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A New Class of Petri Nets for Modeling and Control of Ratio-enforced Resource Allocation Systems

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***Abstract*—This paper presents a class of Petri nets that can well model ratio-enforced resource allocation systems (RASs). Such RASs are required to enforce certain ratio among their processes to meet a desired production plan. A Ratio-enforced System of Sequential Systems with Shared Resources (RS^4R) can model an RAS whose processes are described by state machines. Both a ratio-enforcing controller and liveness-enforcing supervisor must be designed to fulfil the deadlock-free and ratio-constrained production of products. We theoretically prove that their design can be separately performed.**

***Index Terms*—Petri nets; Resource allocation systems; Ratio enforcement; Liveness enforcement; Automated manufacturing system**

I. Introduction

Both ratio specification and liveness enforcement are of significance for a resource allocation system (RAS) [2], [6]–[8]. In the framework of Petri nets, their corresponding supervisors in the form of two sets of control places (monitors) can be derived to respectively implement these two kinds of requirement. This causes an interesting question: whether these two kinds of monitors can be separately designed without considering one's impact on the other, which in turn implies some significant reduction of computational burden.

In practice, different RAS is of different characteristics, which may result in the development of specific solutions to a specific class of RASs [4]. Thus, it is of significance to divide RASs into several categories such that their problems can be coped with independently. According to the taxonomy stated in [4], such categorization depends on: 1) whether an involved process is allowed to have a concurrent nature; 2) whether a process can contain flexible routes during its execution; and 3) whether multiple-resource acquisitions are allowed for each involved operation stage.

In [9], we address the design for ratio and liveness-enforcing supervisors by establishing a new Petri net class, namely Ratio-enforced weighted Augmented Marked Graphs (RAMGs). RAMG features with: 1) each job process allowing concurrent but not choice operations; 2) multiple-resource acquisitions for each operation stage; and 3) ratio enforcement among different processes. Evidently, such a net model does not allow flexible route in each of its processes. Motivated by the need to deal with flexible routes and the success of Systems of Simple Sequential Processes with Resources (S^3PR) and System of Sequential Systems with Shared Resources (S^4R) in modeling automated manufacturing systems, this work proposes and investigates another new class of Petri nets, i.e., Ratio-enforced Systems of Sequential Systems with Shared Resources (RS^4R). They can incorporate the most general resource allocation schemes except concurrent operations in each job process and be viewed to be complementary to RAMG. It is noted that S^4R is a class of general nets evolving from S^3PR in which only single resource can be utilized by each operation stage [3].

The rest of this paper is structured as follows. Section II gives the basic definitions and notations that are necessary for the modeling and analysis of the considered RAS. Section III formulates a Petri net class, and subsequently an algorithm through which an arbitrary ratio allocation among various processes can be implemented as a ratio-enforced net. Section IV analyzes the liveness of these net classes and its relation with the ratio-enforced net. Section V uses two examples to validate the legitimacy of our approach. Section VI concludes with the obtained results and directions for future work.

II. Definition and Notation

A Petri net is $N{=}(P,\ T,\ F,\ W)$ where P and T are finite, nonempty, and disjoint sets, $F{\subseteq}(P{\times}T){\cup}(T{\times}P)$. Specifically, P (resp., T, F) is the set of places (resp., transitions, directed arcs). $W{:}F{\rightarrow}\mathbb{N}^+$ is the weight function, where $\mathbb{N}^+{=}\{1,\ 2,\ \ldots\}$. If $(x,\ y){\notin}F$, $W(x,\ y){=}0$. The preset (resp., postset) of a node $x{\in}P{\cup}T$ is defined as ${}^\bullet x{=}\{y{\in}P{\cup}T|(y,\ x){\in}F\}$ (resp., $x^\bullet{=}\{y{\in}P{\cup}T|(x,\ y){\in}F\}$). $N{=}(P,\ T,\ F,\ W)$ is a state machine iff $W{:}F{\rightarrow}\{1\}$ and $|{}^\bullet t|{=}|t^\bullet|{=}1$. The input (resp., output) incidence matrix $[N^-]{=}\{a^-_{ij}\}$ (resp., $[N^+]{=}\{a^+_{ij}\}$) of the Petri net N are defined as $a^-_{ij}{=}W(p_i,\ t_j)$ (resp., $a^+_{ij}{=}W(t_j,\ p_i)$) when $(p_i,\ t_j){\subseteq}F$ (resp., $(t_j,\ p_i){\subseteq}F$); otherwise, $a^-_{ij}{=}0$ (resp., $a^+_{ij}{=}0$). The incidence matrix $[N]{=}\{a_{ij}\}$ of the Petri net N is defined as $[N]{=}[N^+]{-}[N^-]$. $[N_{p_j}]$(resp., $[N^-_{p_j}]$, $[N^+_{p_j}]$) is the j-th row of $[N]$ (resp., $[N^-]$, $[N^+]$).

