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Design of Petri Net-based Deadlock Prevention Controllers for Flexible Manufacturing Systems

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***Abstract*—This paper presents a novel method to design Petri net-based deadlock prevention controllers for flexible manufacturing systems. It starts from the computation of the complete deadlock markings by utilizing the conservativeness property of a Petri net model and the necessary and sufficient condition for deadlock. Then, it verifies a small state space including the dangerous and bad markings only by combining one-step look-forward through the original net and one-step look-backward via its reverse net. Subsequently, it defines the set of place invariants from the subset of marked operation places for the so called "elementary controlled bad markings". Finally, it synthesizes a deadlock prevention controller by a simplified invariant-based method. Its obtained deadlock-free controller allows more behavior of the closed-loop system than those obtained via a siphon-based control method. Its computational efficiency is higher than those based on a complete reachability graph-based control method.**

***Keywords*—Deadlock prevention, Petri net, Flexible manufacturing system, Supervisory control**

I. INTRODUCTION

The investigations on deadlock resolution in flexible manufacturing systems (FMS) have received much attention for the two decades. Petri nets (PN) have been widely adopted to describe and analyze FMS because of their ability to reveal behavioral properties of the system such as deadlock freeness and boundedness [1]. Within the PN context, deadlock prevention as one of three main strategies to deal with a deadlock problem in FMS aims to synthesize a PN controller off-line such that the system reaches no deadlock states during its run time. It consists of a set of control places with their initial marking and related arcs. A comprehensive survey and comparison of PN-based deadlock prevention policies for FMS can be found in [2]. Generally speaking, there are mainly two approaches to PN-based deadlock prevention: siphons [3]-[7] and reachability graph-based methods [10]-[13]. In addition, Li *et al*. [8] presents a hybrid approach to address deadlock prevention problem for FMS by combining siphon-based control method and the theory of regions [9].

In this paper, we propose a novel design method for deadlock prevention of FMS. It consists of four main stages. The first stage computes the set of deadlock markings by exploring the conservativeness property of a Petri net and the necessary and sufficient condition for deadlock. The second one identifies dangerous and bad markings by combining one-step look-forward via the original net and one-step look-backward via its reverse net. The third stage defines the set of place invariants from the subset of marked operation places for elementary controlled bad markings. The last one derives the deadlock prevention controller by a simplified invariant-based method. To the best knowledge of the authors, such design approach is for the first time proposed. It avoids the complete reachability graph by just finding the state space including bad and dangerous markings only, which is often much smaller than the complete reachability graph. From a finite number of elementary controlled bad makings, a set of place invariants is defined to characterize the deadlock-freeness of all the bad markings. Compared with reachability graph-based methods, the proposed one has drastically higher computational efficiency while achieves the same control effects as they do. Compared with siphon-based control methods, the proposed one makes the behavior of the controlled system more permissive. Its advantages are illustrated by an example from the literature.

The rest of the paper is structured as follows. Section II presents the preliminaries related to this paper. Section III is devoted to a novel deadlock prevention policy for FMS. Section IV gives an example to illustrate the proposed approach. Finally, Section V concludes this paper and suggests directions for future work.

II. PRELIMINARIES

A. PN

In this section, the basic definitions and properties concerning PN are reviewed. The more detail on PN theory and application is referred to [14].

Definition 1: A PN is a structure $N=(P, T, F, W)$, where $P=\{p_1, p_2,\ldots, p_m\}$ is a finite set of places; $T=\{t_1, t_2,\ldots, t_n\}$ a finite set of transitions; $P\cap T=\varnothing$, $P\cup T\neq\varnothing$; $F\subseteq(P\times T)\cup(T\times P)$ is called the set of directed arcs; $W:(P\times T)\cup(T\times P)\rightarrow\mathcal{N}$ is a mapping that assigns a weight to an arc in F and zero to any $(x, y)\notin F$ where $\mathcal{N}=\{0, 1, 2, \ldots\}$ and $x, y\in P\cup T$. Specially, N is called an ordinary net if $W:F\rightarrow 1$. A marked PN system (N, M_0) is a PN structure N with an initial marking M_0. The incidence matrix is denoted as $C=C^+-C^-$, where $C^+(p,t)=W(t,p), C^-(p,t)=W(p,t)$.

