A New Approach Based on Particle Filter for Target Tracking with Glint Noise

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Abstract—In radar target tracking application, the observation noise is usually non-Gaussian, which is also referred to as glint noise. The performances of conventional trackers degrade severely in the presence of glint noise. An improved particle filter, Markov chain Monte Carlo iterated extended Kalman particle filter (MCMC-IEKPF), is applied to this problem. The tracking performance of the filter is evaluated and compared to the particle filter (PF) and the Markov chain Monte Carlo particle filter (MCMC-PF) via simulations. It is shown that the MCMC-IEKPF has better tracking performance.

Keywords—radar target tracking; iterated extended Kalman filter; particle filter; glint noise

I. INTRODUCTION

In target tracking, changes in the target aspect with respect to the radar can cause the apparent center of radar reflections to wander significantly. The random wandering of the apparent radar reflecting center gives rise to noisy or jittered angle tracking. This form of measurement noise is called angle fluctuations or target glint. It was found that glint has a long-tailed probability density function (PDF) [1]. The statistical characteristics of the glint noise and its mathematical models have been studied in [2] and [3]. The non-stationary and non-Gaussian measurement noise caused by the glint severely degrades the performance of linear Kalman filter [4]. Many researchers addressed the problem of filtering in non-Gaussian models. The score function based scheme is proposed in [5], but this approach involves a complicated score function calculation. In [6] and [7], the idea of interacting multiple model (IMM) was utilized in the problem of target tracking with glint noise. The IMM approach with extended Kalman filter (EKF) in the tracking problem provides poor performance due to the linearization involved in the EKF.

Recently, a new class of filters based on the sequential Monte Carlo (MC) approach has captured the attention of many researchers in various communities including signal processing, statistics, and econometrics [8]. The most popular realization of the MC approach is the particle filters (PF), which approximate the posterior distribution by a set of random samples with associated weights. In [9], the PF was applied to the problem of tracking in glint noise environment. This strategy uses the state transition density as importance distribution and can fail if the new measurements appear in the tail of the prior or if the likelihood is too peaked in comparison to the prior [10]. In [11], the EKF Gaussian approximation is used as the importance distribution for a PF. In [12], the unscented particle filter (UPF) is developed, which uses the unscented Kalman filter (UKF) [13] to generate the importance proposal distribution. In [14], a Markov chain Monte Carlo (MCMC) based particle filtering algorithm is described for tracking a variable number of interacting targets. It replaces the traditional importance sampling step in the PF with a novel MCMC sampling step to obtain a more efficient MCMC-based multitarget filter.

In this paper, we propose an improved PF for target tracking in the presence of glint noise. The algorithm modifies the iterated extended Kalman filter (IEKF) and then uses the modified IEKF to generate the proposal distribution. Additionally, the Markov chain Monte Carlo (MCMC) step is adopted to counteract the problem of particle impoverishment caused by resampling step and the diversity of the particles is maintained. Based on the glint noise statistical model, the proposed method is compared to the PF and the Markov chain Monte Carlo particle filter (MCMC-PF) via simulations. It is shown that the improved particle filter outperforms both the PF and the MCMC-PF.

The paper is organized as follows. The problem of target tracking with the glint noise is stated in Section II. The Markov chain Monte Carlo iterated extended Kalman particle filter (MCMC-IEKPF), used in this work is described in Section III. Tracking performances of the proposed algorithm are presented in Section IV. Our conclusions are summarized in Section V.

II. PROBLEM FORMULATION

Consider a discrete time linear dynamic system described by a difference equation with additive noise. The dynamic equation is

\[ x_k = Fx_{k-1} + w_{k-1}, \]

where state \( x_k \) consists of the position and the velocity of a moving target at time \( k \) (i.e., \( x_k = [x(k), y(k), \dot{x}(k), \dot{y}(k)]^T \)), and process noise \( w_{k-1} \) is assumed to be white and zero-mean with covariance \( Q \). \( x^T \) denotes the transpose of \( x \).

