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Construction of New Weakening Buffer Operators Based on New Information and Their Applications

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***Abstract*—The grey forecast is one of the important parts in grey systems theory. According to the theory of prior using of new information, based on the present theories of buffer operators and some already existed weakening buffer operators, some new weakening buffer operators are established. Meanwhile, they are compared with the existing weakening buffer operators in effectiveness. The problem that there are some contradictions between quantitative analysis and qualitative analysis in pretreatment for vibration data sequences is resolved effectively. The research result shows the kind of new weakening buffer operators increase the forecast precision of grey forecast model remarkably.**

***Keywords*—Buffer operator; Weakening buffer operator; Average weakening operator; Precision of prediction**

Ⅰ. INTRODUCTION

The gray system theory was put forward by Professor Deng Julong in 1982. The grey forecast is one of the important parts in grey systems theory, and has been widely applied in agriculture, economy, education and science and technology system. Real-life data on the characteristics of various systems has become distorted by a lot of impact factors. In order to be able to grasp the essence of the law of things correctly, we must rule out the role of disturbance factors .Using buffer Operators, generating the gray sequence, weakening the randomness of data, making data to true and regularity, thereby it is easy to grasp the essence of the law of things, and then be able to carry out a reasonable prediction. Typically, the impact factors on the data sequence can be divided into two types, one is to accelerate the development trend of the data or data rate of sequence change of oscillation become larger, the other one is opposite. How to exclude the impact of the outside world a lot of factors and thus make the system characteristics of the data true face is an important research value of one of the issues. In literature [5-8], Liu Sifeng puts forward the concept of buffer operator and constructs a more extensive application of a practical weakening operator. In literature [9], Xie Naiming constructs of a new practical weaken buffer operator. In literature [10-11], Dang Yaoguo constructs a number of strengthening and weakening the buffer operator based on the literature [6-9]. Analyzing comprehensive, we can see, the existing buffer operator can weaken the system characteristics of the data buffer weakened by the impact of factors, improve GM (1, 1) model prediction accuracy in a certain extent, but the results are not ideal. By carrying out structural analysis on the existing weaken buffer operator, the author found that a series of existing weakening buffer operator are unable to utilize the original sequence of the new information $x(n)$ fully, cannot predict the value of interest rate on the new deep excavation, thereby reduces the GM (1,1) model prediction accuracy after weakening the impact of the original sequence. Based on the study, the author puts forward a new weakening buffer operator mode $x(k)d=\frac{x(k)+(n-1)x(n)}{n}$ and of a new weakening buffer operator group based on principle of new information priority and three axioms of buffer operator. The problem which is caused by first part of the growth (decay) too quickly, and latter part of the growth (decay) too slow in reduction process in the modeling prediction that often appear in the results of qualitative analysis and conclusions do not meet has been effectively resolved. The results show that: The prediction accuracy of GM (1,1) which is set up by data sequence which is after first-order weakening buffer operator D_3 to weaken the impact of post-disturbance buffer.

Ⅱ. CONCEPTION

Definition1 Assume that the sequence of data, representing a system's behavior, is given as $X=(x(1),x(2),\cdots,x(n))$

1 If $x(k)-x(k-1)>0$ for each $k=2,3,\cdots,n$, then X is called a monotonous increasing sequence.

2 If $\forall\ k=2,3,\cdots,n,\ x(k)-x(k-1)<0$, then X is called a monotone decreasing sequence.

3If $\exists\ k,k^{'}\in\{2,3,\cdots,n\}$, such that $x(k)-x(k-1)>0$, $x(k^{'})-x(k^{'}-1)<0$,then X is called a monotonic decreasing sequence. If $M=\max\{x(k)|k=1,2,\cdots;n\},m=\min\{x(k)|k=1,2,\cdots;n\}$, then $M-m$ is called the amplitude of X.

Definition2 Assume that $X=(x(1),x(2),\cdots,x(n))$ is a sequence of raw data, D is an operator worked on X, and the sequence, obtained by having D worked on X, is denoted as $XD=(x(1)d,x(2)d,\cdots,x(n)d)$ .Then D is called a sequence operator, and XD the first-order sequence worked on by the operator D.

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A sequence operator can be applied as many times as needed. If $D_1, D_2, D_3 \cdots D_n$ are all sequence operators, we call D1D2 a second-order sequence operator and $XD_1D_2 = (x(1)d_1d_2, x(2)d_1d_2, \cdots, x(n)d_1d_2)$ a sequence worked on by a second-order operator. In the same manner, nth-order sequence operators and sequences worked on by an nth-order operator can be defined.