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Definition 2: A transition t is enabled at marking M iff $M \geq C^{-}(.,t)$. It is denoted as $M[t>$. We define a firing sequence from M_0 as $\sigma = t_1 t_2 \ldots t_k$ and M satisfies the basic state equation: $M = M_0 + C\vec{\sigma}$ as denoted $M_0[\sigma>M$. $\vec{\sigma}$ is called Parikh vector whose ith component is the number of times t_i appears in σ. The set of all reachable markings from M in N is denoted as $R(N, M)$.

Definition 3: (N, M_0) is bounded iff $\exists k \in \mathcal{N}$, $\forall M \in R(N, M_0)$, $\forall \in p \in P$: $M(p) \leq k$ holds.

Definition 4: A marking M is a *deadlock* marking iff there exists no such $t \in T$ that $M[t>$. (N, M_0) is *deadlock-free* iff $\forall M \in R(N, M_0)$, $\exists$ $t \in T$, $M[t>$ holds. (N, M_0) is *live* iff $\forall t \in T$ and $M \in R(N, M_0)$, $\exists$ $M' \in R(N, M)$, $\exists$ $M'[t>$.

Based on the concept of a transitive matrix, some definitions introduced in [16] are presented as follows.

Definition 5: For (N, M_0), let L_{CP} be the labeled place transitive matrix, defined by

$$L_{CP} = C^{-} diag(t_1, t_2, \ldots, t_n)(C^{+})^T$$

Here t_i (i=1, 2,…,n) represents the labels with

$$|t_i| = \begin{cases} 1, & \text{if } t_i \text{ fires;} \\ 0, & \text{otherwise} \end{cases}$$

The elements of L_{CP} describe the directly transferring relation that is from one place to another through one or more transitions.

Definition 6: Let L^{*}_{CP} be the *revised labeled place transitive matrix* of a PN structure N. If t_k appears s times in the same column of t_k, then replace t_k in L^{*}_{CP} by t_k/s.

Definition 7: In (N, M_0), let a reachable marking $M_{R(k+1)}$ from M_k be a $|P|$-vector of nonnegative integers. The transformation is defined by

$$M_{R(k+1)}{}^T = M_k{}^T L^{*}_{CP}$$

where $M_k = [M_k(p_1), M_k(p_2), \ldots, M_k(p_{|P|})]^T$.

Property 1 [17]: Let N be a PN having $|P| \times |P|$ labeled place transitive matrix L_{CP}, then

$$T_R = [\sum_{i=1}^{|P|} L_{CP}(i,1) \quad \sum_{i=1}^{|P|} L_{CP}(i,2) \quad \ldots \quad \sum_{i=1}^{|P|} L_{CP}(i,|P|)]$$

is a row $|P|$-vector of transitions and

$$T_C = [\sum_{j=1}^{|P|} L_{CP}(1,j) \quad \sum_{j=1}^{|P|} L_{CP}(1,j) \quad \ldots \quad \sum_{j=1}^{|P|} L_{CP}(1,j)]$$

is a column $|P|$-vector of transitions.

Changing the direction of all arcs in a PN structure N leads to its reverse net [18] as defined next.

Definition 8: $N_r = (P_r, T_r, F_r, W_r)$ is the reverse net of N if $F_r = \{(x,y) | (y,x) \in F\}$ and $\forall (x,y) \in F_r, W_r(x,y) = W(y,x)$.

Property 2 [19]: For a PN (N, M_0), if there exists a firing sequence $\sigma = (t_1, t_2, \ldots, t_k)$ satisfying $M_0[\sigma\rangle M_k$, then in the corresponding reverse net (N_r, M_k), there exists a firing sequence $\sigma_r = (t_k, t_{k-1}, \ldots, t_1)$ satisfying $M_k[\sigma_r\rangle M_0$.