The measuring equipment considered here is radar which measures range and azimuth. The nonlinear discrete measurement equation is then described as follows.
\[ z_k = h(x_k) + v_k, \quad (2) \]
where \( z_k \) is the measurement vector of the target at time \( k \),
\[ h(x_k) = \begin{bmatrix} \sqrt{(x(k))^2 + (y(k))^2} \\ \arctan(y(k)/x(k)) \end{bmatrix}, \quad (3) \]
\( v_k \) is the glint noise assumed to be independent of \( w_{k+1} \).

As mentioned in [1], the glint noise has a non-Gaussian distribution. A mixture approach is widely used in modeling the glint noise. It was obtained that the glint is Gaussian-like around the mean and has a non-Gaussian, long-tailed nature in the tail region. The data at the tail region represent outliers, caused by the glint spikes. In the proposed tracking algorithm, the glint noise is modeled as a mixture of two zero-mean Gaussians, where the outliers are represented by a zero-mean Gaussian with large covariance matrix:
\[ p(v) = \sigma \Phi(v; 0, P) + (1 - \sigma) \Phi(v; 0, P), \quad (4) \]
where \( \sigma \) is the glint probability, \( \Phi(v; 0, P) \) denotes Gaussian distribution with zero-mean and covariance matrix \( P \).

III. MCMC-IEKF

A. Modifications on IEKF

The main difference between IEKF and EKF lies in the step of measurement update. For the IEKF, once the state prediction \( \hat{x}_{k|k-1} \) and corresponding covariance \( P_{k|k-1} \) are obtained, the following iterates will be recursively carried out
\[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - h(\hat{x}_{k|k-1})), \quad (5) \]
\[ P_{k|k} = (I - K_k H_k) P_{k|k-1}, \quad (6) \]
where
\[ H_k = \begin{bmatrix} \frac{\partial h(x)}{\partial x} \end{bmatrix}_{|x = \hat{x}_{k|k-1}}, \quad (8) \]
\[ K_k = P_{k|k-1} (H_k)^T \left( H_k P_{k|k-1} H_k^T + R_k \right)^{-1}, \quad (9) \]
i is the iterates number, \( i = 1, 2, \ldots, N \).

The termination condition may be varied in different cases, but a commonly used criterion to terminate the iterate is that the inequality \( \| \hat{x}_{k|k-1} - \hat{x}_{k|k} \| \leq V_{th} \) can be satisfied, where \( \| y \| = \sqrt{y^T y} \) is the usual norm on \( y \in \mathbb{R}^n \), \( V_{th} \) is the predetermined threshold.

It was pointed out in [15] that the sequence of iterates generated by the IEKF and that generated by the Gauss-Newton method were identical, thus global convergence of the iterates was guaranteed. Theoretically, the IEKF is surely superior to EKF and modified covariance EKF; however, this is not always true. One reason is that the conclusion in [15] was drawn under the assumption that the local linearization condition is unconditionally met, i.e., the state estimate is close enough to the true value. In many applications, however, this assumption does not always hold because the initial estimate errors may be very large. The other reason is that the Gauss-Newton method is not guaranteed to go up the likelihood surface [16], though it is guaranteed to be globally convergent. Additionally, the threshold \( V_{th} \) is crucial to successfully using the iterated algorithm, but a proper choice of \( V_{th} \) is not easy.

To this end, there is a need to modify the IEKF. In the first modification, according to improving the estimation in each iteration, we use the last estimated state in the state update equation. This means that we use following equation instead of (6):
\[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - h(\hat{x}_{k|k-1})). \quad (10) \]
This modification reduces the impact of the initial estimate errors and removes the term \( H_k (\hat{x}_{k|k-1} - \hat{x}_{k|k}) \) from residual computations in (6). Also, according to Kalman filtering techniques, we get the covariance update formula in terms of the Kalman gain, i.e., substitution of \( P_{k|k} \) for \( P_{k|k-1} \) in (7) yields
\[ P_{k|k} = \left( I - K_k H_k \right) P_{k|k-1}, \quad (11) \]
where
\[ K_k = P_{k|k-1} \left( H_k^T \left( H_k P_{k|k-1} H_k^T + R_k \right)^{-1} \right)^{-1}, \quad (12) \]
holds according to the matrix inversion lemma, as can be shown by multiplying on the left by \( H_k^T R_k^{-1} H_k^T \) and on the right by \( H_k^T P_{k|k-1} H_k^T + R_k \).

