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A Spatio-Temporal Fuzzy Logic System for Process Control

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***Abstract* — A novel application of type-2 fuzzy system is presented by developing a spatio-temporal fuzzy logic controller (FLC) for the distributed parameter system (DPS). The novel difference to the type-2 fuzzy system is to apply the secondary MF for a different physical variable – space domain. Using a number of sensors located on the spatial domain, the 3D fuzzy membership function can be obtained that contains spatio-temporal information. The 3D inference will include spatial T-norm operation and the traditional rule inference. The type-reduction will become the spatial reduction before the traditional defuzzification. This spaitio-temporal FLC is successfully applied to a catalytic reaction rod to demonstrate its effectiveness and potential to a wide range of engineering applications.**

I. INTRODUCTION

Many of industrial processes are inherently distributed parameter systems (DPS) that their states, controls, outputs and process parameters may vary temporally and spatially [1], such as the thermal, fluid flow and chemical reactor process. The traditional approach, which popularly exists in many engineering applications, is to ignore the distributed nature of the system and derive the simplified model based on the assumption of 'lumped' parameters. Thus the classical control theory, which relies on the accurate mathematical model of the process, can be applied.

As an intelligent method, the fuzzy logic controller (FLC) is widely used in industrial processes due to its inherent robustness [2-4]. Though some traditional FLCs have been used for DPS control [5-8], however, these traditional fuzzy approaches are not inherently designed for the spatially distributed dynamic process because of their two-dimensional (2D) feature. The type-2 fuzzy system introduces a secondary membership function (MF) for dealing with more complex processes [9-11]. Unfortunately, the common practice is to apply both primary and secondary MF to describe the same variable, which has limited its potential applications.

Recently, a novel three-dimensional (3D) FLC, based on the concept of the type-2 fuzzy set has been developed for spatially distributed dynamic systems [12-13]. In difference to the type-2 fuzzy system, it applies the secondary MF to a different physical domain – space, while the primary MF still for the temporal variable. Thus, a spatio-temporal fuzzy system is developed for the real application in the process industry.

Since the spatial information is difficult to obtain for building the secondary MF, the actual implementation of this 3D MF relies on the sensors located on the space domain, which will provide the spatial information. This 3D MF is actually a virtual type that consists of a group of 2D MF. After the 3D fuzzification, the 3D MF will be formed with the help of sensors. The 3D inference will include the spatial T-norm operation and the rule inference. Finally, the spatial reduction needs to be carried out, which is equivalent to the type-reduction in type-2 fuzzy systems, before the traditional defuzzification.

II. THREE-DIMENSIONAL FUZZY LOGIC CONTROLLER

The nonlinear distributed parameter system discussed above with one actuator ($l=1$) is considered as shown in Fig. 1, where p point measurement sensors are located at $z_1, z_2, \cdots, z_p$ in the one-dimensional space domain and an actuator u acts on the distributed process.

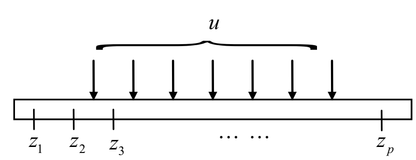

Fig. 1: Sketch of a DPS with a distributed actuator u and p point measurement sensors.

For controlling such a spatio-temporal process, a completely new fuzzy control method should be developed with the inherently designed features for the spatio-temporal process. One of the essential elements of this type of fuzzy system is the 3D fuzzy set used for modeling the 3D uncertainty.

A. Three-dimensional fuzzy set

A 3D fuzzy set is introduced in Fig.2 by developing a third dimension for spatial information from the traditional fuzzy set [24].

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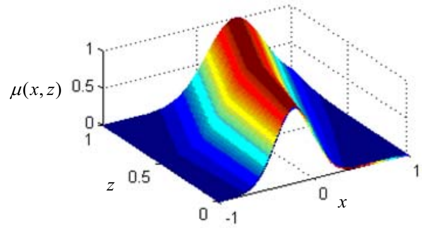


Fig. 2: A three-dimensional fuzzy set.