Axiom 1 (Axiom of Fixed Points) [6] Assume that X is a sequence of raw data and D an arbitrary sequence operator. Then D must satisfy $x(n)d = x(n)$.

This axiom of fixed points says that under the effect of a sequence operator of our choice, the last datum in the sequence of the raw data must be kept unchanged. This axiom is established on the fact or understanding that $x(n)$ is a starting point or foundation for any future development and is an objective reality.

Axiom2 (Axiom on Sufficient Usage of Information) When a sequence operator is applied, all the information contained in each datum $x(k), k = 1,2,\cdots,n$ of the sequence X of the raw data should be sufficiently applied.

The axiom on sufficient usage of information implies that when we define a sequence operator in an application, we must build or define the operator on the foundation of the data available, and not other way around.

Axiom 3 (Axiom of Analytic Representations) For any $x(k), (k = 1,2,\cdots,n)$ can be described with a uniform and elementary analytic representation in $x(1), x(2), \cdots; x(n)$.

The axiom of analytic representations requires that the procedure to obtain a new sequence from the original raw data by applying an operator is clear, standardized, unified, and simplified as much as possible so that the actual computation of the new sequence can be relatively easily implemented on a computer.

Definition 3 Axiom 1,2 and 3 are jointly called three axioms of buffer operators. All sequence operators, satisfying these three axioms, are called buffer operators; and the sequences, obtained by applying first, second, third, ..., orders of buffer operators, are referred to as a first-, second-,third-, ..., order buffer sequences, respectively.

Definition 4 Assume that X is a sequence of raw data, D a buffer operator, When X is respectively a monotonic increasing, decreasing, or vibrational sequence,

1. If the buffer sequence XD increases or decreases more slowly or vibrates with a smaller amplitude than the original sequence X, the buffer operator D is termed a weakening operator;

2. If the buffer sequence XD increases or decreases more rapidly or vibrates with a greater amplitude than the original sequence X, the buffer operator D is termed a strengthening operator.

Ⅲ THE QUALITY OF BUFFER OPERATOR

Theorem 1 [6] Assume that $X = (x(1), x(2), \cdots, x(n)),\ x(i) > 0, i = 1,2,\cdots,n$ is a sequence of raw data.

（1）When X is a monotonic increasing sequence, XD_1 is one of its buffer sequences. Then

①. D_1 is a weakening operator 〈 = 〉 $x(k) \le x(k)d_1, k = 1,2,\cdots,n$;

②. D_1 is a strengthening operator 〈 = 〉 $x(k) \ge x(k)d_1, k = 1,2,\cdots,n$.

That is, the data in a monotonic increasing sequence expand when a weakening operator is applied, and shrink when a strengthening operator is applied.

（2）When X is a monotonic decreasing sequence, XD_1 is one of its buffer sequences. Then

①. D_1 is a weakening operator 〈 = 〉 $x(k) \ge x(k)d_1, k = 1,2,\cdots,n$

②. D_1 is a strengthening operator 〈 = 〉 $x(k) \le x(k)d_1, k = 1,2,\cdots,n$.

That is, the data in a monotonic decreasing sequence shrink when a weakening operator is applied, and expand when a strengthening operator is applied.

（3）When X is a vibrational sequence, XD_1 is one of its buffer sequences. Then

①. D_1 is a weakening operator , then

$$\max_{1\le k\le n}\{x(k)\} \ge \max_{1\le k\le n}\{x(k)d_1\}$$

$$\mathrm{mix}_{1\le k\le n}\{x(k)\} \le \mathrm{mix}_{1\le k\le n}\{x(k)d_1\}$$

②. D_1 is a strengthening operator, then

$$\max_{1\le k\le n}\{x(k)\} \le \max_{1\le k\le n}\{x(k)d_1\}$$

$$\mathrm{mix}_{1\le k\le n}\{x(k)\} \ge \mathrm{mix}_{1\le k\le n}\{x(k)d_1\}$$

The proof is omitted here.

ⅣCONSTRUCTION OF A NEW WEAKENING BUFFER OPERATORS

Theorem 2 Assume that $X = (x(1), x(2), \cdots, x(n)),\ x(i) > 0, i = 1,2,\cdots,n$ is a sequence of raw data. $XD_2 = (x(1)d_2, x(2)d_2, \cdots, x(n)d_2)$ and

$$x(k)d_2 = \frac{x(k) + (n-1)x(n)}{n} .$$

When X is a monotonic increasing sequence, a monotonic decreasing sequence or a vibrational sequence, D_2 is a weakening buffer operator.