The second modification deals with the iterative termination condition. For convenience, we lump the current observation and state estimate into a single “observation” vector. Thus, we form the augmented observation and measurement function
\[ Z = \begin{bmatrix} z \\ \hat{x} \end{bmatrix}, \quad g(x) = \begin{bmatrix} h(x) \\ x \end{bmatrix}. \quad (13) \]

Based on optimal estimation, we get the following maximum likelihood estimate for \( x \)
\[ x^* = \arg \max \Lambda(\hat{\xi}), \quad (14) \]
where \( \Lambda(\hat{\xi}) \) is the likelihood function,
\[ \Lambda (\xi) = cn \exp \left[ -\frac{1}{2} (\mathbf{Z} - g(\xi))^\top \mathbf{Q}^{-1} (\mathbf{Z} - g(\xi)) \right], \] (15)

\( cn \) is a constant.

Assuming that \( \Lambda (\xi_{i+1}) > \Lambda (\xi_i) \), we say that \( \Lambda (\xi_{i+1}) \) is closer to the maximum likelihood surface than \( \Lambda (\xi_i) \), equivalently \( \xi_{i+1} \) has a more accurate approximation than \( \xi_i \) to the optimal solution. Substituting \( \hat{x}_{ik} \) for \( \xi_i \) we obtain the following iterative condition:

\[ (\hat{x}_i)^\top (F_{ik}^{-1})^{-1} \hat{x}_i + (\tilde{z}_i)^\top R_i^{-1} \tilde{z}_i < (\hat{z}_i)^\top R_i^{-1} \hat{z}_i, \] (16)

where

\[ \hat{x}_i = x_i - \hat{x}_{ik}, \] (17)

\[ \tilde{z}_i = z_i - \hat{h} (\hat{x}_{ik}). \] (18)

### B. Particle filtering method

The sequential importance sampling (SIS) algorithm is a Monte Carlo (MC) method that forms the basis for most sequential MC (SMC) filters developed over the past decades [17]. This SMC approach is also known as particle filtering. It is a technique for implementing a recursive Bayesian filter by a set of random samples with associated weights. As the number of samples becomes very large, this MC characterization becomes an equivalent representation to the usual functional description of the posterior PDF, and the SIS filter approaches the optimum Bayesian estimate.

Let \( \{x_k^{(n)}, \omega_k^{(n)}\}_{n=1}^{N_t} \) denote a random measure that characterizes the posterior PDF \( p(x_k | z_{1:k}) \), where \( \{x_k^{(n)}, n = 1, \ldots, N_t\} \) is a set of support points with associated weights \( \{\omega_k^{(n)}, n = 1, \ldots, N_t\} \). The weights are normalized such that \( \sum_n \omega_k^{(n)} = 1 \). Then the posterior density at time \( k \) can be approximated as

\[ p (x_k | z_{1:k}) = \sum_{n=1}^{N_t} \omega_k^{(n)} \delta (x_k - x_k^{(n)}). \] (19)

The weights are chosen using the principle of importance sampling [18]. Suppose one can obtain a weighted set of samples \( \{x_k^{(n)}, \omega_k^{(n)}\}_{n=1}^{N_t} \) approximately distributed according to \( p (x_k^{(n)} | z_{1:k}) \) and the new samples generated from a suitable designed proposal distribution, i.e., \( x_k^{(n)} \sim q (x_k | x_k^{(n)} - z_k) \), \( n = 1, \ldots, N_t \). Then, the weights can be calculated by

\[ \omega_k^{(n)} \propto \omega_k^{(n)} p (x_k^{(n)} | x_k^{(n)} - z_k) p (x_k^{(n)} | z_{1:k}) \] (20)

It can be shown that as \( N_t \to \infty \), the approximation (19) approaches the true posterior density \( p (x_k | z_{1:k}) \). A more detailed description of this algorithm can be found in [8].

A common problem with the PF is the degeneracy phenomenon, where after a few iterations, all but one particle will have negligible weight. This degeneracy implies that a large computational effort is devoted to updating particles whose contribution to the approximation to \( p (x_k | z_{1:k}) \) is almost zero. There are two methods to deal with the problem.

