Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Novel Direct Adaptive Fuzzy Control System Design

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***Abstract*—In our earlier study, the property of L_2-gain robust control is incorporated into the direct adaptive fuzzy control design with the use of the compensative controller. The tracking control performance is guaranteed. In the study, we further employ the L_2-gain property to improve the adaptive law design. Due to the L_2-gain control property, the approximate error of the fuzzy system is estimable, and thus, an adaptive law with the approximate error feedback is proposed in the study. The novel adaptive law improves the learning speed so that the learning performances can be better. Since the L_2-gain based compensative controller can guarantee the tracking control performance and remove the effect of the gain function in the adaptive law. The direct adaptive fuzzy control system design of the paper not only has the satisfactory tracking control performance but also has the better learning performance. Various simulations are conducted to demonstrate the effectiveness of the proposed design.**

***Keywords*—adaptive fuzzy control, L_2-gain, learning control**

I. Introduction

Fuzzy control has been successfully employed in solving many nonlinear control problems. A significant feature of fuzzy control is the use of linguistic fuzzy control rules in the controllers. However, the fuzzy rules are difficult to obtain, especially for complicated systems. In order to obtain fuzzy control rules systematically, a learning (adaptive) control scheme, termed as the adaptive fuzzy control has been proposed in literature [14]. By making use of the fuzzy universal approximation theories [3][13], Lyapunov stable theory, feedback linearization control technique [8]-[10], and backstepping control technique [5][8], various adaptive fuzzy control systems have been reported in recent years. However, it is known that a fuzzy system with a finite number of rules (the finite fuzzy system) can only estimate a function within a limited accuracy. The approximate error of the finite fuzzy system is generally inevitable. The existence of approximate errors may often create problems for adaptive fuzzy control systems, such as the final system stability [9][11][19]. Although the Lyapunov theorem guarantees the stability of the control system, but the existence of the approximate error may make the derivative of the Lyapunov function being possibly positive so that the stability of the system may be destroyed.

An extra robust controller is often used to guarantee the system stability in such a case, such as the sliding-mode based controllers [11][12][21][23], supervisory controllers [12], adaptive proportional-integral robust controller [16], and adaptive sliding proportional-integral robust controller [17]. In addition, there are approaches using other robust theories to redesign the adaptive fuzzy control systems, such as small-gain theorem [22], and H-infinite tracking design [4]. In literature [15], the authors proposed an approach of considering the approximate errors into the Lyapunov function. However, these approaches are usually be used for indirect adaptive fuzzy control because it is easy to define approximate errors. For direct adaptive fuzzy control, it is difficult to define the approximate error, such as the unknowingness of the gain function (g) is a well-known cause. In our study, due to the proposed compensative controller, the finite L_2-gain property is introduced into the direct adaptive fuzzy control system design. Because the finite L_2-gain property was a criterion of the estimator design [6], the proposed compensative controller is also a control error estimator and then the approximate error of fuzzy system is estimable. Therefore, the estimated approximate error feedback term is designed and added into the adaptive law design to enhance learning performance of the direct adaptive fuzzy control system. The proposed approach is shown to have satisfactory performance as expected.

This paper is organized as follows. After this introduction session, in section II, the background of the direct adaptive fuzzy control for an nth-order SISO nonlinear system is introduced. The proposed approach and corresponding proof are presented in section III. In section IV, various simulations with the commonly-used inverted pendulum system are conducted to demonstrate the effectiveness of the proposed approach. Finally, the discussion is given in section V.

II. Background

Consider an nth-order SISO nonlinear system in a companion form as

$$\begin{cases} x^{(n)} = f(\mathbf{x}) + g(\mathbf{x})u + d = f + gu, \\ y = x, \end{cases} \tag{1}$$

where f and g are two unknown continuous system functions, u is the control input, and y is the system output. Define the system state vector as $\mathbf{x} = [x, \dot{x}, \cdots, x^{(n-1)}]^T = [x_1, x_2, \cdots, x_n]^T$. In this study, we assume that the sign of g is known and also the upper bound (g_{up}) and lower bound (g_{low}) of $|g|$ are known. In practice, g_{up} and g_{low} are not necessary to be known because they are used in obtaining the stability proof only not in the proposed controller. We assume that all system states are