Proof: It is easy to proof that D_2 meets three axioms of buffer operators. So D_2 is a weakening buffer operator.

(1) If X is a monotonic increasing sequence, then

$$x(k)d_2 - x(k) = \frac{x(k)+(n-1)x(n)}{n} - x(k) \geq \frac{x(k)+(n-1)x(k)}{n} - x(k) = 0$$

So $x(k)d_2 \geq x(k)$, that is D_2 is a weakening buffer operator.

(2) If X is a monotonic decreasing sequence, then

$$x(k)d_2 - x(k) = \frac{x(k)+(n-1)x(n)}{n} - x(k) \leq \frac{x(k)+(n-1)x(k)}{n} - x(k) = 0$$

So $x(k)d_2 \leq x(k)$, that is D_2 is a weakening buffer operator.

(3) If X is a vibrational sequence, suppose

$$x(a) = \max\{x(k) | k = 1,2,\cdots,n\}$$

$$x(b) = \min\{x(k) | k = 1,2,\cdots,n\}$$

$$x(a)d_2 = \frac{x(a)+(n-1)x(n)}{n}$$

$$x(a)d_2 - x(a) = \frac{x(a)+(n-1)x(n)}{n} - x(a) \leq \frac{x(a)+(n-1)x(a)}{n} - x(a) = 0$$

So, $x(a)d_2 \leq x(a)$, that is $\max\limits_{1\leq k\leq n}\{x(k)\} \geq \max\limits_{1\leq k\leq n}\{x(k)d\}$. In the same matter $\mathrm{m}i\mathrm{x}_{1\leq k\leq n}\{x(k)\} \leq \mathrm{m}i\mathrm{x}_{1\leq k\leq n}\{x(k)d\}$. So D_2 is a weakening buffer operator.

Inference 1 For D_2 the weakening buffer operator defined in theorem 2, let $XD_2^2 = XD_2D_2 = (x(1)d_2^2, x(2)d_2^2, \cdots, x(n)d_2^2)$

$$x(k)d_2^2 = \frac{x(k)d_2 + (n-1)x(n)d_2}{n}$$

$$k = 1,2,\cdots,n$$

D_2 is always a second-order weakening operator for monotonic increasing, monotonic decreasing, and vibrational sequences.

Theorem 3 Assume that $X = (x(1), x(2), \cdots, x(n))$, $x(k) > 0, k = 1,2,\cdots,n$; is a sequence of raw data. $XD_3 = (x(1)d_3, x(2)d_3, \cdots, x(n)d_3)$ and

$$x(k)d_3 = \frac{\sum_{i=k}^{n} \frac{x(i)+(n-1)x(n)}{n}}{n-i+1}, \quad k = 1,2,\cdots,n;$$

When X is a monotonic increasing sequence, a monotonic decreasing sequence or a vibrational sequence, D_3 is a weakening buffer operator.

Proof: It is easy to proof that D_3 meets three axioms of buffer operators. So D_3 is a weakening buffer operator.

(1) If X is a monotonic increasing sequence, then

$$x(k)d_3 - x(k) = \frac{\sum_{i=k}^{n} \frac{x(i)+(n-1)x(n)}{n}}{(n-i+1)} - x(k)$$

$$= \frac{\sum_{i=k}^{n} \frac{x(i)+(n-1)x(n)}{n} - (n-i+1)x(k)}{(n-i+1)}$$

$$\geq \frac{(n-i+1)\frac{x(k)+(n-1)x(k)}{n} - (n-i+1)x(k)}{(n-i+1)}$$

$$\geq x(k)+(n-1)x(k)-nx(k)=0$$

So $x(k)d_3 \geq x(k)$, that is D_3 is a weakening buffer operator. We call D_3 average weakening buffer operator.

(2) If X is a monotonic decreasing sequence, then

$$x(k)d_3 - x(k) = \frac{\sum_{i=k}^{n} \frac{x(i)+(n-1)x(n)}{n}}{(n-i+1)} - x(k)$$

$$= \frac{\sum_{i=k}^{n} \frac{x(i)+(n-1)x(n)}{n} - (n-i+1)x(k)}{(n-i+1)}$$

$$\leq \frac{(n-i+1)\frac{x(k)+(n-1)x(k)}{n} - (n-i+1)x(k)}{(n-i+1)}$$

$$\leq x(k)+(n-1)x(k)-nx(k)=0$$

So $x(k)d_3 \leq x(k)$, that is D_3 is a weakening buffer operator.