B. PN Models for FMS

There are various PN models to describe the different types of FMSs. We use S^3PR [3] to illustrate the proposed approach.

Definition 9: An S^3PR $N = (P \cup P^0 \cup P_R, T, F, W)$ is defined as the union of a set of nets $Ni = (P_i \cup P_i^0 \cup P_{Ri}, T_i, F_i, W_i)$ sharing common places, where the following statements are true.

1) P_i^0 is called the process idle place of N_i. Places in P_i and P_{Ri} are called operation and resources ones, respectively.

2) $P_i \neq \emptyset$; $p_i^0 \notin P_i$; $P_{Ri} \neq \emptyset$; $(P_i \cup \{p_i^0\}) \cap P_{Ri} = \emptyset$; $\forall p \in P_i$, $\forall t \in {}^{\bullet}p, \forall t' \in p^{\bullet}, \exists r_p \in P_{Ri}, {}^{\bullet}t \cap P_{Ri} = t'^{\bullet} \cap P_{Ri} = \{r_p\}; \forall r \in P_{Ri}$, ${}^{\bullet\bullet}r \cap P_i = r^{\bullet\bullet} \cap P_i \neq \emptyset; \forall r \in P_{Ri}, {}^{\bullet}r \cap r^{\bullet} = \emptyset; {}^{\bullet\bullet}(p_i^0) \cap {}^{\bullet\bullet}(p_i^0) \cap P_{Ri} = (p_i^0)^{\bullet\bullet} \cap P_{Ri} = \emptyset$.

3) For $r \in P_{Ri}, H(r) = ({}^{\bullet\bullet}r) \cap P_i$, the operation places that use r are called the set of holders of r.

4) $\forall p \in P_i, \exists$ unique resource $r \in P_{Ri}$ such that $p \in H(r)$.

5) N_i' is a strongly connected state machine, where $N_i' = (P_i \cup \{p_i^0\}, T, F)$ is a resultant net after the places in P_{Ri} and related arcs are removed from N_i. Every circuit of N_i' contains place P_i^0.

6) Any two N_i's are composable when they share a set of resource places and every shared place must be a resource one.

The conservativeness property of a marked S^3PR is stated as follows.

Property 3 [3]: Let (N, M_0) be a marked S^3PR, then

1) N is conservative.
2) $\forall M \in R(N, M_0)$, we have that

$$\forall i, \sum_{p \in P_i \cup P_i^0} M(p) = M_0(p_i^0) \quad (1)$$

$$\forall r \in P_R, \sum_{p \in H(r) \cup r} M(p) = M_0(r) \quad (2)$$

III. DESIGN METHOD

A. Approach to Computing Deadlock States

Definition 10: In (N, M_0) with $|P| = m, |T| = n$, a reachable marking M_k from M_0 is completely dead, iff for i=1, 2, … , m, the following equation holds.

$$M_{R(k+1)}(p_i) = M_k{}^T L^{*}_{cpi} = \sum_{j=1}^{n} t_j f_j[M_k(p_l)] = 0$$

where $f_j[M_k(p_l)]$ corresponds to a linear combination of $M_k(p_l)$. Formally, $f_j[M_k(p_l)] = \sum_{l=1}^{m} a_{jl} M_k(p_l)$, a_{jl} is the coefficient of $M_k(p_l)$.

Definition 11: A transition $\vec{t_j}$ is dead under M_k denoted as

$$\vec{t_j} f_j[M_k(p_l)] = \vec{t_j}\sum_{l=1}^{m} a_{jl} M_k(p_l) = 0$$

iff $\forall a_{jl} > 0, \prod M_k(p_l) = 0$.

In terms of the sufficient and necessary condition for a complete deadlock to occur in a PN, the following theorem is given.