The first method is to choose a good importance density. In [18], the optimal importance density function that minimizes the variance of the true weights has been shown to be

\[ q (x_k | x_{k-1}^{(n)}, z_k) = p (x_k | x_{k-1}^{(n)}, z_k) \]

\[ \approx p (x_k | x_{k-1}^{(n)}, z_k) \] (21)

Substitution of (21) into (20) yields

\[ \omega_k^{(n)} \propto \omega_k^{(n)} \int p (z_k | x_k^{(n)}) p (x_k | x_{k-1}^{(n)}) dx_k. \] (22)

However, this optimal importance density suffers from two major drawbacks. It requires the ability to sample from \( p (x_k | x_{k-1}^{(n)}, z_k) \) and to evaluate the integral over the new state. In general, this may not be straightforward to do either of these things.

The basic PF uses the transition density as importance density

\[ q (x_k | x_{k-1}^{(n)}, z_k) = p (x_k | x_{k-1}^{(n)}). \] (23)

Substitution of (23) into (20) then yields

\[ \omega_k^{(n)} \propto \omega_k^{(n)} p (z_k | x_k^{(n)}). \] (24)

This would seem to be the most common choice of importance density since it is intuitive and simple to implement. However, because it does not incorporate the most recent observations, the degeneration of samples is very serious which can often lead to poor performance.

The second method by which the effects of degeneracy can be reduced is to use resampling whenever a significant degeneracy is observed. The basic idea of resampling is to eliminate particles that have small weights and to concentrate on particles with large weights. The resampling step involves generating a new set \( \{x_k^{(n)}, n = 1, \ldots, N_t\} \) by resampling \( N_t \) times from

\[ \{x_k^{(n)}\}_{n=1}^{N_t} \] so that \( \Pr (x_k^{(n)} = x_k^{(n)}) = \omega_k^{(n)} \). The resulting sample is in fact an i.i.d. sample from the discrete density
$p(\mathbf{x}_k|z_{k|k})$; therefore, the weights are now reset to $\omega_k^{(i)} = 1/N_s$. Several resampling schemes have been proposed in the literature [19] [20] [21].

Although the resampling step reduces the effects of the degeneracy problem, it introduces other practical problem, known as sample impoverishment. The particles that have high weights are statistically selected many times. This leads to a loss of diversity among the particles as the resultant sample will contain many repeated points.

C. MCMC-IKPF

Here, we use the modified IEKF to generate the importance density function, which integrates the latest observation into system state transition density, so the proposal distribution can approximate the posterior distribution reasonably well. In addition, after the resampling step we increase the MCMC step to introduce sample variety to solve the problem of sample impoverishment. The improved PF implementation is as follows.

Step 1 Initialization: $k = 0$

Draw a set of particles $x_0^{(i)} (i = 0,...,N_s)$ from the prior $p(x_0)$.

Step 2 Importance Sampling

For $i = 1:N_s$

—Update the particles with the modified IEKF:

Evaluate the state estimate $\hat{x}_{ik}$ and corresponding covariance $\mathbf{P}_{ik}$ through (10) and (11). Use inequality (16) as the iterative termination condition.

—Sample $x_{ik}^{(i)} \sim q(x_{ik}^{(i)} | x_{ik-1}^{(i)}, z_k) = N(\hat{x}_{ik}^{(i)}, \mathbf{P}_{ik}^{(i)})$. (19)

—Evaluate the importance weights

$\omega_k^{(i)} \propto \frac{p(z_k | x_{ik}^{(i)})p(x_{ik}^{(i)} | x_{ik-1}^{(i)})}{q(x_{ik}^{(i)} | x_{ik-1}^{(i)}, z_k)}$, (20)

and normalize the importance weights

$\omega_k^{(i)} = \frac{\omega_k^{(i)}}{\sum_i \omega_k^{(i)}}$. (21)

Step 3 Resampling

Resample particles $\{x_{ik}^{(i)}, i = 1,2,\ldots,N_s\}$ using residual resampling[12] strategy to obtain a new set of particles $\{\hat{x}_{ik}^{(i)}, j = 1,2,\ldots,N_s\}$, and set $\omega_k^{(i)} = \frac{1}{N_s}$.

