

Gain-Scheduling Control of a Teleoperation System

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Abstract—This paper presents a control design framework for robust teleoperation. The teleoperation system with uncertainties is modeled as a linear parameter varying plant to employ the gain-scheduling control framework. The time-varying transmission delay, disturbances, and uncertainty of the environment are considered in the design of the control. The usual assumption that the human operator and the environment are passive is not made in this method. The control scheme employs an online estimation of the environment including disturbance and transmission time delay between master and slave. Simulation results show that position and force tracking errors are small, and the system shows stable behavior in spite of the uncertainties. A transparency analysis in the frequency domain shows that the developed control scheme guarantees perfect transparency in the low frequencies.

Keywords—teleoperation, gain-scheduling control, robust stability, transparency

I. INTRODUCTION

Uncertainty, difference between a model and the reality [1], makes it difficult to design control of teleoperation systems. Transmission time delay, nonlinearities and time-varying properties are the main uncertainties of the teleoperation systems. Stability robustness is also important when there are uncertainties in the system. But the uncertainties have not been considered much in the previous researches on the teleoperation systems. The human operator and environment are usually assumed passive or linear time-invariant (LTI). Time delays are assumed constant or neglected as well as disturbances.

Stability of the teleoperation system has been checked using absolute stability criterion or passivity condition [2]–[7]. The absolute stability and passivity, however, assume that the human operator and the environment are passive. Moreover, the disturbance is difficult to consider in this control framework. Therefore, a potential instability exists due to active behaviors of human operator or environment and unconsidered disturbance.

It is well known that the transmission time delay may destabilize the teleoperation system. There is a passive control law with constant time delay [5], [7]. However, the

transmission time delay is not a constant. In addition, the transmission time delay from master to slave may differ from the time delay from slave to master. Although there are recent approaches that time delays are random time-varying, still the robust control design is needed to consider lots of uncertainties of the system.

H_∞ control and μ -synthesis framework have been employed to deal with the disturbance and uncertainty of the teleoperation system [8]–[12]. The uncertain region of interest is, however, treated as an uncertainty block, and it is often infeasible to obtain a stationary controller satisfying both the stability and the performance objectives. Therefore, there is a need of less conservative controller while considering disturbance and uncertainty of the system.

For linear parameter varying (LPV) plants, the gain-scheduling control provides a reasonable compromise between stability robustness and performance [13]–[15]. The gain-scheduling control also provides the control design framework considering the disturbance. However, the framework can be applied with the LPV plants. In this paper, the uncertain teleoperation system is modeled as a LPV plant. The parameters are known or computed in real-time. There are already few studies applying the gain-scheduling control framework to the teleoperation systems. However, the uncertainty of the whole teleoperation system is not the main concern of those papers. Time delay and motion scaling factor are discussed in [16], [17], respectively.

In this paper, time-varying delay, disturbances, and time-varying properties or incorrect models of the environment are considered as the uncertainty of the system. The usual assumptions that the human operator and the environment are passive are not made in this gain-scheduling control framework. The disturbances to the system are muscle force of human operator and environment disturbance force. To obtain the environment parameters, online estimation is employed. The estimation error is also considered as a disturbance. The transmission time delay varies randomly in certain ranges and known in real-time. Time delay is linearized with Pade approximation model. Although recent studies do not use the approximation model of time delay [18], [19] and some studies handle stochastic nonlinear teleoperation system [20]–[22], the control design of uncertain teleoperation system is beyond their goal. Moreover, performance-optimized stable controller is automatically obtained in this paper.

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II. GAIN-SCHEDULING CONTROL

In this paper, the gain-scheduling control framework using linear fractional transformation is adapted to design robust controller of the teleoperation system. There are some preliminaries of the gain-scheduling control [14], [15].

A linear time varying plant can be governed by following state space equations.