Theorem 1: In (N, M_0) with $|P| = m, |T| = n$, a reachable marking M_k is a complete deadlock marking, iff for j=1,2,…,n, $t_j f_j[M_k(p_l)] = t_j \sum_{l=1}^{m} a_{jl} M_k(p_l)$, the following relation holds.

$$\begin{cases} \forall a_{jl} > 0, \prod M_k(p_l) = 0 \\ M_k(p_l) \geq 0, M_k(p_l) \in \mathbb{Z}, l = 1,2,...,m \end{cases} \qquad (3)$$

The Theorem is applicable to any ordinary PN.

Corollary 1: If t_j is dead under marking M_k, then

1) If $\exists l$, a_{jl} =1, then $M_k(p_l)$=0.

2) If $\exists l = 1,2,...,s, a_{jl} = \frac{1}{s}$, then $\prod_{l=1}^{l=s} M_k(p_l) = 0$.

By exploiting the conservativeness property of S^3PR and the sufficient and necessary conditions for a deadlock to occur in it, we can compute the set of its deadlock markings.

Theorem 2: If a S^3PR net (N, M_0) is not deadlock-free, then there must exist a solution satisfying (1)-(3). Furthermore, the solution is the set of deadlock markings.

B. Computation of Bad and Dangeous Markings

Definition 12: A marking is called a partial deadlock marking if it inevitably leads to a complete deadlock.

Definition 13: A marking M_{bi} is called a bad one if it is a complete or partial deadlock marking.

From the above definition, it is evident that the set of bad markings M_b includes all the complete and partial deadlock markings.

Definition 14: A marking M_{di} is defined as a dangerous marking, if $M_{di}[t_j>M_{bi}$, where M_{bi} is a bad marking but M_{di} is not.

According to Property 1, we can obtain the enabling condition for transitions in a marked PN, which is represented as follows.

Property 4: A transition t_j is enabled, if it belongs to a finite linear combination of transitions $M_k^T T_c$ for any reachable marking M_k and the following conditions are satisfied

1) $t_j \in T_R(p_i), 1 \leq i \leq |P|$; and

2) The number of times that t_j appears in $M_k^T T_c$ exactly equals coefficient c_j of $t_j \in T_R(p_i), 1 \leq i \leq |P|$ in the linear combination, where c_j is a positive integer.

The above property can be used to check whether the transitions are enabled under a given state M_k and the set of these enabled transitions is denoted as T_{ek}, while the next marking M_{k+1} from M_k through the firing of the set of enabled transitions under M_k is computed by the following property.

Property 5: For any enabled transition $t_j \in T_c(p_i)$ for $M_k(P_i) \geq 1$, then

$$M_{k+1} = M_{k'} + T_{Rk}$$
$$M_{k'}(p_i) = M_k(p_i) - 1$$
$$T_{Rk}(p_i) = \begin{cases} 1, \ t_j \in T_R(p_i) \\ 0, \ \text{otherwise} \end{cases}$$

When a PN reaches some state M_k, we expect to know where the system will go after some transition (event) fires. In other words, we focus on how to obtain the set of next markings M_{k+1} from M_k by firing some transition. Here, an algorithm called "one-step look-forward" is provided as follows.

Algorithm 1 (One-step look-forward)
Input: (N, M_0) and $M_k \in R(N, M_0)$
Output $M_{k+1} = \{M_{k+1}^i \,|\, M_k[t_j\rangle M_{k+1}^i, t_j \in T, M_k \in R(N, M_0)\}$

1. Compute the labeled place transitive matrix L_{CP} of N by Definition 5;
2. Check whether a transition t_j is enabled under M_k by Property 4 and obtain the set of enabled ones T_{ek} under M_k;
3. Compute $M_{k+1} = \{M_{k+1}^i \,|\, M_k[t_j\rangle M_{k+1}^i, t_j \in T_{ek}, i \in \mathbb{N}\}$ by Property 5.