B. Configuration of 3D FLC

Since the 3D fuzzy set is inherently defined to express the spatial information, it would be useful to develop fuzzy control for the distributed parameter process that has spatio-temporal nature. Theoretically, the 3D fuzzy set or 3D global fuzzy MF is the assembly of 2D traditional fuzzy sets at every spatial location. Practically, this 3D fuzzy MF can be approximately constructed by 2D fuzzy MF at each sensing location. The more sensors are used in the space domain, the better 3D global fuzzy MF could be obtained.

As there is only one control output and no precise or quantitative information that can be used for design, the feasible action is to control the overall behavior of the spatial domain instead of accurately manipulating each spatial location. Thus, a centralized rule base might be more appropriate, which can also avoid the exponential explosion of rules when sensors increase.

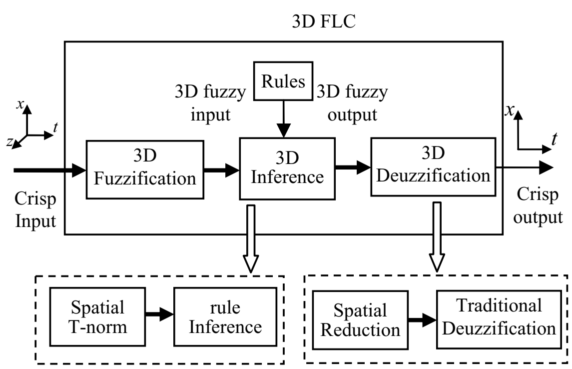


Fig. 3: Basic structure of 3D FLC

The new FLC will have the same basic structure as the traditional one, which is composed of fuzzification, inference and defuzzification as shown in Fig. 3. Due to its unique 3D nature, some detailed operations of this new FLC will be different from the traditional one as shown in Fig. 3 and Fig. 4 for spatial information processing and compressing.

1) Fuzzification

Similar to the traditional FLC, there are two different fuzzifications: singleton fuzzifier and non-singleton fuzzifier [14]. If finite sensors are used, this 3D fuzzification can be considered as the assembly of the traditional 2D fuzzification at each sensing location. Therefore, for p discrete measurement sensors located at $z_1, z_2, \cdots, z_p$ shown in Fig.1, $x(z_i) = [x_1(z_i), \cdots, x_J(z_i)]$ is defined as J crisp spatial input variables in space domain $Z = \{z_1, \cdots, z_p\}$, where $x_j(z_i) \in X_j \subset IR$ ($j = 1, \cdots, J$) denotes the crisp input at the measurement location $z = z_i$ for the spatial input variable $x_j(z)$, X_j denotes the domain of $x_j(z_i)$. The fuzzification for each crisp spatial input variable $x_j(z)$ is uniformly expressed as one 3D fuzzy input $\overline{A}_{Xj}$ in the discrete form as follows

$$\overline{A}_{X_1} = \sum_{z\in Z} \sum_{x_1(z)\in X_1} \mu_{X_1}(x_1(z), z) / (x_1(z), z)$$
$$\overline{A}_{X_2} = \sum_{z\in Z} \sum_{x_2(z)\in X_2} \mu_{X_2}(x_2(z), z) / (x_2(z), z)$$
$$\vdots$$
$$\overline{A}_{X_J} = \sum_{z\in Z} \sum_{x_J(z)\in X_J} \mu_{X_J}(x_J(z), z) / (x_J(z), z)$$

2) Rule Inference

a) Rule Base

Using the 3D fuzzy set, the l^{th} rule in the rule base can be expressed as follows

$$R^l: \text{ if } x_1(z) \text{ is } \overline{C}_1^l \text{ and } \ldots \text{ and } x_J(z) \text{ is } \overline{C}_J^l \text{ then } u(z) \text{ is } \overline{G}^l. \quad (1)$$

where $\overline{R}^l$ denotes the l^{th} rule, $l = 1, 2, \cdots, N$; $x_j(z)$ denotes spatial input variable, $j = 1, \cdots, J$; $\overline{A}_j^l$ denotes 3D fuzzy set; $u(z)$ denotes the control action at spatial location z, $u(z) \in U \subseteq IR$, and $\overline{G}^l$ denotes a output 3D fuzzy set; N is the number of fuzzy rules.