A marking of N is a mapping $M{:}P{\rightarrow}\mathbb{N}$, where $\mathbb{N}{=}\{0\}{\cup}\mathbb{N}^+$. $(N,\ M_0)$ is a net system with an initial marking M_0. t is enabled, denoted by $M[t\rangle$, iff $\forall p{\in}{}^\bullet t$, $M(p){\geq}W(p,\ t)$. M' is reachable from M, denoted by $M[\sigma\rangle M'$, iff there exists a firing sequence $\sigma{=}{<}t_1\ldots t_n{>}$ so that $M[t_1\rangle M_1\ldots[t_n\rangle M'$. The firing sequence vector of σ is a $|T|$-dimensional vector,

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namely $\overrightarrow{\sigma}$, where $\overrightarrow{\sigma}(t)$ states the appearance count of t in σ. The set of all markings reachable from M_0 is denoted by $R(N, M_0)$. A place $p \in P$ is said to be k-bounded iff $M(p) < k$, $\forall M \in R(N, M_0)$, where k is a finite positive integer. N is k-bounded iff all its places are k-bounded. Furthermore, p is bounded iff it is k-bounded for certain k while N is bounded iff each place in P is bounded. Given (N, M_0), $t \in T$ is live under M_0 iff $\forall M \in R(N, M_0)$, $\exists M' \in R(N, M)$, $M'[t\rangle$ holds. (N, M_0) is live iff $\forall t \in T$, t is live.

Vector I becomes a P-semiflow iff $I > \mathbf{0}$ and $[N]^T \cdot I = \mathbf{0}$ holds. $\|I\| = \{p \in P | I(p) \neq 0\}$ is called the support of I. For economy of space, $\sum_{p \in P} M(p) \cdot p$ (resp., $\sum_{p \in P} I(p) \cdot p$) is used to denote vector M (resp., I), e.g., [1 2 0 0] is written as $p_1 + 2p_2$. (N, M_0) is pure iff $\forall (x, y) \in (P \times T) \cup (T \times P)$, $W(x, y) > 0 \Rightarrow W(y, x) = 0$.

A string $x_1, x_2, \ldots, x_n$ is called a path of N iff $\forall i \in \{1, 2, \ldots, n-1\}$, $x_{i+1} \in x_i^{\bullet}$. An elementary path is a path whose nodes are all different (except for, perhaps, x_1 and x_n), which is denoted by $EP(x_1, x_n)$. A path $x_1, \ldots, x_n$ is called a circuit iff it is an elementary path and $x_1 = x_n$.

A nonempty set $S \subseteq P$ is a siphon iff ${}^{\bullet}S \subseteq S^{\bullet}$ holds. A strict minimal siphon is a siphon containing no other siphon and P-semiflow. $M(p)$ indicates the number of tokens in p under M. p is marked by M iff $M(p) > 0$. The sum of tokens in S is denoted by $M(S)$, where $M(S) = \sum_{p \in S} M(p)$. A subset $S \subseteq P$ is marked by M iff $M(S) > 0$. A siphon is undermarked iff $\nexists t \in S^{\bullet}$ can fire.

III. Ratio-enforced RASs and Their Petri Net Modeling

An RAS considered in this work is assumed that flexible routes and multiple-resource acquisitions are allowed while concurrent operations in a job process are forbidden. Following [8], such an RAS can be partitioned into a set of resource types $\mathcal{R} = \{R_i, i = 1, 2, \ldots, L\}$ and a set of process types $\mathcal{J} = \{J_j, j = 1, 2, \ldots, K\}$. Every resource type R_i is further characterized by its capacity $C_i \in \mathbb{IN}^+$. Processing requirements of process type J_j are defined by a set of stages. Each process stage modeled by a place p_{jk} is associated with a conjunctive resource requirement, expressed by an L-dimensional vector $a_{p_{jk}}$ with $a_{p_{jk}}[i]$, $i = 1, 2, \ldots, L$, indicating how many units of resource R_i are required to support the execution of stage p_{jk}.

To model this category of RASs, the work in [1] proposes S^4R. For the self-completeness of this paper, its definition is included as follows.

Definition 1 *An S^4R is a connected generalized pure Petri net $N = (P, T, F, W)$ where: (1) $P = P_0 \cup P_A \cup P_R$ is a partition such that: a) P_0, P_A, and P_R are called idle, operation (or activity), and resource places, respectively; b) $P_0 = \cup_{i \in \mathbb{IN}_K} \{p_{0_i}\}$, where $\mathbb{IN}_K = \{1, 2, \ldots, K\}$; c) $P_A = \cup_{i \in \mathbb{IN}_K} P_{A_i}$, where for each $i \in \mathbb{IN}_K$, $P_{A_i} \neq \emptyset$, and for each $i, j \in \mathbb{IN}_K$, $i \neq j$, $P_{A_i} \cap P_{A_j} = \emptyset$; and d) $P_R = \{r_1, r_2, \ldots, r_l\}$, $l > 0$. (2) $T = \cup_{i \in \mathbb{IN}_K} T_i$, where for each $i \in \mathbb{IN}_K$, $T_i \neq \emptyset$, and for each $i, j \in \mathbb{IN}_K$, $i \neq j$, $T_i \cap T_j = \emptyset$. (3) For each $i \in \mathbb{IN}_K$, subnet $\overline{N}_i = N|(\{p_{0_i}\} \cup P_{A_i}, T_i)$ is a strongly connected state machine such that every cycle contains p_{0_i}. (4) For each $r \in P_R$, $\exists$ a unique minimal P-invariant $X \in \mathbb{IN}^{|P|}$ such that $\{r\} = \|X\| \cap P_R$, $P_0 \cap \|X\| = \emptyset$, $P_A \cap \|X\| \neq \emptyset$, and $X(r) = 1$. (5) $P_A = \cup_{r \in P_R} (\|X\| \setminus \{r\})$.*

