PM	PB
PS	NM	NS	ZE	PS	PS	PM	PB
PM	NS	ZE	PS	PM	PM	PM	PB
PB	ZE	PS	PS	PM	PB	PB	PB

B. Membership functions

Figures 1 and 2 show the proposed IT2 membership functions for systems with 9 and 49 rules, respectively.

As can be seen, the membership functions are defined within the interval $[-1, 1]$. Hence, to make the best use of the rule-base system the actual values of e and Δe must be mapped onto $[-1, 1]$ using scaling factors. The scaling factors or gains are introduced in the next section.

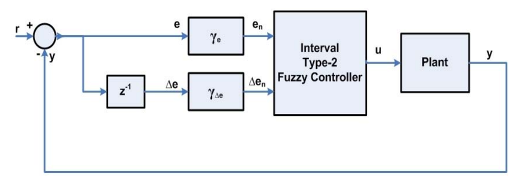

Fig. 3. Structure of the IT2 PD-FLCS.

C. Control structure

Figures 3 and 4 show the structures of IT2 TSK PD-FLCS and IT2 TSK PI-FLCS, respectively. Input to both closed-loop systems is r and the output of the plant is y. Error, e, is defined as the difference between r and y. The inputs to the IT2 TSK PD-FLCS or IT2 TSK PI-FLCS are e_n and Δe_n, and the output is u or Δu (u for PD-FLCS and Δu for PI-FLCS). Scaling factors or normalizing parameters γ_e and $\gamma_{\Delta e}$ map e and Δe onto $[-1, 1]$, respectively, using the following linear relationships:

$$e_n = \gamma_e e \quad (13)$$

$$\Delta e_n = \gamma_{\Delta e} \Delta e \quad (14)$$

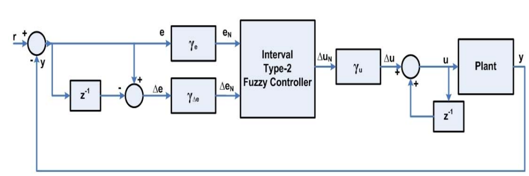

Fig. 4. Structure of the IT2 PI-FLCS.

D. Inference mechanism and control structure

Using the inference engine introduced in (5), the controller outputs, u_n and Δu_n, are, respectively, given as

$$u = m\frac{\sum_{i=1}^{M}\underline{f}^{i}(\boldsymbol{e})\,u_i}{\sum_{i=1}^{M}\underline{f}^{i}(\boldsymbol{e})} + n\frac{\sum_{i=1}^{M}\overline{f}^{i}(\boldsymbol{e})\,u_i}{\sum_{i=1}^{M}\overline{f}^{i}(\boldsymbol{e})} \quad \big(\text{IT2 TSK PD-FLC}\big) \quad (15)$$

$$\Delta u_n = m\frac{\sum_{i=1}^{M}\underline{f}^{i}(\boldsymbol{e})\,\Delta u_i}{\sum_{i=1}^{M}\underline{f}^{i}(\boldsymbol{e})} + n\frac{\sum_{i=1}^{M}\overline{f}^{i}(\boldsymbol{e})\,\Delta u_i}{\sum_{i=1}^{M}\overline{f}^{i}(\boldsymbol{e})} \quad \big(\text{IT2 TSK PD-FLC}\big) \quad (16)$$

where m and n are the controller tuning parameters.

1) Determining the TSK consequent parameters: In this subsection, we present the TSK consequent parameters of IT2 TSK PD-FLC and IT2 TSK PI-FLCs. Since both controllers use the same rule-base in Table I, the following results can be applied to either of them. To find the consequent parameters, i.e., a_1^i's and a_2^i's in (6) and (7), we consider the conditions where e_n and Δe_n correspond to the linguistic terms defined in the rule-base given by Table I, i.e., 'NB,' 'ZE,' and 'PB'.