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measurable. The task is to drive the system state to track a bounded reference trajectory $\mathbf{y}_m = [y_m, \dot{y}_m, \cdots, y_m^{(n-1)}]^T$. Let the tracking error be $e = y_m - y$ and define the error vector as $\mathbf{e} = [e, \dot{e}, \cdots, e^{(n-1)}]^T = [e_1, e_2, \cdots, e_n]^T$. A reference controller is designed based on the feedback linearization as

$$u^* = \frac{1}{g}(-f + y_m^{(n)} + \mathbf{k}^T\mathbf{e}), \tag{2}$$

where the error feedback gain vector $\mathbf{k} = [k_n, k_{n-1}, \cdots, k_1]^T$ is selected such that all roots of the polynomial $h(s) = s^n + k_1 s^{n-1} + \cdots + k_n$ are in the open left-half plane, so called Hurwitz stable [10]. With the controller as (2), the following state error dynamics equation is obtained

$$e^{(n)} = y_m^{(n)} - x^{(n)} = -\mathbf{k}^T\mathbf{e} + g(u^* - u). \tag{3}$$

The vector form is

$$\dot{\mathbf{e}} = \mathbf{\Lambda}\mathbf{e} + \mathbf{B}(u^* - u), \tag{4}$$

where $\mathbf{\Lambda} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ -k_n & -k_{n-1} & \cdots & -k_2 & -k_1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ g \end{bmatrix}$.

Since $\mathbf{\Lambda}$ is a Hurwitz stable matrix, there exists a unique positive definite symmetric $n \times n$ matrix $\mathbf{P}$ satisfying the Lyapunov function $\mathbf{\Lambda}^T\mathbf{P} + \mathbf{P}\mathbf{\Lambda} = -\mathbf{Q}$, where $\mathbf{Q} = q\mathbf{I}$, in which $\mathbf{I}$ is an unit matrix with an order n and $q > 0$ is a constant.

Now, we consider the direct adaptive fuzzy control [12], which uses a fuzzy system to estimate the reference controller (2) directly. The fuzzy approximator is constructed by a set of fuzzy IF-THEN rules as

R^l : IF x_1 is A_1^l, and $\cdots$, and x_n is A_n^l

THEN y_F is θ^l, for $l = 1, 2, \cdots, M$

where $(x_1, x_2, \cdots, x_n)$ and y_F are the input and output of the fuzzy approximator, respectively. $(A_1^l, A_2^l, \cdots, A_n^l)$ and θ^l are the corresponding fuzzy sets, l is the rule index, and M is the number of rules. In this study, for easy computation, θ^l is a fuzzy singleton. The above fuzzy approximator with the center-average defuzzification and product inference can be obtained as

$$y_f(\mathbf{x}) = \frac{\sum_{l=1}^{M} \theta^l (\prod_{j=1}^{n} \mu_{A_j^l}(x_j))}{\sum_{l=1}^{M} (\prod_{j=1}^{n} \mu_{A_j^l}(x_j))} \tag{5}$$

where $\mu_{A_j^l}(\cdot)$ is the membership function of the fuzzy set A_j^l. Equation (5) is an universal approximator [3] and usually is written as

$$y_f(\mathbf{x}|\boldsymbol{\theta}) = \boldsymbol{\theta}^T\boldsymbol{\omega}, \tag{6}$$

where $\boldsymbol{\theta} = [\theta^1, \theta^2, \cdots, \theta^M]^T$ is an adjustable parameter vector, and $\boldsymbol{\omega} = [\omega^1, \omega^2, \cdots, \omega^M]^T$ is a fuzzy degree vector. The elements of $\boldsymbol{\omega}$ are

$$\omega^l = \frac{\prod_{j=1}^{n} \mu_{A_j^l}(x_j)}{\sum_{l=1}^{M} (\prod_{j=1}^{n} \mu_{A_j^l}(x_j))}, \; l = 1, \cdots, M. \tag{7}$$

As mentioned earlier, approximator (6) is employed to estimate the reference controller u^* directly. In order to cope with the approximate error, a compensative controller (u_c) is added into the control system as

$$u = u_d + u_c = \boldsymbol{\theta}_d^T\boldsymbol{\omega}_d + u_c, \tag{8}$$

Based on the universal approximator theory [13], the optimal fuzzy approximator is defined as follows.