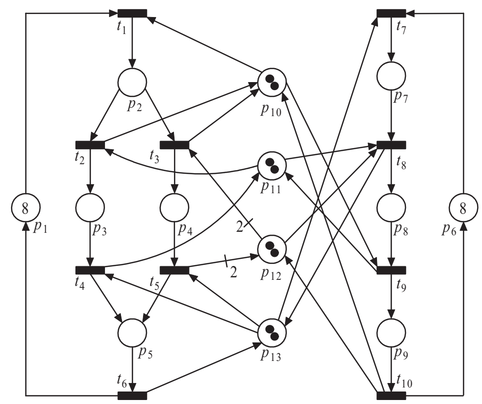


Fig. 1. An example S^4R.

The subsequent theoretical developments involve the notation of acceptable initial markings, formally defined as follows.

Definition 2 *Let $N = (P_0 \cup P_A \cup P_R, T, F, W)$ be an S^4R. (N, M_0) is said to be acceptably marked iff: 1) $M_0(p_0) \geq 1$, $\forall p_0 \in P_0$; 2) $M_0(p) = 0$, $\forall p \in P_A$; 3) $M_0(r) \geq X(p)$, $\forall r \in P_R$, $\forall p \in P_A$, where X is r's minimal P-semiflow as defined in Definition 1.*

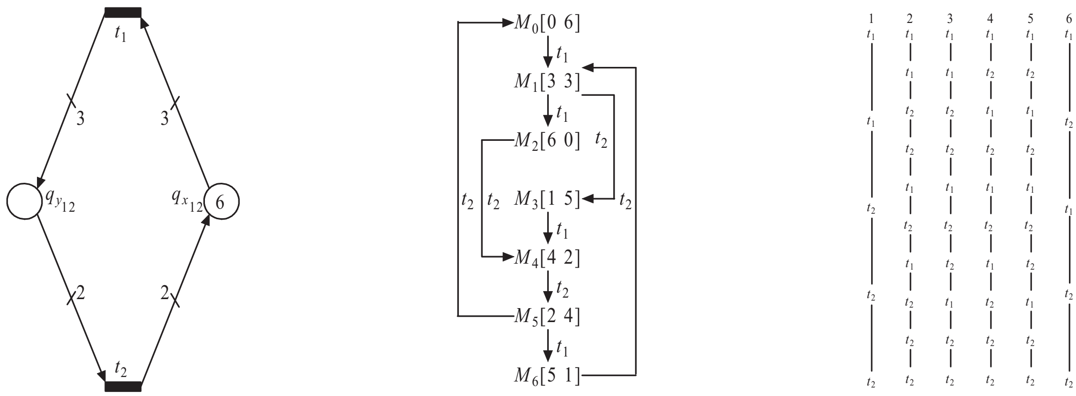
In the sequel, given an S^4R, $N = (P_0 \cup P_A \cup P_R, T, F, W)$, we denote $P = P_0 \cup P_A \cup P_R$, where P_0, P_A, and P_R are called idle, operation (or activity), and resource places, respectively.

Evidently, an S^4R N is composed of K state machines $\overline{N}_i$, $i \in \mathbb{IN}_K$, which are connected via resource places in P_R. An acceptably marked S^4R ensures that any product can be produced if all resources are dedicated to its production. In the sequel, when mentioning an S^4R, we assume it is acceptably marked.

With the aforementioned notions, we are now ready to approach our problem of enforcing ratios among different processes. Suppose that $h_i \in \mathbb{IN}^+$, $i \in \mathbb{IN}_K$ are K mutually primary integers, i.e., their largest common divisor is one. For example, 4, 3, and 2 are mutually primary while 4, 8, and 2 are not as they have 2 as their largest common divisor. We are required to enforce ratios $h_1{:}h_2{:}\ldots{:}h_K$ among K processes.

Algorithm 1 ([9]) *Ratio Enforcement Net (REN) Synthesis*

1) Input: Enforce ratios $h_1{:}h_2{:}\ldots{:}h_K$ among K processes;
2) Output: An REN $(P_Q, T_Q, F_Q, W_Q, M_0)$;
3) Begin
4) $P_Q = \{q_{x_{12}}, q_{x_{23}}, \ldots, q_{x_{(K-1)K}}, q_{y_{12}}, q_{y_{23}}, \ldots, q_{y_{(K-1)K}}\}$;
5) $T_Q = \{t_1^0, t_2^0, \ldots, t_K^0\}$;
6) $F_Q = \{(q_{x_{i(i+1)}}, t_i^0), (t_i^0, q_{y_{i(i+1)}}), (q_{y_{i(i+1)}}, t_{i+1}^0), (t_{i+1}^0, q_{x_{i(i+1)}}), i \in \mathbb{IN}_{K-1}\}$;



(a) (b) (c)

Fig. 2. Illustrations of the ratio-enforced constraint.

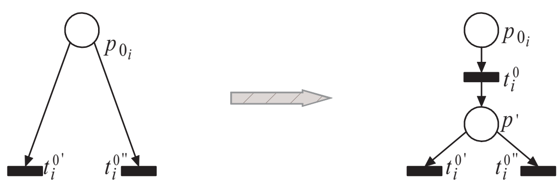


Fig. 3. Transformation from multiple source transitions to a single one.