b) Inference

The inference engine of the 3D FLC is expected to transform a 3D fuzzy input into a traditional fuzzy output. Thus, the inference engine is the same as traditional inference operation. The only difference is that this 3D fuzzy inference is at each spatial location. The inference process is about the operation of 3D fuzzy set including union, intersection and complement operation. Considering the fuzzy rule expressed as in (1), the rule presents a fuzzy relation

$$R^l: \overline{C}_1^l \times \cdots \times \overline{C}_J^l \to \overline{G}^l, \quad l = 1, 2, \cdots, N.$$

Thus, a traditional fuzzy set is generated via combining the 3D fuzzy input and the fuzzy relation represented by rules.

Spatial T-norm Operation

This first operation in the inference is to transform the 3D fuzzy input $\overline{A}_X$ into a 3D set W^l, Through the sup-star composition operation on the input set and antecedent set, W^l is denoted by

$$W^l_{\overline{I}\circ\overline{C}^l\times\overline{D}^l} = \overline{I} \circ \overline{C}^l \times \overline{D}^l$$

with the grade of the MF derived as

$$\begin{aligned}\mu_{W^l} &= \mu_{\bar{I}\circ\bar{C}^l\times\bar{D}^l}(e,ce,z)\\ &= \sup_{e\in E,ce\in CE}[\mu_{\bar{I}}(e,ce,z)*\mu_{\bar{C}^l\times\bar{D}^l}(e,ce,z)]\\ &= \sup_{e\in E,ce\in CE}[\mu_E(e,z)*\mu_{CE}(ce,z)*\mu_{\bar{C}^l}(e,z)*\mu_{\bar{D}^l}(ce,z)]\\ &= \{\sup_{e\in E}[\mu_E(e,z)*\mu_{\bar{C}^l}(e,z)]\}*\{\sup_{ce\in CE}[\mu_{CE}(ce,z)*\mu_{\bar{D}^l}(ce,z)]\}\end{aligned}$$

Fig. 4 gives a demonstration of spatial information fusion in the case of two crisp inputs from the space domain Z, i.e. $x(z)=[x_1(z),x_2(z)]$.

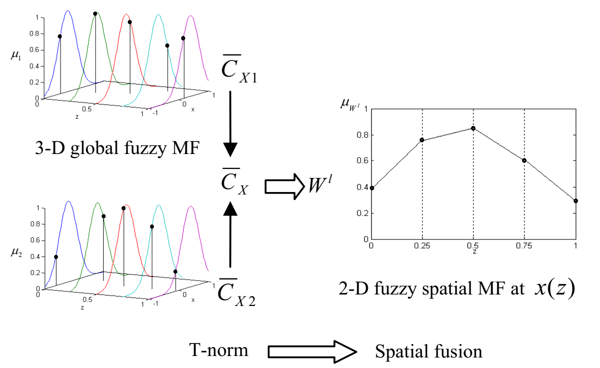


Fig. 4: Spatial T-norm operation

3D Inference Operation

The implication operation is executed through the following equation

$$\mu_{\bar{V}^l}(u,z)=\mu_{\bar{C}_1^l}(x,z)*\cdots\mu_{\bar{C}_J^l}(x,z)\rightarrow\mu_{\bar{G}^l}(u,z)$$

where Mamdani implication is used; $*$ stands for a t-norm; $\mu_{\bar{V}^l}(u,z)$ is the membership grade of the consequent set of the fired rule R^l; $\bar{V}^l$ is the output 3D fuzzy set of the fired rule R^l. Then, $\bar{V}^l$ is defined by the union of relations R^l

$$\mu_{\bar{V}}(u,z)=\bigcup_{l=1,\cdots,N}\mu_{\bar{V}^l}(u,z) \tag{2}$$

where N is the number of rules in the rule base.