If e is NB ($e \longrightarrow -1$), then
Rule 1. If Δe is NB, $\Delta e \longrightarrow -1$: $u_1 = -a_1^1 - a_2^1 \longrightarrow -1$
Rule 2. If Δe is ZE, $\Delta e \longrightarrow 0$: $u_2 = -a_1^2 \longrightarrow -1$
Rule 3. If Δe is PB, $\Delta e \longrightarrow 1$: $u_3 = -a_1^3 + a_2^3 \longrightarrow 0$

If e_n is ZE ($e_n \longrightarrow 0$), then
Rule 4. If Δe_n is NB, $\Delta e_n \longrightarrow -1$: $u_4 = -a_2^4 \longrightarrow -1$
Rule 5. If Δe_n is ZE, $\Delta e_n \longrightarrow 0$: $u_5 \longrightarrow 0$
Rule 6. If Δe_n is PB, $\Delta e_n \longrightarrow 1$: $u_6 = a_2^6 \longrightarrow 1$

If e is PB ($e \longrightarrow 1$), then
Rule 7. If Δe is NB, $\Delta e \longrightarrow -1$: $u_7 = a_1^7 - a_2^7 \longrightarrow 0$
Rule 8. If Δe is ZE, $\Delta e \longrightarrow 0$: $u_8 = a_1^8 \longrightarrow 1$
Rule 9. If Δe is PB, $\Delta e \longrightarrow 1$: $u_9 = a_1^9 + a_2^9 \longrightarrow 1$

where '$a \longrightarrow b$' denotes 'a' approaches/equals 'b'. From the above 9 conditions we can solve for a_1^i and a_2^i, $i = 1, \cdots, 9$, to find the consequent parameters of the IT2 TSK PD-FLC or IT2 TSK PI-FLC models in (6) and (7). These TSK consequent parameters are summarized in Table III. It is worth noting that the above constraints do not *explicitly* determine all of the TSK parameters. For instance, a_1^1, a_2^1, a_1^4, a_1^6, and a_2^8 can not be determined. As a result, we restrict these parameters to be in $[0, 1]$, and allow the designer the freedom to choose their values within this range to satisfy the required design criteria. Furthermore, the computed TSK consequent parameters can be used for T1 TSK FLCs. For a system with 49 rules, a similar approach can be used. These parameters for this rule base structure are given in the Appendix.

TABLE III
TSK CONSEQUENT PARAMETERS FOR A SYSTEM WITH 9 RULES

a_1^1	a_1^2	a_1^3	a_1^4	a_1^5	a_1^6	a_1^7	a_1^8	a_1^9
0.1	1	0.5	0.5	0.3	0.2	0.9	1	0.8
a_2^1	a_2^2	a_2^3	a_2^4	a_2^5	a_2^6	a_2^7	a_2^8	a_2^9
0.9	0.01	0.5	1	0.9	1	0.9	0	0.2

E. Design guidelines

The following steps are recommended for designing the controller and tuning the associated parameters.

1) To tune the error gain, γ_e must be selected such that e_n falls into $[-1, 1]$ to make the best use of the rule-bases. To tune the change of error gain, $\gamma_{\Delta e}$ should be chosen so that the desired transient response is achieved. This process relies on trial and error. Depending on the plant nonlinearities or/and uncertainties, γ_e or/and $\gamma_{\Delta e}$ should be changed (if needed) concurrently so that the overall required performance is obtained.
2) To tune the parameters of the controller, m and n, it is recommended to start with small gains [high gains result in overshoot and undesired response], i.e., $0 < m, n <= 1$, and increment m and/or n, if necessary. If the design requirements or/and stability are not satisfied, m and n should be changed to the values greater than 1 or less than 0. The advantage of this approach is that it does not require changing/tuning the TSK parameters.

Observe that, unless for a particular design problem requiring specific membership functions or footprint of uncertainty, there is no need to redefine the rules or/and the proposed membership functions. Furthermore, the TSK consequent parameters given in Subsection *C* are not unique, and there are no constraints on some of these parameters. Hence, the option also exists to select those free parameters to fit the design requirements.

IV. RESULTS

In this section, the simulation results of the designed controllers are presented. These results are reported in two parts. In the first part, the performance of IT2 TSK PD-FLC and IT2 TSK PI-FLC are presented. In addition, we compare the performance of IT2 TSK FLCSs with their T1 counterpart, i.e., T1 TSK FLCs. In the second part, the effect of the increasing the rule numbers on the performance of IT2 TSK FLCSs is studied. To make an unbiased comparison, we keep the TSK consequent parameters of both T1 and T2 FLCs the same, and only redesign the T1 membership functions that are obtained by removing the uncertainties of the T2 fuzzy membership functions.

A. Performance comparison of T1 and IT2 TSK FLCSs

The transient response specifications that are used to compare the controllers performance are *Rise time,* t_r, *Settling time,* t_s, *Percent overshoot,* $OS\%$, *Steady-state output,* y_{ss}, and *Steady-state error,* e_{ss}.