7) $W_Q(q_{x_{i(i+1)}},\ t_i^0)=W_Q(t_i^0,\ q_{y_{i(i+1)}})=h_{i+1}$, $W_Q(q_{y_{i(i+1)}},\ t_{i+1}^0)=W_Q(t_{i+1}^0,\ q_{x_{i(i+1)}})=h_i$;
8) $M_0(q_{x_{i(i+1)}})=h_i{\cdot}h_{i+1}$, and $M_0(q_{y_{i(i+1)}})=0$;
9) End

By assuming $t_1^0=t_1$ and $t_2^0=t_2$, Fig. 7(a) gives an example to synthesize an REN that enforces 2:3 between t_1 and t_2. Its reachability graph is shown in Fig. 7(b) which comprises six circuits passing over M_0, as indicated in Fig. 2(c). Obviously, in each firing circuit, the ratio between the occurrence counts of t_1 and t_2 is exactly 2:3.

The following definition is used to compose two Petri nets through their common transitions.

Definition 3 *A Petri net $N=(P,\ T,\ F,\ W)$, obtained by composing two Petri nets $\{N_i|N_i=(P_i,\ T_i,\ F_i,\ W_i),\ i\in\{1,\ 2\}$, where $P_1\cap P_2=\emptyset$, $T_1\cap T_2\neq\emptyset$, and $F_1\cap F_2=\emptyset\}$, denoted by $N=N_1\circ N_2$, through their common transitions is a Petri net such that: $P=P_1\cup P_2$, $T=T_1\cup T_2$, $F=F_1\cup F_2$, $W(p,\ t)=W_i(p,\ t)$ if $(p,\ t)\in F_i$, and $W(t,\ p)=W_i(t,\ p)$ if $(t,\ p)\in F_i$.*

Definition 4 *A Petri net $N=N_1\circ N_2$ is an RS^4R iff :*
1) *N_1 is an S^4R such that $t_i^0=p_{0_i}^{\bullet}$, $i\in\mathbb{N}_K$;*
2) *N_2 is an REN derived according to Algorithm 1 given ratios $h_1{:}h_2{:}\ldots{:}h_K$.*

The definition indicates that an RS^4R is obtained as the composition of one RS^4R and one REN. It can be easily proved that the above defined RS^4R is bounded. As a convention, t_i^0 is called source transition of $\overline{N}_i$. Without loss of generality, we assume that each $\overline{N}_i$ has only one source transition. This assumption makes sense thanks to the fact that multiple source transitions in $\overline{N}_i$ can be conveniently transformed to a single one, as shown in Fig. 3. In the sequel, it is reasonable to suppose that any $\overline{N}_i$ has only one source transition.

Definition 5 *Let $N=N_1\circ N_2$ be an RS^4R, where N_1 and N_2 are an S^4R and REN, respectively. Then, $(N,\ M_0)$ is said to be acceptably marked RS^4R if $(N_1,\ M_{0_1})$ is acceptably marked by M_{0_1} and $(N_2,\ M_{0_2})$ is initially marked by M_{0_2} as defined in Algorithm 1.*

Given an RS^4R with $N=(P_0\cup P_A\cup P_R\cup P_Q,\ T,\ F,\ W)$, P is referred to $P_0\cup P_A\cup P_R\cup P_Q$, where P_Q is called a ratio place set.

Thanks to its special structure, an RS^4R can describe RASs in which flexible routes and multiple-resource acquisitions are allowed, respectively. Specifically, it is composed of a set of subsets $\overline{N}_i$, $i\in\mathbb{N}_K$, which has one-to-one correspondence with a process that allows sequential operations. More specifically, each $\overline{N}_i$ can be decomposed into an acyclic graph and an idle place p_{0_i}. The operations together with their interactions required by a process are represented by the operation places and transitions involved in the respective acyclic graph of $\overline{N}_i$. The initial marking of p_{0_i} corresponds to the number of type i process instances, e.g., products, that are allowed in the system at a time. As a convention, p_{0_i} is also designated as the final destination of all finished process instances so that a repetitive process is well modeled. Places in P_R are used to model various resource types in an RAS. Their marking during the evolution of a Petri net corresponds to the number of available resources in the modeled system. In particular, their initial

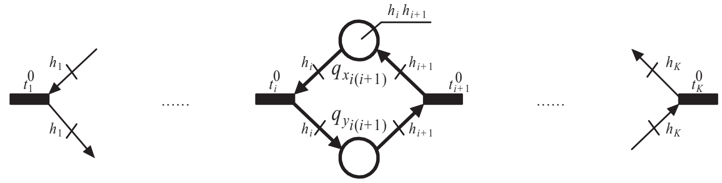


Fig. 4. An Illustration for the Proof of Theorem 1.

markings define the capacities of the corresponding resource types. In such RASs, different processes proceed independently but must follow a given ratio.

IV. Separability of Ratio and Liveness Enforcements

Liveness is one of the most important behavioral properties, which corresponds to the absence of global and local deadlocks. In many cases, deadlock can be attributed to the presence of some undermarked siphons. Since the number of siphons in a Petri net increases exponentially, a challenge is to reduce their number to be considered, when possible. In this section, we prove that the liveness of an RS^4R can be established without considering its REN. We show that an RS^4R is live when all the siphons in its corresponding S^4R are max-controlled. For the self-completeness of this paper, we cite the following definitions [2]. In the sequel, when talking about a siphon, we mean a strict minimal one.