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t) \end{aligned} \quad (1)$$

where $A(t)$, $B(t)$, $C(t)$, and $D(t)$ are state-space matrices. $x(t)$, $u(t)$, and $y(t)$ are state, control input, and measured outputs, respectively. If the state-space matrices are functions of parameter vector, $\theta(t)$, then (1) is governed as following.

$$\begin{aligned} \dot{x}(t) &= A(\theta(t))x(t) + B(\theta(t))u(t) \\ y(t) &= C(\theta(t))x(t) + D(\theta(t))u(t) \end{aligned} \quad (2)$$

Then, a gain-scheduling controller depend on the $\theta(t)$ is formulated with

$$\begin{aligned} \dot{x}_k(t) &= A_k(\theta(t))x_k(t) + B_k(\theta(t))y(t) \\ u(t) &= C_k(\theta(t))x_k(t) + D_k(\theta(t))y(t) \end{aligned} \quad (3)$$

Linear fractional transformations (LFT) on the time varying parameters are used to gain-scheduling controller synthesis [14], [15]. Linear parameter varying plants with a linear fractional dependence on $\theta(t)$ can be represented by the upper LFT interconnection.

$$\begin{bmatrix} q \\ y \end{bmatrix} = F_u(P(s), \Theta) \begin{bmatrix} w \\ u \end{bmatrix} \quad (4)$$

where w , u , q , and y are exogenous inputs, control inputs, controlled outputs and measured outputs, respectively. $P(s)$ is a known LTI plant and Θ is block diagonal time-varying operator specifying how θ enters the plant dynamics.

There are some restrictions of LPV systems where the parameter dependence is affine and the time varying parameter θ varies in a polytope θ of vertices w_1, w_2, \dots, w_r ; that is,

$$\theta \in \Theta := \text{Co}\{w_1, w_2, \dots, w_r\} \quad (5)$$

LPV plants mapping w and u to q and y are considered.

$$\begin{aligned} \dot{x} &= A(\theta)x + B_1(\theta)w + B_2(\theta)u, \\ q &= C_1(\theta)x + D_{11}(\theta)w + D_{12}(\theta)u, \\ y &= C_2(\theta)x + D_{21}(\theta)w + D_{22}(\theta)u. \end{aligned} \quad (6)$$

There are three assumptions to obtain a gain-scheduling controller [14].

$$(A1) \quad D_{22}(\theta) = 0,$$

(A2) $B_2(\theta)$, $C_2(\theta)$, $D_{12}(\theta)$ and $D_{21}(\theta)$ are parameter independent,

(A3) the pairs $(A(\theta), B_2)$ and $(A(\theta), C_2)$ are quadratically stabilizable and quadratically detectable over $B_2(\theta)$, respectively.

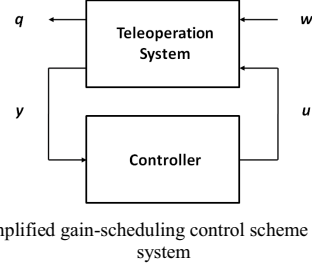


Figure 1. Simplified gain-scheduling control scheme for teleoperation system

(A3) is satisfied if and only if there exist two symmetric matrices (R, S) in satisfying the system of $2r+1$ LMIs.

$$\begin{aligned} & \begin{pmatrix} N_R & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} A_i R + R A_i^T & R C_{ii}^T & B_{ii} \\ C_{ii} R & -\gamma I & D_{ii} \\ B_{ii}^T & D_{ii}^T & -\gamma I \end{pmatrix} \begin{pmatrix} N_R & 0 \\ 0 & I \end{pmatrix} < 0, \quad i=1, \dots, r, \\ & \begin{pmatrix} N_S & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} A_i^T S + A_i S & S B_{ii} & C_{ii}^T \\ B_{ii}^T S & -\gamma I & D_{ii}^T \\ C_{ii} & D_{ii} & -\gamma I \end{pmatrix} \begin{pmatrix} N_S & 0 \\ 0 & I \end{pmatrix} < 0, \quad i=1, \dots, r, \\ & \begin{pmatrix} R & I \\ I & S \end{pmatrix} \geq 0. \end{aligned} \quad (7)$$

where N_R and N_S are denote bases of the null spaces of (B_{ii}^T, D_{ii}^T) and (C_{ii}, D_{ii}) , respectively. From these LMIs, the LPV controller is obtained [14], [15].