$$u_d^*(\mathbf{x}|\boldsymbol{\theta}_d^*) = u_d^* = \boldsymbol{\theta}_d^{*T}\boldsymbol{\omega}_d, \tag{9}$$

where $\boldsymbol{\theta}_d^*$ is the optimal parameter vector which is defined as

$$\boldsymbol{\theta}_d^* \equiv \arg\min_{\boldsymbol{\theta}_d \in \Omega_{\theta_d}} \{\sup_{\mathbf{x} \in \Omega_x} |u^* - u_d^*(\mathbf{x}|\boldsymbol{\theta}_d)|\}, \tag{10}$$

where Ω_{θ_d} and Ω_x are the constant sets of the suitable bounds of $\boldsymbol{\theta}_d$ and $\mathbf{x}$, respectively. In fact, a fuzzy system with a finite number of rules (finite fuzzy system) can only provides a limited accuracy to the approximation of a nonlinear function. Therefore, the approximate error is defined as $\varepsilon_d = u^* - u_d^*$. By substituting control input (8) into (4), we have

$$\begin{aligned} \dot{\mathbf{e}} &= \mathbf{\Lambda}\mathbf{e} + \mathbf{B}(u^* - \boldsymbol{\theta}_d^T\boldsymbol{\omega}_d - u_c + u_d^* - u_d^*) \\ &= \mathbf{\Lambda}\mathbf{e} + \mathbf{B}(\tilde{\boldsymbol{\theta}}_d^T\boldsymbol{\omega}_d + \varepsilon_d - u_c), \end{aligned} \tag{11}$$

where the parameter estimate error is $\tilde{\boldsymbol{\theta}}_d^T = (\boldsymbol{\theta}_d^* - \boldsymbol{\theta}_d)^T$. In equation (11), $\mathbf{\Lambda}\mathbf{e}$ provides an inner linear system, and $(\tilde{\boldsymbol{\theta}}_d^T\boldsymbol{\omega}_d + \varepsilon_d - u_c)$ is a nonlinear input with respect to the system uncertainties. We can design u_c to cause (11) to have some specific form, such as the L_2-gain inequality in Definition 1 given in the next section.

III. Design Results

The general L_2-gain control notions [1][2][7] will be provided in the following definition. The robust control system is considered with the relationship between the tracking error ($\mathbf{e}$) and the total uncertainty (ε_t).

Definition 1: Control system (1) has the finite L_2-gain property if the following inequality is guaranteed.

$$\|\mathbf{e}\|_{L_2} \le \delta \|\varepsilon_t\|_{L_2} + \gamma_{bias}, \tag{12}$$

where ε_t is the total uncertainty, $\delta > 0$ is a controllable finite gain, and $\gamma_{bias} \in \mathrm{R}^+$ is the bias constant with respect to the initial condition.

The L_2-norm for a variable z is defined as $\|z\|_{L_2} = \sqrt{\int_0^\infty z^2\, dt}$. Refer to inequality (12), it is evident that $\delta\|\varepsilon_t\|_{L_2}$ provides an attraction region around the origin of the state space and the attractiveness is controlled by δ . Thus, the L_2-gain property states that the tracking error is bounded around the origin. In this paper, a compensative controller is proposed in Theorem 1 to let the direct adaptive fuzzy control system have the finite L_2-gain property. In Theorem 1, the used 2-norm for a vector $\mathbf{z}_1$ is defined as $\|\mathbf{z}_1\|_2 = \sqrt{\mathbf{z}_1^T\mathbf{z}_1}$.