Definition 6 ([2]) *A siphon S is said to be max-marked at $M \in R(N,\ M_0)$ iff $\exists p \in S$ such that $M(p) \geq max_{p^\bullet}$, where $max_{p^\bullet} = max_{t \in p^\bullet} W(p,\ t)$.*

Definition 7 ([2]) *A siphon S is said to be max-controlled iff S is max-marked at any reachable marking, i.e., $\forall M \in R(N,\ M_0)$, $\exists p \in S$ such that $M(p) \geq max_{p^\bullet}$.*

Definition 8 ([2]) *$(N,\ M_0)$ is said to be satisfying the max-controlled-siphon property (max cs-property, for short) iff each siphon of $(N,\ M_0)$ is max-controlled.*

Proposition 1 ([2]) *Suppose S is a siphon. If there exists a P-invariant I such that $\forall p \in (\|I\|^- \cap S)$, $max_{p^\bullet} = 1$, $\|I\|^+ \subseteq S$, and $\sum_{p \in P} I(p) \cdot M_0(p) > \sum_{p \in S} I(p) \cdot (max_{p^\bullet} - 1)$, then S is max-controlled.*

Theorem 1 *The firing ratio among $t_1^0 - t_K^0$ in REN is $h_1 : h_2 : \ldots : h_K$ and REN is bounded and live.*

Proof: Given an REN as composed by Algorithm 1, it is trivial that the vector $I = \mathbf{1}$ is a P-semiflow. This consequently leads to $M(q_{x_{12}}) + M(q_{x_{23}}) + \ldots + M(q_{x_{(K-1)K}}) + M(q_{y_{12}}) + M(q_{y_{23}}) + \ldots + M(q_{y_{(K-1)K}}) = M_0(q_{x_{12}}) + M_0(q_{x_{23}}) + \ldots + M_0(q_{x_{(K-1)K}}) + M_0(q_{y_{12}}) + M_0(q_{y_{23}}) + \ldots + M_0(q_{y_{(K-1)K}}) = \sum_{i=1}^{K-1} h_i \cdot h_{i+1} < +\infty$. Therefore, $\forall i \in \mathbb{N}_{K-1}$, $M(q_{x_{i(i+1)}}) < +\infty$ and $M(q_{y_{i(i+1)}}) < +\infty$. We can conclude that REN is bounded.

To prove the liveness of an REN, we first prove the liveness of its each subnet as shown in bold lines in Fig. 3. As known, the sum of markings in $q_{x_{i(i+1)}}$ and $q_{y_{i(i+1)}}$ remain constant such that $M(q_{x_{i(i+1)}}) + M(q_{y_{i(i+1)}}) = h_i \cdot h_{i+1}$. Then, two cases must be considered. Initially, we have $M_0(q_{x_{i(i+1)}}) = h_i \cdot h_{i+1}$ and $W(q_{x_{i(i+1)}},\ t_i^0) = h_i$. Since $h_{i+1} \geq 1$, we have $M_0(q_{x_{i(i+1)}}) \geq W(q_{x_{i(+1)}},\ t_i^0)$. Thus, t_i^0 can fire under the initial marking. In the case t_i^0 fires several times, the two following cases must be considered.

Case 1: $M(q_{i(i+1)}) \geq h_i$. Apparently, t_i^0 remains fireable in the case.

Case 2: $M(q_{i(i+1)}) \leq h_i - 1$. Since $M(q_{y_{i(i+1)}}) = h_i \cdot h_{i+1} - M(q_{x_{i(i+1)}})$, the minimum value for $M(q_{y_{i(i+1)}})$ is $M(q_{y_{i(i+1)}}) = h_i \cdot h_{i+1} - (h_i - 1) = h_{i+1}(h_i - (h_i - 1)/h_{i+1})$. This obviously leads to $M(q_{y_{i(i+1)}}) \geq h_{i+1}$ since $h_i \geq 1$ and $h_{i+1} \geq 1$. Thus, t_{i+1}^0 can fire under this case.

Therefore, we know that any similar subnet in an REN is live. The liveness of the REN can subsequently be verified by contradictively assuming the existence of a dead transition t_i^0 under a marking M. Since t_i^0 is dead, we must have $M(q_{y_{(i-1)i}}) < W(q_{y_{(i-1)i}},\ t_i^0)$ and $M(q_{x_{i(i+1)}}) < W(q_{x_{i(i+1)}},\ t_i^0)$ hold at $\forall M \in R(N,\ M_0)$. This contradicts our above discussion. Thus, we can conclude that REN is live. ■

Subsequently, we prove that the composition of an REN and S^4R cannot lead to a siphon or the constituted siphon cannot be undermarked.

Theorem 2 *If S is a newly composed siphon in RS^4R by the composition of an REN and S^4R with $S \cap P_Q \neq \emptyset$, $S \cap P_A \neq \emptyset$, $S \cap P_R \neq \emptyset$, we have that S cannot be a siphon that can be undermarked.*

Proof: (by sketch) According to Theorem 1, $S \not\subseteq P_Q$ since otherwise S cannot be a strict minimal siphon. Obviously, we have $S \cap P_A \neq \emptyset$ and $S \cap P_R \neq \emptyset$ hold. To prove that S cannot be undermarked, two cases should be considered separately.