From the preliminaries, the gain-scheduling control scheme of the teleoperation system is simplified as shown in Fig. 1. The teleoperation system must be modeled with a LPV plant, which parameter dependence is affine and (A1) – (A3) are satisfied, to obtain a gain-scheduling controller.

III. MODEL OF THE TELEOPERATION SYSTEM

A. Master, Slave, and Human Operator

In this paper, the master device and slave robot are modeled as a one degree-of-freedom LTI model. The degree-of-freedom of the master and slave are easily extended if it is modeled as LTI. The nonlinearity of the device is neglected to simplify the problem. To deal with the nonlinearity of the device, disturbance observer method or adaptive control schemes can be applied to the system. It is, however, beyond our scope. The master device and slave robot are modeled with lumped mass and damping coefficients.

$$Z_m = M_m s + B_m \quad (8)$$

$$Z_s = M_s s + B_s \quad (9)$$

Z_m and Z_s stand for impedances of the master and the slave devices. M_m and M_s are inertia coefficients of master and slave devices. B_m and B_s are damping coefficients of master and slave devices, respectively. Although the human operator has highly nonlinear dynamics and unmodeled muscular and neural systems, it can be simplified with LTI passive dynamics and active muscle force and is generally used in many haptic interfaces. F_{op} means active muscle force from the human operator. Z_h is the passive human arm impedance and it can be generally modeled as a LTI second-order model. Eventually,

the operator generates F_m which is the active force command minus the passive force.

$$F_m = F_{op} - Z_h \dot{x}_m . \quad (10)$$

$$Z_h \dot{x}_m = M_{op} \ddot{x}_m + B_{op} \dot{x}_m + K_{op} x_m . \quad (11)$$

where M_{op} , B_{op} and K_{op} are mass, damping and stiffness coefficients of the human operator, respectively.

B. Environment

The interaction between slave robot and environment is actually nonlinear and time-varying in nature. There are many ways to describe the interaction between slave robot and environment, in example, the interaction can be modeled as a sum of nominal model and uncertainty. In this paper, we choose the method of [23]. The authors of [23] used first-order linear model with time-varying parameters to illustrate the insertion force between soft tissue and needle.

$$F_s = k(t)x_s + \lambda(t)\dot{x}_s . \quad (12)$$

where $k(t)$ and $\lambda(t)$ are time-varying coefficients of damping and stiffness, respectively. Using only time-varying stiffness and damping coefficients, the interaction force F_s is well described. Moreover, the time-varying parameters are estimated in real-time. An estimation error term is added to Barbe's model to consider the effects of the estimation error F_{est} . The disturbance force is always in the environment, F_{dist} is included as well. The environment is not passive anymore.

$$F_s = K_{env}x_s + B_{env}\dot{x}_s + F_{dist} = K_e(t)x_s + B_e(t)\dot{x}_s + F_{est} + F_{dist} . \quad (13)$$

$B_e(t)$ and $K_e(t)$ are estimated value of the environment coefficients at particular time t . The real coefficients of the environment are B_{env} and K_{env} . Recursive least squares algorithm similar with that of [23] is used for the estimation process. It is assumed that the estimation error F_{est} and F_{dist} are bounded. The sum of F_{est} and F_{dist} is represented as F_{err} .