Theorem 1: Consider the direct adaptive fuzzy control system as mentioned earlier. The proposed adaptive law for the fuzzy system is

$$\dot{\boldsymbol{\theta}}_d = \alpha_m \operatorname{sgn}(g)\mathbf{e}^T\mathbf{P}\mathbf{B}_1\boldsymbol{\omega}_d + \beta_m\|\mathbf{e}\|_2(u_c\boldsymbol{\omega}_d^{-1})^T, \quad (13)$$

and the proposed compensative controller is

$$u_c = k_{c1}\operatorname{sgn}(g)\mathbf{e}^T\mathbf{P}\mathbf{B}_1, \quad (14)$$

where $\alpha_m > 0$ is an adaptive constant, $\beta_m > 0$ is a scale constant, $\mathbf{B}_1^T = [00\cdots01]_{1\times n}$, and $k_{c1} > 0$ is a selected constant. If $\mathbf{P}$ satisfies $\boldsymbol{\Lambda}^T\mathbf{P} + \mathbf{P}\boldsymbol{\Lambda} = -\mathbf{Q}$ with $\mathbf{Q} = q\mathbf{I}$ and $q > 0$, then the compensative controller (14) guarantees that the system have the finite L_2-gain property with a finite gain $\delta = \sqrt{q^{-1}}(g_{up}/\sqrt{2k_{c1}g_{low}})$ and bias constant $\gamma_{bias} = \sqrt{q^{-1}(\mathbf{e}(0)^T\mathbf{P}\mathbf{e}(0) + (1/2\alpha)\tilde{\boldsymbol{\theta}}_d^T(0)\tilde{\boldsymbol{\theta}}_d(0))}$, where $\alpha > 0$ is a constant.

Proof of Theorem 1: Consider a continuous differentiable Lyapunov function

$$V = \mathbf{e}^T\mathbf{P}\mathbf{e} + (1/2\alpha)\tilde{\boldsymbol{\theta}}_d^T\tilde{\boldsymbol{\theta}}_d, \quad (15)$$

where α is a positive constant. Using equation (11), then the derivative of (15) is

$$\dot{V} = -\mathbf{e}^T\mathbf{Q}\mathbf{e} + 2\mathbf{e}^T\mathbf{P}\mathbf{B}\varepsilon_d - 2\mathbf{e}^T\mathbf{P}\mathbf{B}\,u_c + (1/\alpha)\tilde{\boldsymbol{\theta}}_d^T(2\alpha\mathbf{e}^T\mathbf{P}\mathbf{B}\,\boldsymbol{\omega}_d - \dot{\boldsymbol{\theta}}_d). \quad (16)$$

From equation (16), the normal adaptive law is

$$\dot{\boldsymbol{\theta}}_d = 2\alpha\mathbf{e}^T\mathbf{P}\mathbf{B}\boldsymbol{\omega}_d. \quad (17)$$

Since g is required in $\mathbf{B}$ and is unknown, adaptive law (17) cannot be used. We redesign the adaptive law as follows.

$$\begin{aligned}\dot{\boldsymbol{\theta}}_d &= 2\alpha k_g \operatorname{sgn}(g)\mathbf{e}^T\mathbf{P}\mathbf{B}_1\boldsymbol{\omega}_d + \beta_m\|\mathbf{e}\|_2\tilde{\boldsymbol{\theta}}_d \\ &= \alpha_m \operatorname{sgn}(g)\mathbf{e}^T\mathbf{P}\mathbf{B}_1\boldsymbol{\omega}_d + \beta_m\|\mathbf{e}\|_2\tilde{\boldsymbol{\theta}}_d, \end{aligned} \quad (18)$$

where $k_g > 0$ is an arbitrary constant to replace $|g|$, and $\alpha_m = 2\alpha k_g > 0$ is an extend adaptive constant. $\tilde{\boldsymbol{\theta}}_d$ is estimable, and the estimation method will be provided later. Substituting new adaptive law (18) into (16), we have

$$\begin{aligned}\dot{V} &= -\mathbf{e}^T\mathbf{Q}\mathbf{e} + 2\mathbf{e}^T\mathbf{P}\mathbf{B}\varepsilon_d - 2\mathbf{e}^T\mathbf{P}\mathbf{B}u_c \\ &\quad + (1/\alpha)\tilde{\boldsymbol{\theta}}_d^T(2\alpha\mathbf{e}^T\mathbf{P}\mathbf{B}\,\boldsymbol{\omega}_d - 2\alpha k_g \operatorname{sgn}(g)\mathbf{e}^T\mathbf{P}\mathbf{B}_1\boldsymbol{\omega}_d - \beta_m\|\mathbf{e}\|_2\tilde{\boldsymbol{\theta}}_d) \\ &= -\mathbf{e}^T\mathbf{Q}\mathbf{e} + 2\mathbf{e}^T\mathbf{P}\mathbf{B}\varepsilon_t - 2\mathbf{e}^T\mathbf{P}\mathbf{B}u_c - \beta_m\|\mathbf{e}\|_2\tilde{\boldsymbol{\theta}}_d^T\tilde{\boldsymbol{\theta}}_d, \end{aligned} \quad (19)$$