Case 1: $\nexists \overline{N}_i$ where $\exists p \in P_i$ such that $|p^\bullet| \geq 2$. To constitute S with $S \cap P_Q \neq \emptyset$, $S \cap P_A \neq \emptyset$, and $S \cap P_R \neq \emptyset$, a circuit $\mathcal{C}$ containing elements from the sets P_Q, P_A, and P_R must be established owing to the structural property of RS^4R. These involved resource places together with their adjacent operation places can either lead to a strict minimal P-invariant or a strict minimal siphon. Neither of these two situations can imply that S is a strict minimal siphon.

Case 2: $\exists \overline{N}_i$ where $\exists p \in P_i$ such that $|p^\bullet| \geq 2$. Assume that there are two paths beginning from p, which are denoted by l^j and l^k, where $j \neq k$. To establish S, a circuit $\mathcal{C}$ must be constituted with a path from $p_{0_i}^\bullet$ to ${}^\bullet p_{0_i}$ among which l^j is

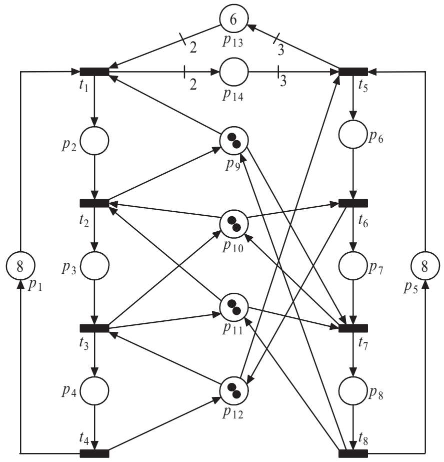


Fig. 5. An RS^4R with no flexible routes.

supposed to be involved. If a place p in $\mathcal{C}$ has an outgoing transition belonging to l^k, where $j \neq k$, a siphon is constituted since $^{\bullet}S \subset S^{\bullet}$. Nevertheless, S can neither be strict nor minimal since either a P-invariant or strict minimal siphon can be introduced by related $p_r \subseteq P_R$ together with their corresponding operation places. ■

Theorems 1 and 2 are of significance since we only need to consider siphons constituted by operation and resource places while ignoring all the others, i.e., P_0 and P_Q. This proves the principle of separation, i.e., ratio control and liveness-enforcing supervision of RS^4R can be performed separately without affecting each other.

Theorem 3 *Let (N, M_0) be a marked RS^4R be a marked net composed of an REN and S^4R (N', M_0'). If each siphon in (N', M_0') is max-controlled, (N, M_0) is live.*

Since no new undermarked siphon can be induced owing to the composition between REN and S^4R. To derive a live RS^4R, only the siphons in S^4R need to be controlled. Thus, Theorem 3 evidently holds. Note that only few methods, e.g., [2], [6], exist to derive the needed control places for the proposed net classes. Others, e.g., [3], do not work without non-trivial extensions.

V. Synthesis Examples

This section contains some synthesis examples, through which the effectiveness of the synthesis algorithm and the correctness of the theoretical results are verified. We show two examples, among which the first one corresponds to Case 1 considered in the proof of Theorem 2 while the other one corresponds to Case 2. For brevity and simplicity, both systems involve four resource types $R_1 - R_4$ and can support two job types J_1 and J_2. The ratio between their two process flows corresponding to J_1 and J_2 is 3:2.

A. Example 1—An RS^4R with no Flexible Routes

The example is shown in Fig. 5, which describes an RAS whose process flows can be described by state machines with no flexible routes. This corresponds to Case 1 since no operation place has more than two succeeding transitions. Job type J_1 (resp., J_2) is defined by the set of strictly ordered job stages $\{p_2 - p_4\}$ (resp., $\{p_6 - p_8\}$). The conjunctive resource requirements associated with various job stages are as follows: a_{p_2}=[1 0 0 0]^T, a_{p_3}=[0 1 1 0]^T, a_{p_4}=[0 0 0 1]^T, a_{p_6}=[0 0 0 1]^T, a_{p_7}=[0 1 0 0]^T, and a_{p_8}=[1 0 1 0]^T. The P-semiflows corresponding to the system resources are: $X_1 = p_2 + p_8 + p_9$, $X_2 = p_3 + p_7 + p_{10}$, $X_3 = p_3 + p_8 + p_{11}$, and $X_4 = p_4 + p_6 + p_{12}$. This system involves four siphons, which are: $S_1 = \{p_4, p_8, p_{10}, p_{11}, p_{12}\}$, $S_2 = \{p_4, p_7, p_{10}, p_{12}\}$, $S_3 = \{p_4, p_8, p_9, p_{10}, p_{12}\}$, and $S_4 = \{p_3, p_8, p_9, p_{10}\}$. Since none of $S_1 - S_4$ contains p_{13} or p_{14} as a member, no siphon can be attributed to the composition of the specific RS^4R and REN.