When an input x' is applied, the composition of the fuzzy set W to which x' belongs (2) can be written as

$$\mu_{\bar{V}}(u,z)=\bigcup_{l=1,\cdots,N}\mu_W(x,z)*\mu_{\bar{G}}(u,z) \tag{3}$$

3) 3D Defuzzification

Spatial Reduction

The set $\bar{V}^l$ shows an approximate fuzzy spatial distribution for each output $u(z)$, in which physical information such as the sensor locations and their relative distance between each other are approximately embedded. The spatial reduction operation is to compress the spatial distribution information $\mu(u,z)$ into 2D information $\mu(u)$.

The 3D set $\bar{V}^l$ could be simply regarded as a 2D spatial MF on the plane (μ,z) for each output $u(z)$. Thus the centroid operation in (4) might be a good option to compress this 3D set $\bar{V}^l$ into a 2D set φ^l that can approximately describe the overall impact of the spatial distribution with respect to the output $u(z)$.

$$\mu_{\varphi^l}=\frac{\int_\zeta\mu_{\bar{V}^l}(u,z)dz}{\int_\zeta dz} \tag{4}$$

where ζ denotes a continuous plane curve and $\int_\zeta dz$ denotes the arc length of ζ.

Traditional Defuzzification Operation

After the inference, a traditional fuzzy set is produced. Then, the same defuzzification used in the traditional FLC will be taken to produce a crisp output. Since there is no standard defuzzifier in theory, the center-of-sets [15] is chosen as the defuzzifier in this paper due to its simple computation.

In center-of-sets defuzzification, the consequent set of each fired rule R^l ($l=1,2,\cdots,N'$) is replaced by a singleton situated at its centroid, whose amplitude equals the firing level, then the result of defuzzification is the centroid of these singletons. The output expression is given as follows

$$u=\sum_{l=1}^{N'}c^l\mu_{\varphi^l}\Big/\sum_{l=1}^{N'}\mu_{\varphi^l}$$

where $c^l\in U$ is the centroid of the consequent set of the fired rule R^l ($l=1,2,\cdots,N'$), which represents the consequent set G^l in (1), and N' is the number of fire rules, $N'\le N$.

C. Design of 3D FLC for spatio-temporal processes

If only error and error in change are used as the crisp spatial inputs for 3D FLC, then a 3D two-term FLC as shown in Fig. 5 will be designed for a distributed parameter system. $E(z)$ and $R(z)$ are the scaled spatial inputs for 3D two-term FLC, where $E(z)$ denotes the scaled spatial error, $R(z)$ denotes the scaled spatial error in change, $E(z)=K_e(z)e(z)\subset IR$, $R(z)=K_d(z)\dot{e}(z)\subset IR$, $K_e(z)$ and $K_d(z)$ denote the scaling factors of actual error $e(z)$ and error in change $\dot{e}(z)=de(z)/dt$, respectively. Then, a simple 2D rule base is generated with $J=2$ in (1). The 3D FLC properly utilizes the spatial information only with a simple rule base, taking full advantages of the traditional fuzzy control described in subsection B of section II.

Currently, there is no mature method that could guide design of 3D FLC for the DPS. Without the well-developed approach, all these have to be considered using the practical knowledge of the process when designing for the distributed parameter process [16]. In general, design procedure of 3D FLC can be summarized as follows.

1) Properly design the overall behavior measurement of the spatial domain according to control requirement.
2) Properly allocate measurement sensors in spatial domain according to the process nature and overall

behavior measurement.

3) Properly design 2D fuzzy MFs for each input from the spatial domain. These 2D fuzzy MFs will form a 3D global fuzzy MF on the entire spatial domain.
4) Design and tune scaling gains for all inputs and output for the overall behavior measurement, and the control performance.
5) Design control rule base according to the control requirement. This design is similar to those in the traditional FLC [17-18].

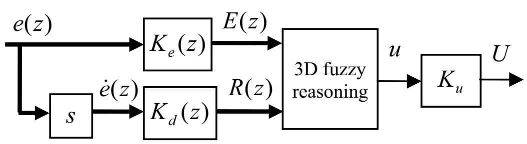


Fig. 5: Structure of 3D FLC system.