where $\varepsilon_t = \varepsilon_d + \tilde{\boldsymbol{\theta}}_d^T\boldsymbol{\omega}_d\{1-(k_g/|g|)\} \in L_\infty$ and is regarded as the total uncertainty. Now, the compensative controller is designed as (14). Substituting compensative controller (14) into (19) and after some algebraic manipulations, we have

$$\begin{aligned}\dot{V} &= -\mathbf{e}^T\mathbf{Q}\mathbf{e} - (\sqrt{2k_{c1}|g|}\,\mathbf{e}^T\mathbf{P}\mathbf{B}_1 - \frac{\varepsilon_t g}{\sqrt{2k_{c1}g}})^2 \\ &\quad + (\frac{g}{\sqrt{2k_{c1}|g|}})^2\varepsilon_t^2 - \beta_m\|\mathbf{e}\|_2\tilde{\boldsymbol{\theta}}_d^T\tilde{\boldsymbol{\theta}}_d. \end{aligned} \quad (20)$$

The following inequality is verified.

$$\dot{V} \le -\mathbf{e}^T\mathbf{Q}\mathbf{e} + \delta_o^2\varepsilon_t^2, \quad (21)$$

where $\delta_o = g_{up}/\sqrt{2k_{c1}g_{low}}$ with $g_{low} \le |g| \le g_{up}$. It can be found that the negative term $-\beta_m\|\mathbf{e}\|_2\tilde{\boldsymbol{\theta}}_d^T\tilde{\boldsymbol{\theta}}_d$ in (20) provides more negative edge for (21). By assuming $V(\mathbf{e}(0),\tilde{\boldsymbol{\theta}}_d(0)) \ge 0$, and we known $V(\mathbf{e}(\infty),\tilde{\boldsymbol{\theta}}_d(\infty)) \ge 0$ because $\varepsilon_t \in L_\infty$, inequality (21) is rewritten as

$$\int_0^\infty \mathbf{e}^T\mathbf{Q}\mathbf{e}\,dt \le \int_0^\infty \delta_o^2\varepsilon_t^2\,dt + \mathbf{e}(0)^T\mathbf{P}\mathbf{e}(0) + \frac{1}{2\alpha}\tilde{\boldsymbol{\theta}}_d^T(0)\tilde{\boldsymbol{\theta}}_d(0). \quad (22)$$

Due to $\mathbf{Q} = q\mathbf{I}$, inequality (22) becomes

$$\int_0^\infty \mathbf{e}^T\mathbf{e}\,dt \le q^{-1}(\int_0^\infty \delta_o^2\varepsilon_t^2\,dt + \mathbf{e}(0)^T\mathbf{P}\mathbf{e}(0) + \frac{1}{2\alpha}\tilde{\boldsymbol{\theta}}_d^T(0)\tilde{\boldsymbol{\theta}}_d(0)). \quad (23)$$

If $V(\mathbf{e}(0),\tilde{\boldsymbol{\theta}}_d(0)) = 0$, then following inequality is obtained.

$$\frac{\|\mathbf{e}\|_{L_2}^2}{\|\varepsilon_t\|_{L_2}^2} \le \delta^2\Big|_{V(\mathbf{e}(0),\tilde{\boldsymbol{\theta}}_d(0))=0}, \quad (24)$$

where $\delta = \sqrt{q^{-1}}(g_{up}/\sqrt{2k_{c1}g_{low}}) = \delta_o\sqrt{q^{-1}}$. It is easy to verify that $\sqrt{z_1^2 + z_2^2} \le (z_1 + z_2)$ is always true for any $z_1 > 0$ and $z_2 > 0$. If $V(\mathbf{e}(0),\tilde{\boldsymbol{\theta}}_d(0)) \ne 0$, then we have