B. Example 2—An RS^4R with Flexible Routes

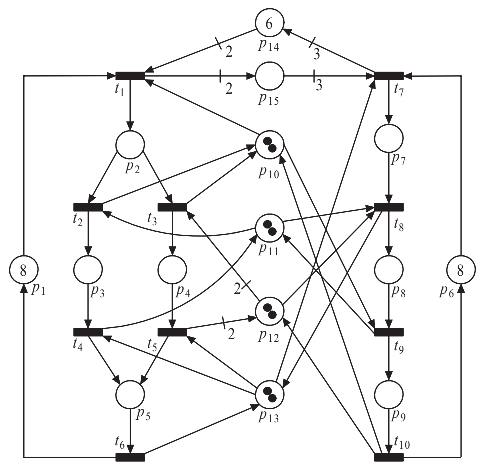
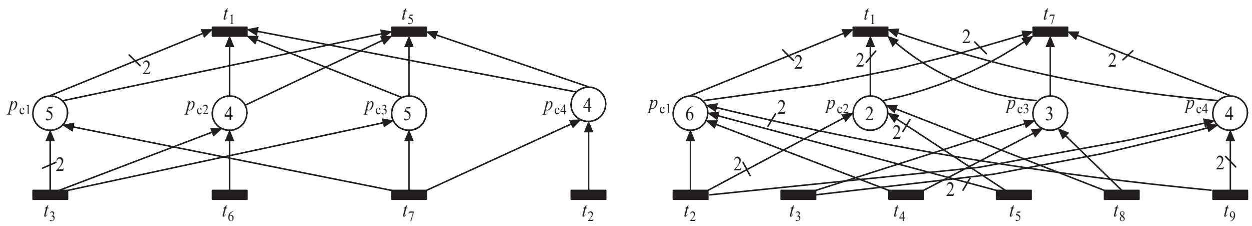


Fig. 6. An RS^4R with flexible routes.

The example is shown in Fig. 6, which describes an RAS whose process flows have flexible routes, i.e., there exists an operation place, say p_2, from which several paths can be introduced. Moreover, for p_5, we have $|^{\bullet}p_5|=2$ while $^{\bullet\bullet}p_5 \cap P_R = \{p_{13}\}$. This corresponds to Case 2 in the proof of Theorem 2. Job type J_1 (resp., J_2) is defined by $\{p_2 - p_5\}$ (resp., $\{p_7 - p_9\}$). The resource requirements are as follows: a_{p_2}=[1 0 0 0]^T, a_{p_3}=[0 1 0 0]^T, a_{p_4}=[0 0 2 0]^T, a_{p_5}=[0 0 0 1]^T, a_{p_7}=[0 0 0 1]^T, a_{p_8}=[0 1 1 0]^T, and a_{p_9}=[1 0 1 0]^T. The resource-related P-semiflows are: $X_1 = p_2 + p_9 + p_{10}$, $X_2 = p_3 + p_8 + p_{11}$, $X_3 = 2p_4 + p_8 + p_9 + p_{12}$, and $X_4 = p_5 + p_7 + p_{13}$. There are four siphons: $S_1 = \{p_5, p_9, p_{10}, p_{11}, p_{12}, p_{13}\}$, $S_2 = \{p_5, p_8, p_9, p_{12}, p_{13}\}$, $S_3 = \{p_5, p_8, p_{11}, p_{13}\}$, and $S_4 = \{p_3, p_4, p_9, p_{10}, p_{11}, p_{12}\}$. As anticipated in Theorem 2, none of $S_1 - S_4$ contains p_{14} or p_{15} as a member. Thus, no new siphon is introduced owing to the composition of RS^4R and REN.



(a) Supervisor for Example 1.

(b) Supervisor for Example 2.

Fig. 7. The liveness-enforcing supervisors for the Petri nets in Figs. 5 and 6.

C. Synthesis of Liveness-enforcing Supervisor for the two examples

According to Theorem 3, a live RS^4R can be obtained when all the siphons in its corresponding S^4R are max-controlled. Such a problem is well investigated in [2] as shown by Proposition 1, which states that a siphon S can be max-controlled by a monitor denoted by p_c. Subsequently, we review some results on how p_c can be obtained to control S in an S^4R.

For any S^4R, we have $S{=}S_A{\cup}S_R$, where $S_A{=}S{\cap}P_A$ and $S_R{=}S{\cap}P_R$. $\exists$ P-invariant I_S and monitor p_c such that S is max-controlled if $\sum_{p\in P} I_S(p){\cdot}M_0(p){>}\sum_{p\in S} I_S(p){\cdot}(max_{p^\bullet}-1)$, where P includes all places including p_c. For $r{\in}S_R$, there exists a minimal P-semiflow $X{\in}\mathbb{N}^{|P|}$ such that $\{r\}{=}\|X\|{\cap}P_R$, $P_0{\cap}\|X\|{=}\emptyset$, $P_A{\cap}\|X\|{\neq}\emptyset$, and $X(r){=}1$. We have $g_S{=}\sum_{r\in S_R} X$ is a P-invariant and $\|g_S\|\backslash S{\subset}P_A$. A new place, namely p_c, is introduced so as to produce a new P-semiflow h_S with $\forall p{\in}\|g_S\|\backslash S$, $h_S(p){=}g_S(p)$, $h_S(p_c){=}1$, and $\|h_S\|\backslash\{p_c\}{\subseteq}P_A$. Considering that g_S and h_S are both P-invariants, we can conclude that $I_S{=}g_S{-}h_S$ is also a P-invariant. Due to their special constructions, we have $\|I_S\|^+{=}\|g_S\|\backslash\|h_S\|{=}S$, $\|I_S\|^-{=}\{p_c\}$, $\|I_S\|^-{\cap}S{=}\emptyset$.