Corollary 2: For a reverse net N_R corresponding to (N, M_0), the labeled place transitive matrix of N_R, denoted as L_{CPR}, can be computed as follows:

$$L_{CPR} = C^+ diag(t_1, t_2, ..., t_n)(C^-)^T$$

In contrast to Algorithm 1, the so called "one-step look-backward" algorithm is to obtain the information where the given marking M_{k+1} comes from. More specifically, the set of previous markings M_k leading to the known marking M_{k+1} by firing some transition is computed by the following algorithm.

Algorithm 2 (One-step look-backward):
Input: (N, M_0) and $M_{k+1} \in R(N, M_0)$.
Output: $M_k = \{M_k^i \,|\, M_k^i[t_j\rangle M_{k+1}, t_j \in T, M_{k+1} \in R(N, M_0)\}$

1. Construct reverse net N_R corresponding to N;
2. Compute L_{CPR} by Corollary 2;
3. For PN(N_R, M_{k+1}), check whether t_j is enabled under M_{k+1} by Property 4 and obtain the set of enabled transitions $T_{e(k+1)}$ under M_{k+1};
4. Compute $M_k = \{M_k^i \,|\, M_{k+1}[t_j\rangle M_k^i, t_j \in T_{e(k+1)}, i = 1,2,...\}$ by Property 5.

Assume that the set of deadlock markings is known. The set of bad and dangerous markings are expected to obtain by the following algorithm:

Algorithm 3:

Input: (N, M_0) and the set of deadlock markings $M_{D(k+1)} = \{M_{Di(k+1)} \mid M_{Di(k+1)} \in R(N, M_0), i = 1, 2, ..., D\}$

Output: a set of dangerous markings M_d and set of bad markings M_b

1. Initialization: $M_b = M_{b(k+1)} = M_{D(k+1)}$, $M_d = 0$;
2. i=1;
3. Compute $M_{dik} = \{M^h_{dik} \mid M_{bi(k+1)}[t_j\rangle M^h_{dik}, t_j \in T_{e,k+1}, h = 1, 2, ..., m_k\}$ from $M_{bi(k+1)}$ by Algorithm 2, where m_k denotes the number of all the markings in M_{dik};
4. h=1;
5. Compute $M^{hF}_{di(k+1)} = \{M^f_{di(k+1)} \mid M^h_{dik}[t_l\rangle M^f_{di(k+1)}, t_l \in T_{ek}, f = 1, ..., p_k\}$, by Algorithm 1, where $M^{hF}_{di(k+1)}$ denotes the set of markings from M^h_{dik} by one-step look-forward, and p_k is the number of the markings in $M^{hF}_{di(k+1)}$.
6. If $\exists t_l, t_l \neq t_j, M^f_{di(k+1)} \notin M_b$, then $M_d = M_d \cup \{M^h_{dik}\}$;
   Else $M_{bik} = M^h_{dik}$, $M_b = M_b \cup \{M_{bik}\}$, go to Step 9;
7. h=h+1;
   If h=m_k+1, go to Step 8;
   Else go to Step 5;
8. i=i+1;
   If i=D+1, go to Step 9;
   Else go to Step 3;
9. k=k-1;
   If $\exists k = k - q, \forall i, h, M^h_{di(k-q+1)}[t_j\rangle M^h_{di(k-q)}, t_j \in T_{e,k-q+1}$ $M^h_{di(k-q+1)} \in M_b, M^h_{di(k-q)} \notin M_b$, go to Step 10;
10. Output M_d and M_b.

Theorem 3: Algorithm 3 can terminate in a finite number of steps for a bounded PN.

C. Synthesis of Controllers for Deadlock Prevention

For the sake of convenience, let $M_d = \bigcup_{K=k-q}^{k} M_{dK}$, $M_b = \bigcup_{K=k-q+1}^{k} M_{bK}$ be the set of dangerous markings and bad markings obtained by Algorithm 3, respectively.