To compare the performance of an IT2 TSK FLCS with its T1 counterpart, the following plants are considered:

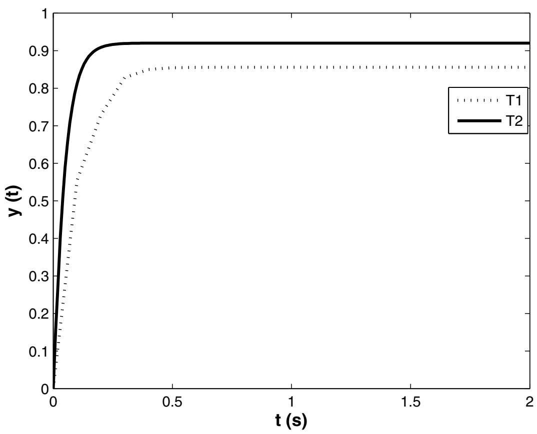

Fig. 5. Unit step responses of T1 and IT2 TSK PD-FLCs for plant 1. m and n are 1.5 and 0.5, respectively; γ_e and $\gamma_{\Delta e}$ are 10 and 1, respectively, for both controllers.

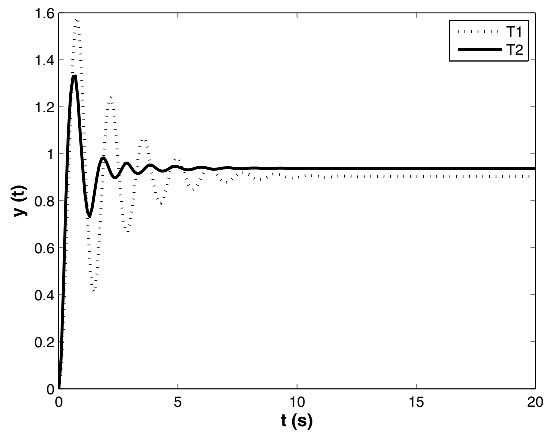

Fig. 6. Unit step responses of T1 and IT2 TSK PD-FLCs for plant 2. The values of m and n are -0.6 and 3.9, respectively. γ_e and $\gamma_{\Delta e}$ for T1 are 18 and 1 , respectively, and for T2 are 9 and 5.

1) $\dot{y}(t) = u(t) - y(t) - 0.8y^2(t)$
2) $\ddot{y}(t) = u(t) - \dot{y}(t) - y^{1.8}(t) - y(t)$

For each plant, the unit step is considered as an input to the closed-loop system.

1) PD-FLC:

a) First plant: As shown in Figure 5, both controllers have output-tracking errors. However, the IT2 controller outperforms its T1 counterpart by revealing a significantly faster response as well as the reduced steady-state error. The transient response characteristics of both controllers are shown in Table 5, which verifies the enhanced performance of the IT2 controller.

TABLE IV
PERFORMANCE COMPARISON OF T1 AND IT2 TSK PD-FLCS FOR THE FIRST PLANT

	$t_r(s)$	$t_s(s)$	$OS\%$	y_{ss}	e_{ss}
T_1	0.227	0.249	0	0.856	0.144
IT_2	0.110	0.139	0	0.920	0.080

b) Second plant: As shown in Figure 6, both controllers have output-tracking errors. However, the IT2 controller outperforms its T1 counterpart by revealing a significantly faster response as well as the reduced steady-state error. Note that the T1 controller has a poor tracking performance. More importantly, the T1 controller has undesired overshoot compared to the IT2 controller. The transient response characteristics of both controllers are shown in Table V, which verifies the enhanced performance achieved by the IT2 controller.

TABLE V
PERFORMANCE COMPARISON OF T1 AND IT2 TSK PD-FLCS FOR THE SECOND PLANT

	$t_r(s)$	$t_s(s)$	$OS\%$	y_{ss}	e_{ss}
T_1	0.264	7.790	57.981	0.903	0.097
IT_2	0.241	2.927	39.302	0.938	0.062

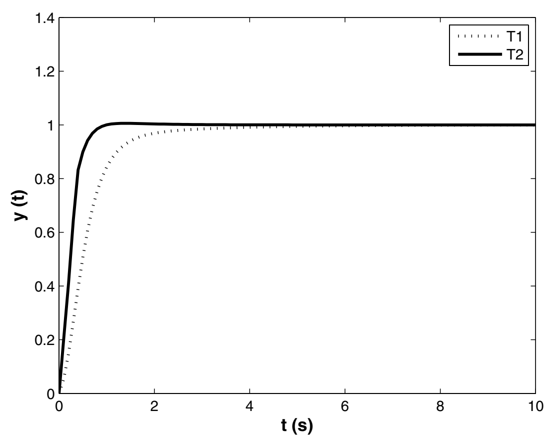

Fig. 7. Unit step responses of T1 and IT2 TSK PI-FLCs for plant 1. The values of m and n are 0.3 and 3.5, respectively. γ_e and $\gamma_{\Delta e}$ are 0.5 and 1, respectively, for both controllers.