$$\|\mathbf{e}\|_{L_2} \le \delta\|\varepsilon_t\|_{L_2} + \gamma_{bias}, \quad (25)$$

where $\gamma_{bias} = \sqrt{q^{-1}(\mathbf{e}(0)^T\mathbf{P}\mathbf{e}(0) + (1/2\alpha)\tilde{\boldsymbol{\theta}}_d^T(0)\tilde{\boldsymbol{\theta}}_d(0))}$. Now, we assume a suitable δ is used. Then the compensative controller leads $\lim_{t\to t_f}\{(u_c + u_d) - u^*\} \cong 0\Big|_{t_f<\infty}$ even thought $u_d^* \ne u^*$ (the fuzzy system is finite) and $\varepsilon_t \in L_\infty$. In other words, the tracking error is bounded densely around the origin. That implies $u_c \cong u^* - u_d = \tilde{\boldsymbol{\theta}}_d^T \boldsymbol{\omega}_d$. From [6], it is known that u_c gives a referable learning (approximate) error estimator for the finite fuzzy system based on the L_2-gain property so that $u_c \cong \tilde{\boldsymbol{\theta}}_d^T \boldsymbol{\omega}_d$ is guaranteed. Therefore, $\tilde{\boldsymbol{\theta}}_d$ is designed as

$$\tilde{\boldsymbol{\theta}}_d = \beta_m (u_c \boldsymbol{\omega}_d^{-1})^T , \tag{26}$$

where $\beta_m > 0$ is a scale constant. By substituting (26) into (18) and then (13) is obtained. Intuitively, the learning of the fuzzy system will be faster owing to the external learning information (26). Finally, the 2-norm of error vector ($\|\mathbf{e}\|_2$) of (13) is a simple adaptation scheme to enhance the learning stability.

Q.E.D.

IV. SIMULATIONS

In this section, two simulations are considered to demonstrate effectiveness of the proposed approach. The MATLAB function "ode45" is used to emulate the system dynamics with the time step size 0.01 (sec.). An often-used example, the inverted pendulum system, is considered and its model is described as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f + gu \end{aligned} ,$$

$$f = \frac{g_r \sin x_1 - \dfrac{m_t L x_2^2 \sin x_1 \cos x_1}{m_c + m_t}}{L\left(\dfrac{4}{3} - \dfrac{m \cos^2 x_1}{m_c + m_t}\right)} , \quad g = \frac{\dfrac{\cos x_1}{m_c + m_t}}{L\left(\dfrac{4}{3} - \dfrac{m \cos^2 x_1}{m_c + m_t}\right)} . \tag{27}$$

where $x_1(rad)$ is the angle of the pole, x_2 $(rad/\text{sec.})$ is the angular velocity of the pole and g_r is the acceleration due to gravity ($9.8 m/s^2$), m_c is the mass of the cart $(1.0kg)$, m_t is the mass of the pole $(0.1kg)$, and $L = 1$ (m) is the length of the pole. g is a continuous positive function and $0.6 < |g| < 1.5$ [12]. The initial states are $x_1(0) = 30(\text{deg.})$ and $x_2(0) = 0$. The fuzzy system consists of 25 fuzzy rules. The corresponding fuzzy sets and their membership functions are shown in Fig. 1. The initial parameter values are all set to zeros; that is, $\boldsymbol{\theta}_d(0) = 0$. The selected tracking trajectory is $y_m = \sin(t)$ (rad). $\mathbf{k} = [5 \quad 1]^T$ and $\mathbf{Q} = 10\mathbf{I}$ (q=10). These parameters are used in all simulations.

Notice: The adaptive rate α_m and learning error feedback term ($\beta_m \|\mathbf{e}\|_2 (u_c \boldsymbol{\omega}_d^{-1})^T$) of (13) are the targets for testing. It should be noted that in order to demonstrate the difference of the learning speed. The sense of the persistently exciting condition [9] is considered to ascertain the learning situation based on a learning performance index e_L, such as if the following learning condition (28) is satisfied, the learning target is achieved.

$$\|\mathbf{e}_{(i)}\|_{L_2} = \sqrt{\int_{(i-1)t_c}^{it_c} \mathbf{e}^T \mathbf{e} \, dt} < e_L \Big|_{i=1,2,\cdots} , \tag{28}$$

where t_c is the cyclic time of y_m. We let $e_L = 5 \times 10^{-3}$. The cyclic time of the tracking trajectory $y_m = \sin(t)$ (rad) is about 6.28 seconds, we have $t_c = 6.28$. The achievement time of the learning condition is the focus.