Definition 15: A marking M_{ebi} is defined as an elementary bad marking, if $M_{di}[t_j\rangle M_{ebi}, M_{di} \in M_{d(k-q)}, M_{ebi} \in M_{b(k-q+1)}$.

Lemma 1: For an elementary bad marking M_{ebi}, there is at least two operation places marked. Furthermore, a place invariant PI_i to prevent the system from reaching M_{ebi} can be defined as $\sum_{l=1}^{I} M(p_{ml}) \leq b_i$, where p_{ml} is a marked operation place, I denotes the number of marked operation ones, $b_i = \sum_{l=1}^{I} M_{ebi}(p_{ml}) - 1$. In other words, PI_i can also be expressed as $L_{PIi} M_{PIi} \leq b_i$, where $L_{PIi} = [1 \ \ ... \ \ 1]_{1\times I}$, $M_{PIi} = [M(p_{m1}) \ \ ... \ \ M(p_{mI})]^T_{I\times 1}$.

Given the set of dangerous and bad markings, the place invariants from some bad markings can be defined to prepare for the synthesis of a deadlock-free controller. The algorithm for implementing this function is given as follows

Algorithm 4:

Input: the set of dangerous markings M_d and bad ones M_b;

Output: the set of place invariants PI;

1. Obtain the set of the elementary bad markings $M^0_{eb} = \{M_{ebi}, i \in \mathbb{N}\}$ according to Definition 15;
2. Initialization: c=0, $M^c_{br} = M_b$, $M^c_{eb} = M^0_{eb}$, $M^c_{bv} = \varnothing$, where c denotes the number of iterations needed in the algorithm, M^c_{br} the remaining bad markings after the cth iteration, M^c_{eb} the selected elementary bad marking at the cth iteration, and M^c_{bv} the bad markings violating the place invariants defined at the cth iteration.
3. Define the set of place invariants PI^c from M^c_{eb} based on Lemma 1;
4. Check $\forall M_{bl} \in M^{c+1}_{br} = M^c_{br} - M^c_{bv} - M^c_{eb}, l \in \mathbb{N}$ whether violates PI^c;
   If yes, then denote these bad markings M^{c+1}_{bv} and go to Step 5;
   Else, go to Step 7;
5. $M^{c+1}_{eb} = \{M^{c+1}_{ebk} \mid M^c_{ebi}[t_j\rangle M^{c+1}_{ebk}, \forall M^c_{ebi} \in M^c_{eb}, k \in \mathbb{N}\}$, c=c+1;
6. If $M^c_{br} = \varnothing$, go to Step 7; Else, go to Step 3;
7. Output $PI = \bigcup_c PI^c$.

Theorem 4: Algorithm 4 terminates in a finite number of steps for a bounded PN of an FMS.

Corollary 3: $|PI| = FI$ where PI is the set of place invariants obtained by Algorithm 4 and FI is a finite integer.

Definition 16: The set of bad markings $M_{ecb} = \bigcup_c M^c_{eb}$ obtained by Algorithm 4 is called the set of elementary controlled bad markings.

Lemma 2 [15]: Given a place-invariant related net, i.e., a set of places with their input-output arcs and constraint, i.e., L_{PIi} and b_i, if $L_{PIi}\ M_{PIi0} \leq b_i$, then the supervisor with incidence matrix $C_{ci} = -L_{PIi} C_{PIi}$ and initial marking $M_{ci0} = b_i - M_{PIi0}$ to enforce the constraint $L_{PIi}\ M_{PIi} \leq b_i$ when included in the closed-loop system. Furthermore, the supervision is the least restrictive.

For an S[3]PR with deadlock states, the following algorithm can be adopted to ensure the system to be deadlock-free.

Algorithm 5:

Input: an S[3]PR (N, M_0)

Output: a deadlock-free controlled system with (N_c, M_{c0})

1. Obtain the reduced net S[3]PR (N', M_0') corresponding to (N, M_0) by using PN reduction technique [21];
2. Compute the set of deadlock markings M_D in (N', M_0') by Theorem 3;
3. Compute the set of dangerous markings M_d and bad markings M_b according to Algorithm 3;
4. Define place invariants from elementary controlled bad markings by Algorithm 4;
5. Synthesize a controller (N_c, M_{c0}) based on Lemma 2.