III. Application

A. Catalytic reaction rod

Consider the problem of a catalytic reaction in a reactor as shown in Fig. 6 [1], where the reactor is fed with pure species A and a zero-th order exothermic catalytic reaction of the form $A \rightarrow B$ takes place on a rod. The reaction is exothermic, so that a cooling medium that is in contact with the rod is used for cooling. Under the assumptions of constant density, constant heat capacity of the rod, constant conductivity of the rod, constant temperature at both ends of the rod, and excess of species A in the furnace, the mathematical model describing the spatio-temporal evolution of the dimensionless rod temperature is given as the following parabolic PDE:

$$\frac{\partial y(z,t)}{\partial t} = \frac{\partial^2 y(z,t)}{\partial z^2} + \beta_T \exp\left(-1 + y(z,t)\right) + \beta_U\left[b(z)U(t) - T(z,t)\right] - \beta_T \exp\left(-\tau\right) \quad (5)$$

subject to the boundary conditions and initial condition:

$$\begin{cases} y(z,t) = 0, & z = 0 \\ y(z,t) = 0, & z = 0 \\ y(z,t) = y_0(z), & z = 0 \end{cases} \quad (6)$$

where $\beta_T = 50$ denotes a dimensionless heat of reaction; $\tau = 4$ denotes a dimensionless activation energy; $\beta_U = 2$ denotes a dimensionless heat transfer coefficient; $x(z,t)$ with $z \in [0, \pi]$ denotes the dimensionless temperature of the rod, which is spatially dependent; $y_0(z)$ denotes the initial temperature, which is spatially dependent; $b(z)U(t)$ denotes a spatially distributed cooling source with $b(z)$ as the distribution and $U(t)$ as the manipulated input.

In this case, there is one point sources that are used to cool the reactor, i.e. $b(z) = \delta(z - 0.5\pi)$, where $\delta(\cdot)$ denotes Dirac delta function. It is verified that the spatially uniform operating steady-state $y(z,t) = 0$ is an unstable one [1] as shown in Fig. 7. For any initial condition with a deviation from $y(z,t) = 0$, the process will move to another spatially non-uniform steady state with the peak in the middle of the rod without any control. Therefore, the control objective is to stabilize the rod temperature along the rod length.

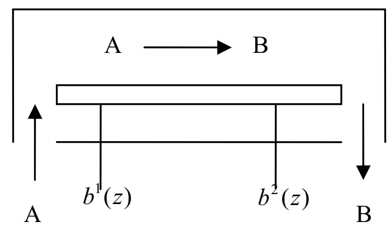


Fig. 6. Catalytic rod.

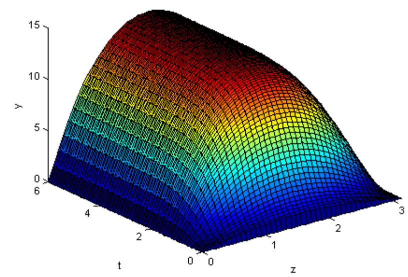


Fig. 7. Unstable rod temperature evolution from initial condition $y_0(z) = \sin(z)$.

B. Deign of 3D FLC

For simplicity and convenience, seven point sensors are uniformly located along the length of the reactor with $z_p = p\pi/8$ with $p = 1, \cdots, 7$ for collecting the spatial distribution of the temperature $y(z,t)$. Therefore, at each sampling time k, seven spatial temperatures are measured as $y(z_p, k)$ with $p = 1, \cdots, 7$. Two spatial inputs for 3D FLC are error and error in change, i.e. $e(z) = \{e(z_p)\}$ and $\Delta e(z) = \{\Delta e(z_p)\}$, where $e(z_p) = y_d(z_p, k) - y(z_p, k)$, $\Delta e(z_p) = e(z_p, k) - e(z_p, k-1)$, k and $k-1$ are the k th and $k-1$ th sample time. Initially, the scaling factor for each $e(z_p)$ is set to be 1, the scaling factor for each $\Delta e(z_p)$ is set to be 1.27, and the scaling factor for u is set to be 4.25. All the scaled input variables $e(z_p)$, $\Delta e(z_p)$ and output variable u are normalized into [-1, 1].