Simulation 1: The general control performances of Theorem 1 are showed in this simulation. These parameters $k_{c1} = 15$, $\alpha_m = 50$, and $\beta_m = 20$ are used. The simulation time is 50 seconds. Fig. 2 is the tracking control performance, and the final tracking error is small. Fig. 3 is the control action of the compensative controller (14). It is evident that the compensative control action is convergent and is not large in the final stage. Fig. 4 is the control action of the fuzzy system, which is also the learning result. Fig. 5 is the total control action. It can be found that the control action of the fuzzy system (Fig. 4) is similar to the control action (Fig. 5) so that the learning of the fuzzy system is successful. Fig. 6 is the learning results of $(\theta^1, \theta^2, \cdots, \theta^{25})$. The learning is stably convergent and the so-called parameter drifting problem is not obvious. Fig. 7 shows the L_2-norm of the error vector in each cycle and the condition $\|\mathbf{e}_{(i)}\|_{L_2} < e_L$ is reached at 18.84 seconds or at 3*th* learning cycle (i=3). The L_2-norm of the error vector is still bounded in further learning cycles.

Simulation 2: The learning speed tests are illustrated in this simulation. $k_{c1} = 15$ is still used. The other parameters are collected in Table 1. Among them, β_m will be changed according to whether the learning error feedback $\beta_m \|\mathbf{e}\|_2 (u_c \boldsymbol{\omega}_d^{-1})^T$ in (13) is used or not (if not, then the normal adaptive law is used). The value of α_m is also increased to illustrate the improvement of the learning speed. From the simulation results (Table 1), it is evident that the adaptive law with the learning error feedback has faster learning speed, and the reached cycle of each simulations are close even if the used adaptive rates are different. It can be found that the selection of the adaptive rate can be relaxed because the use of the novel adaptive law (13) and compensative controller (14).

V. DISCUSSIONS

Based on the finite L_2-gain property, a novel direct adaptive fuzzy control system design with a compensative controller and improved adaptive law is proposed. The compensative controller does not need the knowledge of the mathematical model of the plant, and the compensative controller provides a referable approximate error estimator for the fuzzy system. The property leads that a learning error feedback term is added into the adaptation scheme. As a result, not only the proposed approaches can provide faster learning speed but also the gain function (g) is removed in the adaptive law design. The adaptive rate does not affect the learning speed too much, that fits the requirement of the adaptive approximation based controls. Even if the learning rate is small, the learning performance condition is still reached quickly. The guarantee of the learning speed is a very important topic to intelligent control systems. The system stability and learning performance of the resultant control system are guaranteed in the study.

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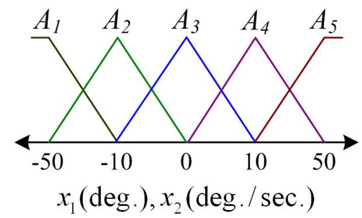

Figure 1. The used membership functions and fuzzy sets.

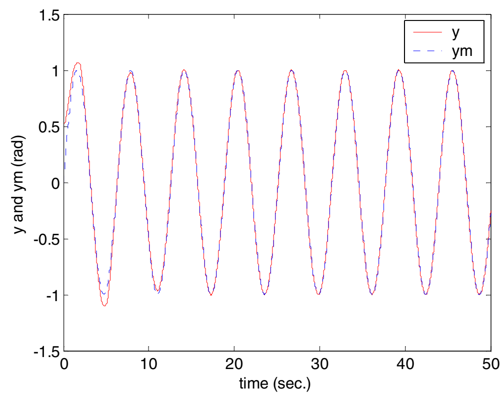

Figure 2. The tracking control performance.