Definition 17: An elementary controlled bad marking $M_{eb}^c \in M_{ecb}$ is optimal iff there exists no such marking $M \in M_d$ that $\forall p \in H(r)$, $M(p) = M_{eb}^c(p)$.

Theorem 5: A controller synthesized by Algorithm 5 can ensure the controlled system to be deadlock-free. If all the elementary controlled bad markings are optimal, then the controller is maximally permissive.

IV. EXAMPLE

Let us consider an example of FMS modeled as an S[3]PR [3], which is shown in Fig.1. We will apply the proposed method to this net system.

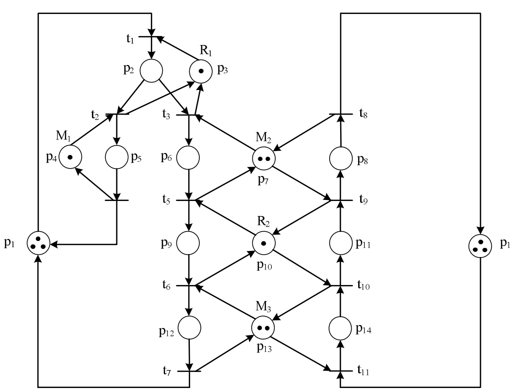

Fig.1 S[3]PR for example FMS

Step 1: Reduce the S[3]PR by PN reduction technique. The reduced one is shown in Fig.2.

Step 2: Compute the deadlock markings in (N', M_0').

According to Property 2, we can obtain the following equations:

$$M_k(p_1)+M_k(p_6)+M_k(p_9)=3$$
$$M_k(p_{11})+M_k(p_{14})+M_k(p_{15})=3$$
$$M_k(p_6)+M_k(p_7)=2$$
$$M_k(p_9)+M_k(p_{10})+M_k(p_{11})=1$$
$$M_k(p_{13})+M_k(p_{14})=2$$

Assume that M_k is a deadlock marking. By Theorem 1, the equations characterizing deadlock markings are as follows:

$$M_k(p_1)M_k(p_7)=0$$
$$M_k(p_6)M_k(p_{10})=0$$
$$M_k(p_9)M_k(p_{13})=0$$
$$M_k(p_{13})M_k(p_{15})=0$$
$$M_k(p_{10})M_k(p_{14})=0$$
$$M_k(p_7)M_k(p_{11})=0$$
$$M_k(p_i)\geq 0, M_k(p_i)\in \mathbb{Z}$$

Solve the above equations leads to two deadlock markings.

$$M_{D1}=[0\quad 1\quad 1\quad 1\quad 0\quad 0\quad 0\quad 2\quad 1]$$
$$M_{D2}=[1\quad 2\quad 0\quad 0\quad 0\quad 1\quad 0\quad 2\quad 0]$$

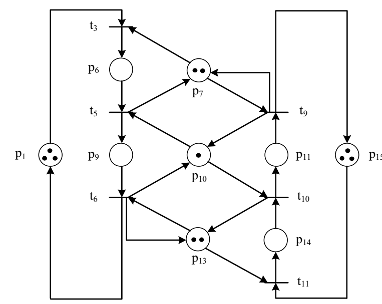

Fig. 2 The reduced S[3]PR

Step 3: Compute the set of dangerous markings M_d and bad markings M_b. Fig. 3 is the search process identifying M_d and M_b by Algorithm 3.