C. Communication Channel

As shown in Fig. 2, the transmission time delay from master to slave and from slave to master are $T_{d1}(t)$ and $T_{d2}(t)$, respectively. In this paper, random time-varying delays within a certain region are considered. $T_{d1}(t)$ and $T_{d2}(t)$ are assumed around zero. If the time delay is around zero, a classical Pade approximation represents the property of time delay very well. Therefore, the model of the transmission time delays in communication channels are described with first order Pade approximation. $TD1$ and $TD2$ are used as time delay models in this paper.

$$TD1 = e^{-T_{d1}(t)s} \cong \frac{1 - (T_{d1}(t)s/2)}{1 + (T_{d1}(t)s/2)} . \quad (14)$$

$$TD2 = e^{-T_{d2}(t)s} \cong \frac{1 - (T_{d2}(t)s/2)}{1 + (T_{d2}(t)s/2)} . \quad (15)$$

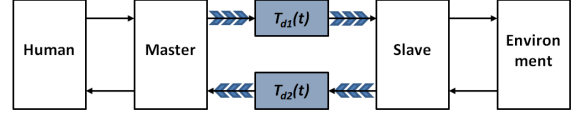


Figure 2. Transmission time delay in communication channel between master and slave

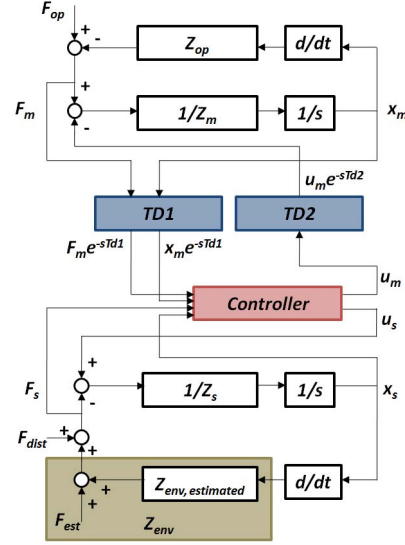


Figure 3. Gain-scheduling control scheme for teleoperation system.

IV. GAIN-SCHEDULING CONTROL OF THE TELEOPERATION SYSTEM

The control architecture of the teleoperation system is shown in Fig. 3. Both position and force information of master and slave are used as inputs of a controller similar with general four-channel (4C) control architecture. The controller is located at slave because the gain-scheduling controller requires the parameters, coefficients of environment and transmission time delays, at every moment. Inputs of the controller from master are delayed due to $TD1$ and controller output to the master is also delayed by $TD2$. In this gain-scheduling control scheme, controller outputs affect the dynamics of master and slave as shown below.

$$\dot{x}_m = \frac{1}{Z_m} (F_m - u_m e^{-sT_{d2}(t)}) . \quad (16)$$

$$\dot{x}_s = \frac{1}{Z_s} (u_s - F_s) . \quad (17)$$

where u_m and u_s are control signals from the controller. The actuator nonlinearities such as friction and backlash are not included for the sake of simplicity. From (8) to (17), following equations are obtained.

$$F_m = F_{op} - (M_{op}\ddot{x}_m + B_{op}\dot{x}_m + K_{op}x_m) . \quad (18)$$

$$F_s = B_e(t)\dot{x}_s + K_e(t)x_s + F_{err} . \quad (19)$$

$$F_m - u_m e^{-sT_{d2}(t)} = M_m \ddot{x}_m + B_m \dot{x}_m . \quad (20)$$

$$u_s - F_s = M_s \ddot{x}_s + B_s \dot{x}_s . \quad (21)$$

The measured outputs, which are the inputs of the controller, are $F_m e^{-sT_{d1}(t)}$, $x_m e^{-sT_{d1}(t)}$, F_s and x_s .