(3) If X is a vibrational sequence, suppose

$$x(a) = \max\{x(k) | k = 1,2,\cdots,n\}$$

$$x(b) = \min\{x(k) | k = 1,2,\cdots,n\}$$

$$x(a)d_3-x(a)=\frac{\sum_{a=k}^{n}\frac{x(a)+(n-1)x(n)}{n}}{(n-a+1)}-x(a)$$

$$=\frac{\sum_{a=k}^{n}\frac{x(a)+(n-1)x(n)}{n}-(n-a+1)x(a)}{(n-a+1)}$$

$$\le\frac{(n-a+1)\frac{x(a)+(n-1)x(a)}{n}-(n-a+1)x(a)}{(n-a+1)}$$

$$\le x(a)+(n-1)x(a)-nx(a)=0$$

$$(a=1,2,\cdots,n)$$

So $x(a)d_3 \le x(a)$, that' s

$\max_{1\le k\le n}\{x(k)\} \ge \max_{1\le k\le n}\{x(k)d_3\}$

In the same matter $\min_{1\le k\le n}\{x(k)\} \le \min_{1\le k\le n}\{x(k)d_3\}$

So when X is a vibrational sequence, D_3 is a weakening buffer operator. We call D_3 average weakening.

Inference 2 For D_3 the weakening buffer operator defined in theorem3, let

$$XD_3^2=XD_3D_3=(x(1)d_3^2,x(2)d_3^2,\cdots,x(n)d_3^2)$$

$$x(k)d_3^2==\frac{\sum_{i=k}^{n}\frac{x(i)d_3+(n-1)x(n)d_3}{n}}{n-i+1}$$

$$k=1,2,\cdots,n$$

D_3 is always a second-order weakening operator for monotonic increasing, monotonic decreasing, and vibrational sequences.

Theorem4 Assume that $X=(x(1),x(2),\cdots,x(n)),\ x(k)>0,k=1,2,\cdots,n$; is a sequence of raw data. $XD_4=(x(1)d_4,x(2)d_4,\cdots,x(n)d_4)$ and

$$x(k)d_4=\frac{(k\frac{x(k)+(n-1)x(n)}{n}+\cdots+nx(n))}{(n+k)(n-k+1)/2}$$

When X is a monotonic increasing sequence, a monotonic decreasing sequence or a vibrational sequence, D_4 is a weakening buffer operator.

Proof: It is easy to proof that D_4 meets three axioms of buffer operators. So D_4 is a weakening buffer operator.

(1) If X is a monotonic increasing sequence, then

$$x(k)d_4-x(k)=\frac{k\frac{x(k)+(n-1)x(n)}{n}+\cdots+nx(n)}{(n+k)(n-k+1)/2}-x(k)$$

$$=\frac{k(\frac{x(k)+(n-1)x(n)}{n}-x(k))+\cdots+n(x(n)-x(k))}{(n+k)(n-k+1)/2}$$

$$\ge\frac{k(x(k)-x(k))+\cdots+n(x(k)-x(k))}{(n+k)(n-k+1)/2}=0$$

so $x(k)d_4 \ge x(k)$, that is D_4 is a weakening buffer operator.

In the same matter, it can be proved when X is a monotonic decreasing sequence or a vibrational sequence, D_4 is a weakening buffer operator. We call D_4 weighted average weakening buffer operator.

Based on the analysis above, we can see, all the weakening buffer operators constructed in this text can play a role of slowing growth (decay) or decreasing the amplitude of the speed for monotonic increasing sequences, monotonic decreasing sequences or vibrational sequences. Because of the buffer operator in the role of the original data sequence, it must satisfy the Axiom of Fixed Points, when X is respectively a monotonic increasing sequence, the weakening buffer sequence XD increases more slowly or vibrates with a smaller amplitude than the original sequence X, when X is respectively a monotonic decreasing sequence, the weakening buffer sequence XD decreases more slowly or vibrates with a smaller amplitude than the original sequence X. The weakening buffer operators constructed in this article are based on principle of new information priority. Acting on the sequence of raw data, they can effectively eliminate data "distortion" phenomenon caused by the impact of interference on the system of interference, in the same time, improve the prediction accuracy effectively.