$$M_d=\{M_{d1k}{}^1, M_{d2k}{}^1, M_{d1(k-1)}{}^1, M_{d2(k-1)}{}^1, M_{d1(k-2)}{}^1, M_{d1(k-2)}{}^2, M_{d2(k-2)}{}^1, M_{d2(k-2)}{}^2\}$$
$$M_b=\{M_{D1(k+1)}, M_{D2(k+1)}, M_{d1k}{}^2, M_{d2k}{}^2, M_{d1(k-1)}{}^2, M_{d1(k-1)}{}^3, M_{d2(k-1)}{}^2\}$$

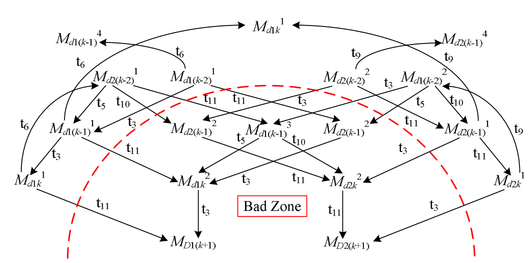

Fig. 3 The bad zone and dangerous markings

Step 4: Define PIs from M_{ecb} by Algorithm 4. The set of elementary controlled bad markings is obtained as follows

$$M_{ecb}=\{M_{d1(k-1)}{}^2, M_{d1(k-1)}{}^3, M_{d2(k-1)}{}^2\}$$
$$PI_1: M(p_6)+M(p_{11})\leq 2$$
$$PI_2: M(p_6)+M(p_{14})\leq 3$$
$$PI_3: M(p_9)+M(p_{14})\leq 2$$

Step 5: Synthesize a controller based on Lemma 2. The controlled deadlock-free system with three control places is shown in Fig.4.

From this example, it is easy to observe that the controller obtained in this paper is the same as that in [11], and the result is optimal, i.e., maximally permissive, owing to the combination of the whole reachability graph based-analysis and the theory of regions. As analyzed in [11], the result in [3] is overly conservative since 76 good states of the state space (232 states) are not reachable in the controlled closed-loop system. In this sense, the approach proposed in this paper can make the behavior of the system more permissive. Moreover, our approach is more efficient than a pure reachability graph based-method as it is not necessary to generate and analyze the complete reachability graph but only a partial one.

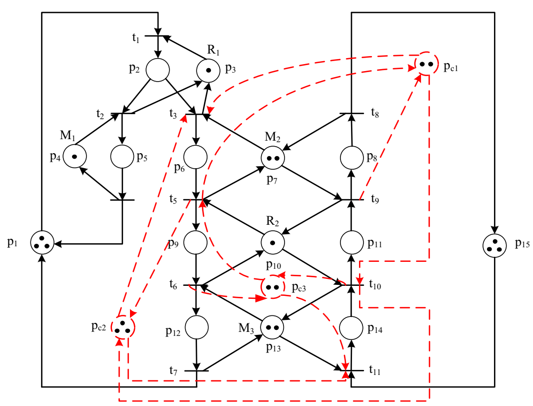

Fig.4 Controlled deadlock-free S^3PR

V. CONCLUSION

In this paper, a novel PN-based design method for deadlock prevention in FMS is presented. It consists of four stages. The first one is to compute the deadlock states based on the marking conservative properties of a reduced PN model and necessary and sufficient condition for deadlock to occur in FMS. From these deadlock markings, the set of dangerous and bad markings are subsequently obtained by combining one-step look-forward and one-step look-backward algorithms. Next, the set of place invariants is defined from the subset of marked operation places for elementary controlled bad markings. Finally, we synthesize a controller to guarantee a deadlock-free system by using the simplified invariant-based method. In comparison with siphon-based control method, the proposed method can lead to more permissive behavior of the controlled closed-loop system. In addition, the whole design process does not need to generate the complete reachability graph of the PN model; therefore the proposed approach has computational advantage over all the reported reachability graph-based methods while enjoys its applicability to other classes of PN models as the latter do.

The proposed method will be extended to more general PN models. In most of the previous studies, a PN plant modeling FMS is assumed to be fully controllable and observable. Therefore, how to deal with the cases with uncontrollable and unobservable transitions should be investigated.

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