2) PI-FLC: In this subsection, we present the results of the closed-loop system for the same plants with a PI-FLC. The number of rules used for simulations are also considered to be 9.

a) First plant: Figure 7 shows the outputs of both controllers for the second plant. While both controllers reveal perfect tracking performances, IT2 controller is a remarkably faster controller. Table VI quantitatively verifies the observed results.

TABLE VI
PERFORMANCE COMPARISON OF T1 AND IT2 TSK PI-FLCS FOR THE FIRST PLANT

	$t_r(s)$	$t_s(s)$	$OS\%$	y_{ss}	e_{ss}
T_1	1.070	1.616	0	1	0
IT_2	0.549	0.789	0	1	0

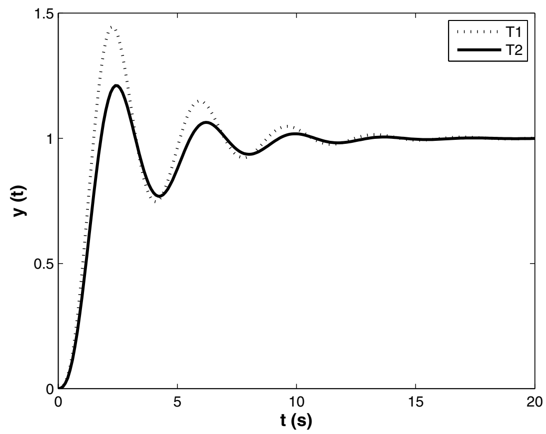

Fig. 8. Unit step responses of T1 and IT2 TSK PI-FLCs for plant 2. The values of m and n are 0.1 and 0.75, respectively. γ_e and $\gamma_{\Delta e}$ are 0.38 and 0.5, respectively, for both controllers.

b) Second Plant: Results are depicted in Figure 8. Both controllers track the unit-step input perfectly. Although the IT2 controller outputs a slightly slower response compared to the T1 controller, it considerably reduces the overshoot. Results in Table VII also show a significant reduction in overshoot using the IT2 controller.

TABLE VII
PERFORMANCE COMPARISON OF T1 AND IT2 TSK PI-FLCS FOR THE SECOND PLANT

	$t_r(s)$	$t_s(s)$	$OS\%$	y_{ss}	e_{ss}
T_1	0.877	11.810	44.549	1.000	0.000
IT_2	1.091	8.950	21.117	1.000	0.000

B. Impact of number of rules on performance of the closed-loop system

In this section, we investigate the performance of the system when the number of rules is increased from 9 to 49. The second plant is chosen for comparison. Figure 9 compares the performances of two IT2 TSK PD-FLCs with 9 and 49 rules. The controller with more rules reveals less overshoot although it has a slightly slower response. Table VIII shows the comparison results of the transient specifications of both controllers.

TABLE VIII
PERFORMANCE COMPARISON OF T1 AND IT2 TSK PD-FLCS FOR THE SECOND PLANT

	$t_r(s)$	$t_s(s)$	$OS\%$	y_{ss}	e_{ss}
IT_2 with 9 rules	0.241	2.927	39.302	0.938	0.062
IT_2 with 49 rules	0.291	3.626	21.443	0.926	0.074

Next, we compare the performance of two IT2 TSK PI-FLCs. Both controllers output perfect tracking performances. Results are depicted in Figure 10. The controller with larger number of rules reduces overshoot. However, it is a slower controller compared to the controller with 9 rules. These results have been tabulated in Table IX.