Step 4 MCMC step

—Sample $\alpha - \nu(0,1), \nu(0,1)$ is uniformly distribution in the interval $[0,1]$.

—Replace $x_{ik+1}^{(j)}$ with $x_{ik+1}^{(j)}$, where $j$ is resampling indexes.

—Update the particles $\{x_{ik+1}^{(j)}, j = 1,2,\ldots,N_s\}$ with the modified IEKF, obtain $\bar{x}_{ik+1}^{(j)}$ and $\mathbf{P}_{ik+1}^{(j)}$.

—Sample

$\bar{x}_{ik+1}^{(j)} \sim q(\bar{x}_{ik+1}^{(j)} | \bar{x}_{ik+1-1}^{(j)}, z_k) = N(\bar{x}_{ik+1}^{(j)}, \mathbf{P}_{ik+1}^{(j)})$. (22)

—Evaluate the ratio of the two importance weights

$\beta = \frac{\omega(\bar{x}_{ik+1}^{(j)})}{\omega(x_{ik+1}^{(j)})}$. (23)

—If $\alpha < \min\{1, \beta\}$, then $x_{ik+1}^{(j)} = \bar{x}_{ik+1}^{(j)}$ else $x_{ik+1}^{(j)} = x_{ik+1}^{(j)}$

end if.

Step 5 $k = k + 1$, go to Step 2.

IV. Simulation Results

In this section, the performance of the proposed algorithm for the target tracking in the presence of glint noise is shown via simulations. Here we consider a target tracking problem where the system models are described in (1) and (2). The numerical values of the parameters of the dynamic system are given as follows.

Initial state $x_0=(5000m,5000m,-20m/s,0m/s)$,  
Sample interval $T = 1s$, 
Glint probability $\varepsilon = 0.05$, 
Covariance matrix for background Gaussian noise $P_1 = diag(2500m^2, 9mrad^2)$, 
Covariance matrix for glint spikes Gaussian noise $P_2 = diag(10000m^2, 400mrad^2)$.

For the PF and MCMC-PF algorithms, the number of particles is set as 1000. In the proposed algorithm, we use 100 particles.

The evaluation metrics of interest here are the root mean squares error (RMSE) which is defined as

$E_k = \left[\frac{1}{M} \sum_{j=1}^{M} \left((x_k - \hat{x}_{ik,j})^2 + (y_k - \hat{y}_{ik,j})^2\right)\right]^{1/2}$ (24)

where $M$ is the Monte Carlo runs ($M = 200$), $(x_k,y_k)$ denotes the true position of the target at time $k$, and $(\hat{x}_{ik,j}, \hat{y}_{ik,j})$ represents the position estimation of the $i$ th Monte Carlo run. The simulations are conducted on the computer (Pentium 4 CPU 3.06GHz).

Figure 1 shows the RMSE of the position with 5% glint in the measurement noise. We also conduct some simulation experiments with different glint probability $\varepsilon$. The mean and variance of RMSE are presented in Table 1. Apparently the proposed MCMC-IKPF algorithm outperforms the PF and
MCMC-PF; however, with the increase of glint probability, they exhibit a different degree of performance loss.

Table 1 shows the average running time of the algorithms. It is obvious that computational efficiency of the proposed algorithm is higher than the PF and MCMC-PF. One major reason for this is that the proposed algorithm uses the modified iterated extended Kalman filter to generate the proposal distribution that can integrate with the current observation and introduces MCMC technique after the resampling step to figure out the problem of sample impoverishment, so it can obtain a relatively good tracking performance by using fewer particles.

V. CONCLUSION

In this paper, we proposed an improved particle filter and considered the use of it in the problem of target tracking when the measurement noise is corrupted by glint. Due to the use of the modified IEKF to generate importance distribution and the increase of MCMC step to introduce sample variety after the resampling step, the MCMC-IEKPF algorithm has better tracking performance than the PF and MCMC-PF.

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Table 2 The average running time of the algorithms

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REFERENCES