For instance, given $S_1{=}\{p_4, p_8, p_{10}, p_{11}, p_{12}\}$ in Example 1, we have $g_S{=}2p_3{+}p_4{+}p_6{+}p_7{+}p_8{+}p_{10}{+}p_{11}{+}p_{12}$, $h_S{=}2p_3{+}p_6{+}p_7{+}p_c$, $I_S{=}g_S{-}h_S{=}p_4{+}p_8{+}p_{10}{+}p_{11}{+}p_{12}{-}p_c$. Since $\|I_S\|^-{\cap}S{=}\emptyset$ and $\|I_S\|^+{\cap}S{=}S$, we can conclude S is max-controlled when $\sum_{p\in P} I_S(p){\cdot}M_0(p){>}\sum_{p\in S} I_S(p){\cdot}(max_{p^\bullet}{-}1)$. Evidently, $\sum_{p\in P} I_S(p){\cdot}M_0(p){=}M_0(p_4){+}M_0(p_8){+}M_0(p_{10}){+}M_0(p_{11}){+}M_0\ (p_{12}){-}M_0(p_c){=}1{\times}0{+}1{\times}0{+}1{\times}2{+}1{\times}2{+}1{\times}2{-}M_0(p_c){=}6{-}M_0(p_c)$. $\sum_{p\in S} I_S(p){\cdot}(max_{p^\bullet}{-}1){=}0$. Thus, $6{-}M_0\ (p_c){>}0$, or equivalently, $M_0(p_c){<}6$, which leads to $M_0(p_c){=}5$. Furthermore, following the method in [3], the outgoing arcs of a monitor derived using [2] are adjusted to the corresponding $t{\in}p_{0_i}^{\bullet}$ so as to structurally simplify the resultant supervisor. By conducting the above derivation process repeatedly, we can obtain the supervisors for Examples 1 and 2 as shown in Fig. 7.

VI. Conclusion

In this paper, we have studied deadlock prevention issue in ratio-enforced resource allocation systems (RASs). In the considered RAS, (a) flexible routes are allowed in any job processes; (b) each process can request and acquire multi-resources; and (c) desired ratios are required among products to be produced. These properties lead to a special Petri net class that is never treated systematically in the literature to the best knowledge of the authors. We prove that the ratio and liveness-enforcing supervisions can be separately performed. In other words, the design of ratio and liveness enforcing supervisors can be executed independently. Its application scope needs to be extended to more general Petri classes such as G-system as well as systems with uncontrollable and unobservable transitions. Another challenging issue is to design supervisory controllers such that the throughput of an RAS with either deterministic or stochastic time variables is optimized while the implementation cost for them is kept minimum.

References

[1] I. B. Abdallah and H. A. ElMaraghy, "Deadlock prevention and avoidance in FMS: a Petri net based approach," *International Journal of Advanced Manufaturing Technology*, vol. 14, no. 10, pp. 704–715, Sep. 1998.

[2] K. Barkaoui and J. F. Peyre, "On liveness and controlled siphons in Petri nets," in *Proceedings of International Conference on Applcaction and Theory of Petri nets*, J. Billington and W. Reisig eds., Berlin: Springer-Verlag, June 1996, vol. 1091, *Lecture Notes in Computer Science*, pp. 57–72.

[3] J. Ezpeleta, J. M. Colom, and J. Martínez, "A Petri net based deadlock prevention policy for flexible manufacturing systems," *IEEE Transactions on Robotics and Automation*, vol. 11, no. 2, pp. 173–184, Apr. 1995.

[4] J. Ezpeleta and R. Valk, "A polynomial deadlock avoidance method for a class of nonsequential resource allocation systems," *IEEE Transactions on Systems, Man, and Cybernetics-Part A*, vol. 36, no. 6, pp. 1234–1243, Nov. 2006.

[5] B. Hruz and M. C. Zhou, *Modeling and Control of Discrete Event Dynamic Systems*, Springer, NYC, NY, 2007.

[6] Z. W. Li and M. Zhao, "On controllability of dependent siphons for deadlock prevention in generalized Petri nets," *IEEE Transactions on Systems, Man, and Cybernetics-Part A*, vol. 38, no. 2, pp. 369–384, Mar. 2008.

[7] H. Ohl, H. Camus, E. Castelain, and J. C. Gentina, "Petri net modeling of ratio-driven flexible manufacturing systems and implications on the WIP for cyclic schedules," in: *Proceedings of IEEE International Conference on Systems, Man, and Cybernetics*, Vancouver, Canada, Oct. 1995, pp. 3081–3086.

[8] J. Park and S. A. Reveliotis, "Deadlock avoidance in sequential resource allocation systems with multiple resource acquisitions and flexible routings," *IEEE Transactions on Automatic Control*, vol. 46, no. 10, pp. 1572–1583, Oct. 2001.

[9] H. S. Hu, M. C. Zhou, and Z. W. Li, "Ratio and Supervisory Control for Automated Manufacturing Systems Using Petri Nets," Technical Report, Laboratory of Discrete Event Systems, ECE, NJIT, 2009.