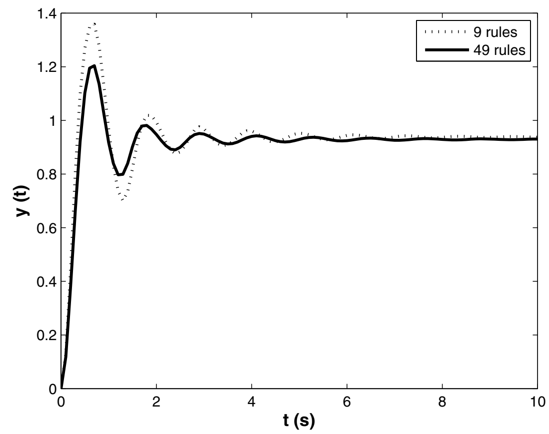

Fig. 9. Performances of the IT2 TSK PD-FLC with different number of rules.

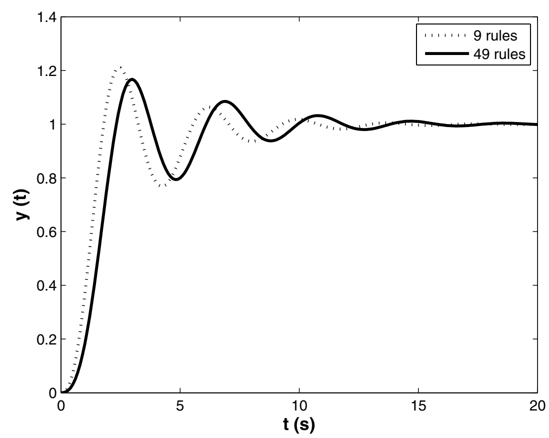

Fig. 10. Performances of the IT2 TSK PD-FLC with different number of rules.

TABLE IX
PERFORMANCE COMPARISON OF T1 AND IT2 TSK PI-FLCS FOR THE SECOND PLANT

	$t_r(s)$	$t_s(s)$	$OS\%$	y_{ss}	e_{ss}
IT_2 with 9 rules	1.091	8.950	21.117	1.000	0.000
IT_2 with 49 rules	1.360	12.801	16.823	1.000	0.00

It is concluded that for the plant studied in the case of IT2 TSK PI-FLC increasing number of rules improves some transient response characteristics, but not all of them.

V. CONCLUSION

In this study, we presented a systematic method to design IT2 TSK FLCSs, which are PD-like or PI-like fuzzy controllers used to achieve desired transient or/and tracking responses. Using the Mac-Vicar Whelan rule-base, we proposed membership functions to minimize the extra effort usually required in their design. In addition, a simple method was presented to derive the TSK consequent parameters to facilitate the use of these systems in control applications. Simulation results showed that for a standard step input, the IT2 TSK FLCs satisfy the required design performance. It was shown that when a well-designed IT2 controller is properly tuned, it can outperform its T1 counterpart. Hence, this approach provides an attractive alternative in PD/PID-FLCSs design. In conclusion, this paper presents a practical and applicable method to effectively design IT2 TSK FLCSs that can be easily used in real-time control applications.

REFERENCES

[1] C. P. Coleman and D. Godbole, "A comparison of robustness: Fuzzy logic, PID, and sliding mode control, " *in Proc. of the third IEEE Conference on World Congress on Computational Intelligence (WCCI)*, Orlando, FL, June 1994, pp. 1655-1659.

[2] Z. Y. Zhao, M. Tomizuka, and S. Isaka, "Fuzzy gain scheduling of PID controllers, " *IEEE Trans. Sys., Man, Cyb.*, vol. 23, no. 5, pp. 1392–1398, 1993.

[3] H. Ying, *Fuzzy Control and Modeling: Analytical Foundations and Applications*, 1st ed. IEEE press, 2000.

[4] J. M. Mendel, "Advances in type-2 fuzzy sets and systems," *Information Sciences*, vol. 177, no. 1, pp. 84-110, 2007.

[5] R. Sepulveda, O. Castillo, P. Melin, A. Rodriguez-Diaz, and O. Montiel, "Experimental study of intelligent controllers under uncertainty using type-1 and type-2 fuzzy logic," *Inf. Sci.*, vol. 177, no. 10, pp. 2023–2048, 2007.

[6] N. S. Nise, Control Systems Engineering, 5th ed. Wiley and Sons, 2007.

[7] S. Chopra, R. Mitra, and V. Kumar, "Fuzzy controller: choosing an appropriate and smallest rule set," *Int. Journal of Computational Cognition*, vol. 3, no. 4, pp. 73–79, 2005.

[8] S. Chopra, R. Mitra, and V. Kumar, "Reduction of fuzzy rules and membership functions and its application to fuzzy PI and PD type controllers," *Int. Journal of Control*, Automation, and Systems, vol. 4, no. 4, pp. 438–447, 2006.