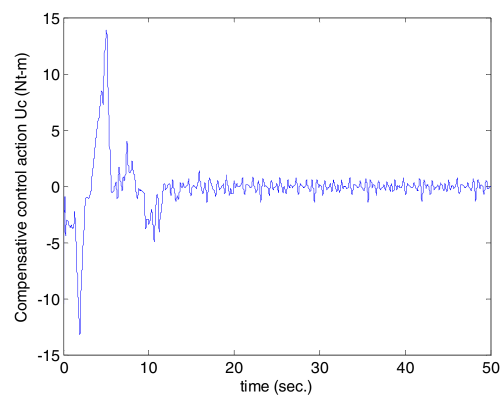

Figure 3. The compensative control action (u_c).

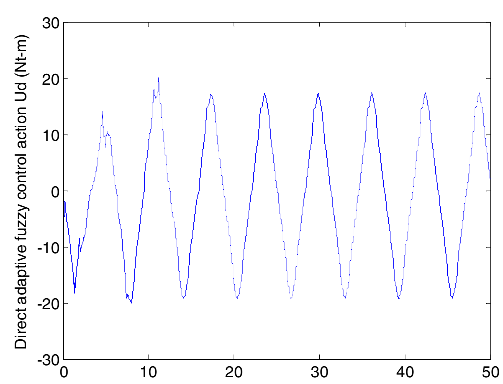

Figure 4. The control action of the direct adaptive fuzzy controller (u_d).

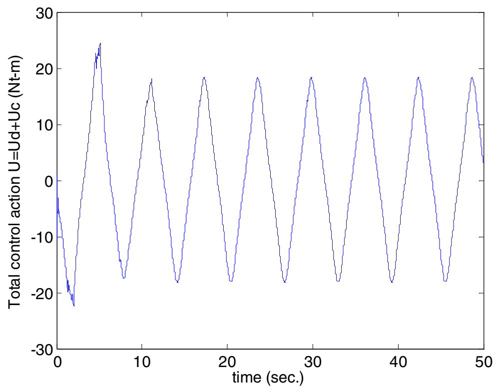

Figure 5. The total control action ($u = u_d + u_c$).

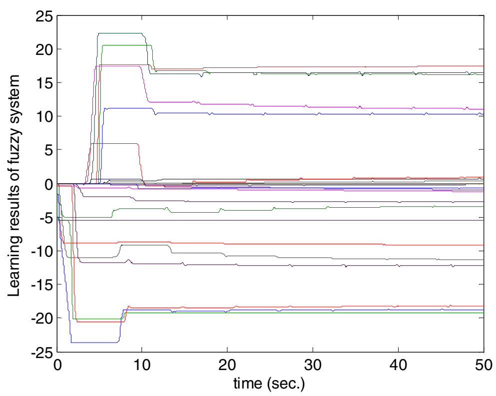

Figure 6. The learning results ($\boldsymbol{\theta}_d$) of the fuzzy system.

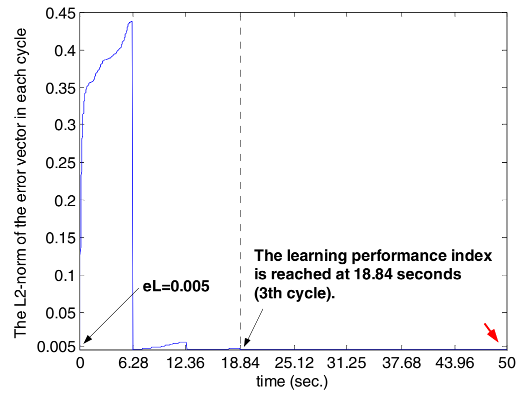

Figure 7. The L_2-norm of the error vector of each cycle.

TABLE I. THE RESULTS OF THE LEARNING SPEED TESTS IN SIMULATION 2

α_m	β_m	**Reached Time (sec.) and Cycle**
1	20	25.12 (4*th cycle*)
1	0	Unstable
10	20	18.84 (*3th cycle*)
10	0	Unstable
20	20	18.84 (*3th cycle*)
20	0	43.96 (*7th cycle*)
30	20	18.84 (*3th cycle*)
30	0	31.40 (*5th cycle*)
40	20	18.84 (*3th cycle*)
40	0	25.12 (*4th cycle*)
50	20	18.84 (*3th cycle*)
50	0	18.84 (*3th cycle*)