V. CASE STUDT

Use a city industrial output data as an example to verify the weakening buffer operator constructed in this article in the GM (1,1) forecast. Select the city's total industrial output value of 1997-2005 for the original data. (Unit: million yuan)

$$X=(187.85,303.79,394.13,498.27,580.43,640.21,702.34,708.86,716.95)$$

Make 1997-2003 data as a modeling data and 2004-2005 data as a simulation test data. From the original data available, the average annual growth rate of 1997- 2003 the city's total industrial output value is 26.06%. It is generally impossible to consider such long-term growth. Use this data to predict the result is hard to believe. Analysis of this situation, mainly in the city's industrialization process, the country gave the city a special industrial policy, making the city's industries to have a very good opportunity for development. But after about two decades of development, the city's industrial strength has been comparatively strong. The country will abolish the special policies, to maintain the pace of development in future has already begun to impossible. We must weaken the original data sequence on the city's total industrial output value of the trend of development to make a reasonable prediction. Using weakening buffer operator with the original data sequence to weaken the deal to eliminate the pre-preferential industrial policies of this factor for the city's post industrial economic systems impact the pace of development, it can allow models to forecast more accurate prediction and the results more in line with the actual situation. Act the new weakening buffer operator in a series structured in this article on the original data in order to weaken one deal, the weakening of the data sequence

are $XD_2 = (628.84, 645.40, 658.31, 673.19, 684.92, 693.46, 702.34)$

$XD_3 = (669.50, 676.27, 682.45, 688.48, 693.58, 697.90, 702.34)$

$XD_4 = (681.75, 683.71, 686.78, 690.66, 694.54, 698.24, 702.34)$

The winterization equations of the $GM(1,1)$ model set up with the weaken sequences above are as follows:

$$\frac{dx^{(1)}}{dt} - 0.016933x^{(1)} = 631.8357$$

$$\frac{dx^{(1)}}{dt} - 0.007519x^{(1)} = 669.6806$$

$$\frac{dx^{(1)}}{dt} - 0.005421x^{(1)} = 677.8087$$

The two-step prediction error of the results of the comparison by calculation (calculation process is abbreviated) such as shown in table 1

Table 1: Weakened before and after the two-step model prediction error on average compared

model	First-order weakening operator role	The average prediction error of two-step（%）
1	none	19.9
2	XD_2	1.45
3	XD_3	0.24
4	XD_4	0.71

Similarly using the weakening buffer operator in literature [7,9,10] into the original data sequence to the second order to weaken the role of buffer, the combination of GM (1,1) model for solving prediction accuracy (calculation process is abbreviated) , the prediction accuracy of the results of the comparison such as shown in table 2

Table2: After the weakening buffer operators in literature [7,9,10] ,comparison of model prediction accuracy

model	second-order weakening operator role	The average prediction error of two-step（%）
5	operators in literature [7]	3.87
6	operators in literature [9]	3.36
7	operators in literature [10]	2.23

As can be seen from Table 1, the original sequence through the three new operators to weaken the role of the buffer, the prediction accuracy has been significantly increased, especially after D_3 to weaken the role of the first order, the prediction accuracy is as high as 99.76%. Compared Table 1and Table 2 data, it can be seen that the original sequence, after the new weakening buffer operator' first-order weaken the model prediction accuracy is higher than the already weakened second-order weakening operator buffer modeling prediction accuracy.

The prediction model set up by the original data used the new operator D_3 is:

$$\hat{x}^{(1)}(k+1) = 89729.66e^{0.007519k} - 89060.160$$

In 2004-2005, the city's industrial output values of the predictive value are 708.51and715.86 billion, which is basically consistent with the actual industrial output value.

VI. Conclusions

In this paper, a new class of weakening buffer operator is constructed on the basis of study already, and take advantage of the new operator, as well as the series have been weakening buffer operators, with the original sequence data who has first half of the growth rate of faster growth in the latter part of slower features buffering effect of weakening, and use the original data and the data sequence after weakened to set up GM (1, 1) model, and compare the accuracy of prediction. The experimental results show that: （1）The prediction accuracy have markedly improved using the new weakening buffer operator to weaken the first-order data than the original data sequence. （2）The prediction accuracy is higher using the three new weakening buffer operators compared to a series of weakening buffer operator. Especially after D_3 to weaken the role of the first order, the prediction accuracy is as high as 99.76%. Embodiment of the new weakening buffer operator process is simple, the effect of significantly weakening the good characteristics.

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