[9] J. Lee, "On methods for improving performance of PI-type fuzzy logic controllers," *IEEE Trans. Fuzzy Set and Systems*, vol. 1, no. 4, pp. 298–301, 1993.

[10] W. Tan and J. Lai, "Development of a type-2 fuzzy proportional controller," in *Proc. Fuzz IEEE Conf.*, Budapest, Hungary, July 2004, pp. 1305–1310.

[11] M. B. Begian, W. W. Melek, and J. M. Mendel, "Stability analysis of type-2 fuzzy systems," in *Proc. IEEE World Congress on Computational Intelligence (WCCI'08)*, Hong Kong, June 2008, pp. 1305–1310.

[12] J. M. Mendel, Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions, Prentice-Hall, Upper Saddle River, *NJ*, 2001.

[13] R. R. Yager and D. P. Filev, Essentials of Fuzzy Modeling and Control, 1st ed. New York, Wiley, 1994.

APPENDIX. TSK CONSEQUENT PARAMETERS

The consequent parameters of the proposed IT2 TSK PD-FLCS or IT2 TSK PI-FLCS for a system with 49 rules are shown as follows (the corresponding value of a_j^i can be found below it):

$$\begin{bmatrix} a_1^1 & a_1^2 & a_1^3 & a_1^4 & a_1^5 & a_1^6 & a_1^7 & a_1^8 & a_1^9 & a_1^{10} \\ 0.5 & 0.5 & 0.75 & 1 & 1 & 0.5 & 0.5 & 1 & 1 & 0 \\ a_1^{11} & a_1^{12} & a_1^{13} & a_1^{14} & a_1^{15} & a_1^{16} & a_1^{17} & a_1^{18} & a_1^{19} & a_1^{20} \\ 1 & 1 & 0.5 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ a_1^{21} & a_1^{22} & a_1^{23} & a_1^{24} & a_1^{25} & a_1^{26} & a_1^{27} & a_1^{28} & a_1^{29} & a_1^{30} \\ 1 & 0.5 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ a_1^{31} & a_1^{32} & a_1^{33} & a_1^{34} & a_1^{35} & a_1^{36} & a_1^{37} & a_1^{38} & a_1^{39} & a_1^{40} \\ 1 & 1 & 0 & 1 & 1 & 1 & 0.5 & 1 & 1 & 0 \\ a_1^{41} & a_1^{42} & a_1^{43} & a_1^{44} & a_1^{45} & a_1^{46} & a_1^{47} & a_1^{48} & a_1^{49} & \\ 1 & 1 & 0.5 & 0.5 & 1 & 1 & 0.75 & 0.75 & 0.5 & \end{bmatrix}$$

$$\begin{bmatrix} a_2^1 & a_2^2 & a_2^3 & a_2^4 & a_2^5 & a_2^6 & a_2^7 & a_2^8 & a_2^9 & a_2^{10} \\ 0.5 & 1 & 1 & 0 & 0 & 1 & 0.5 & 0.5 & 0 & 1 \\ a_2^{11} & a_2^{12} & a_2^{13} & a_2^{14} & a_2^{15} & a_2^{16} & a_2^{17} & a_2^{18} & a_2^{19} & a_2^{20} \\ 1 & 1 & 1 & 0.5 & 0.75 & 0.75 & 0 & 1 & 1 & 1 \\ a_2^{21} & a_2^{22} & a_2^{23} & a_2^{24} & a_2^{25} & a_2^{26} & a_2^{27} & a_2^{28} & a_2^{29} & a_2^{30} \\ 0 & 0.5 & 1 & 1 & 1 & 1 & 1 & 0.5 & 0.5 & 1 \\ a_2^{31} & a_2^{32} & a_2^{33} & a_2^{34} & a_2^{35} & a_2^{36} & a_2^{37} & a_2^{38} & a_2^{39} & a_2^{40} \\ 1 & 1 & 1 & 0 & 0.75 & 0.75 & 0.5 & 1 & 1 & 1 \\ a_2^{41} & a_2^{42} & a_2^{43} & a_2^{44} & a_2^{45} & a_2^{46} & a_2^{47} & a_2^{48} & a_2^{49} & \\ 0 & 0.5 & 0.5 & 1 & 0 & 0 & 1 & 1 & 0.5 & \end{bmatrix}$$