Due to finite sensors used, the MFs for error and change in error are discrete. As described previously, the 3D global fuzzy MF is the assembly of 2D fuzzy MFs at each sensing location. Each $e(z_p)$ and $\Delta e(z_p)$ as well as the control output u are classified into seven linguistic labels as positive large (PL), positive middle (PM), positive small (PS), zero (O), negative small (NS), negative middle (NM), and negative large (NL). For convenience, the MF is chosen as triangular shape. Then, the actual output $U(k)$ is expressed as

$$U(k) = K_u u(k)$$

where k and $k-1$ are the k th and $k-1$ th sample time.

A linear control rule base will be used with the following format

If $e(z_p)$ is $\overline{\text{PS}}$ and $\Delta e(z_p)$ is $\overline{\text{NS}}$ then $u(z_p)$ is $\overline{G}$

where $e(z_p)$ and $\Delta e(z_p)$ are scaled spatial input variables, representing error and error in change respectively; $\overline{\text{PS}}$ and $\overline{\text{NS}}$ denote 3D fuzzy sets, which are assembled by 2D fuzzy sets PS and NS at each sensing location; $u(z_p)$ is the control action; $\overline{G}$ is a 3D fuzzy set.

The rule weight is defaulted as unity. Singleton fuzzifier is used for fuzzification, the minimum is used for the t-norm in the intersection operation, the maximum is used for the *t*-conorm in the union operation, and the Center-of-sets is used for defuzzification.

C. Simulation comparison

The simulation results shown in Fig. 8 and Fig. 9 indicate that the control performance of the proposed 3D FLC is better than that of the 2D FLC.

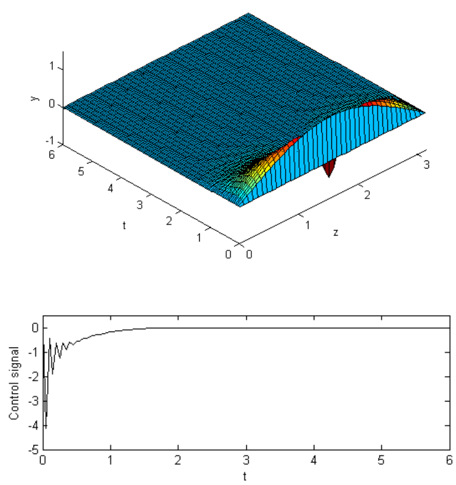
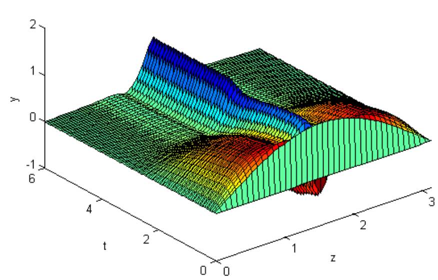


Fig. 8. Rod temperature evolution profile and its control signal from 3D FLC.



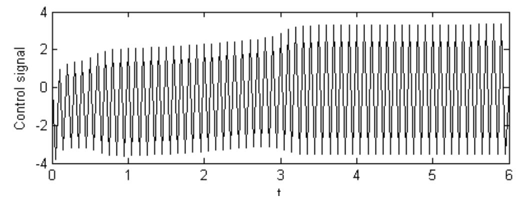


Fig. 9. The traditional FLC fails to stabilize the process

IV. Conclusions

In this paper, a novel application of type-2 fuzzy system is presented by developing a spatio-temporal FLC for controlling a class of spatio-temporal processes. Similar to the type-2 FLC, this 3D FLC also consists of fuzzification, rule inference, spatial reduction and defuzzification. In difference to the type-2 fuzzy system, the secondary MF is used to process spatial variable. With the help of sensors located on the spatial domain, the 3D MF can be obtained. With the novel structure, a simple 2D rule base can still be used for. Thus, rules will not increase as sensors increase for measuring spatial information, which minimizes the computation for real world applications. Finally, the 3D FLC is successfully applied to a catalytic reaction rod.

Acknowledgment

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