The teleoperation system is formulated with state-space equation. The exogenous inputs to the system, w , are the active human force and the sum of environment force estimation error and disturbance force. And the control inputs, u , are the actuator torques of master and slave. Based on the four kinds of information which are used as controller inputs, the measured outputs y are represented as follows.

$$w^T = [F_{op} \quad F_{err}] . \quad (22)$$

$$u^T = [u_m \quad u_s] . \quad (23)$$

$$y^T = [F_m e^{-sT_{d1}(t)} \quad x_m e^{-sT_{d1}(t)} \quad F_s \quad x_s] . \quad (24)$$

The delayed signals are formulated with Pade approximation of time delay as follows.

$$F_m e^{-sT_{d1}(t)} = \frac{1 - (T_{d1}(t)s/2)}{1 + (T_{d1}(t)s/2)} F_m . \quad (25)$$

New assumptions are brought that $T_{d1}(t)$ and $T_{d2}(t)$ are not equal to zero. Then, the equation (25) is formulated with new state X_{F_m} as follows.

$$\dot{X}_{F_m} = -\frac{2}{T_{d1}(t)} X_{F_m} + F_m . \quad (26)$$

$$F_m e^{-sT_{d1}(t)} = \frac{4}{T_{d1}(t)} X_{F_m} - F_m . \quad (27)$$

The other delayed signals $x_m e^{-sT_{d1}(t)}$ and $u_m e^{-sT_{d2}(t)}$ are also formulated in similar way. The time-varying parameters are arranged as parameter vector θ . The parameters include the estimated coefficients and time delays.

$$\theta(t)^T = \left[B_e(t) \quad K_e(t) \quad \frac{1}{T_{d1}(t)} \quad \frac{1}{T_{d2}(t)} \right] . \quad (28)$$

For the description of the teleoperation system, the overall system, without the controller, is presented with a state-space equation. The state vector X has seven states, i.e.,

$$X^T = [X_m \quad X_m \quad \dot{X}_s \quad X_s \quad X_{F_m} \quad X_{x_m} \quad X_{u_m}] . \quad (29)$$

The state-space equations of the teleoperation system with time-varying parameters are obtained. Each component of the state-space equation is described in the Appendix in detail.

$$\dot{X} = A(\theta)X + B_1 w + B_2 u . \quad (30)$$

The control objectives of the teleoperation system are taken as position and force tracking errors. To prevent infinitely large control actions, the penalty functions must be included as similar with μ -synthesis case [12]. The controlled outputs q are also formulated with X , w and u :

$$q = \begin{bmatrix} x_m - x_s \\ F_m - F_s \\ u_m \\ u_s \end{bmatrix} = C_1(\theta)X + D_{11}w + D_{12}u . \quad (31)$$

Weighting factors are multiplied to the controlled outputs before designing the controller.

$$W \cdot q = \begin{bmatrix} W_p \\ W_f \\ W_m \\ W_s \end{bmatrix}^T \cdot \begin{bmatrix} x_m - x_s \\ F_m - F_s \\ u_m \\ u_s \end{bmatrix} . \quad (32)$$

The measured signals, y , are written as:

$$y = \begin{bmatrix} F_m e^{-sT_{d1}(t)} \\ x_m e^{-sT_{d1}(t)} \\ F_s \\ x_s \end{bmatrix} = C_2(\theta)X + D_{21}w + D_{22}u . \quad (33)$$

Filters of the control inputs u and measured outputs y to satisfy the assumptions (A1) and (A2) are designed. The filters have to be designed to provide enough bandwidth of the signals. The state space equation is computed after filtering the signals as written below.

$$\begin{aligned} \dot{X}_u &= A_u X_u + B_u \tilde{u}, \\ u &= C_u X_u \\ \dot{X}_y &= A_y X_y + B_y y, \\ \tilde{y} &= C_y X_y \end{aligned} \quad (34)$$

$$\begin{pmatrix} \dot{X} \\ \dot{X}_u \\ \dot{X}_y \end{pmatrix} = \begin{pmatrix} A(\theta) & B_2 C_u & 0 \\ 0 & A_u & 0 \\ B_y C_2(\theta) & B_y D_{22} C_u & A_y \end{pmatrix} \begin{pmatrix} X \\ X_u \\ X_y \end{pmatrix} + \begin{pmatrix} B_1 \\ 0 \\ B_y D_{21} \end{pmatrix} w + \begin{pmatrix} 0 \\ B_u \\ 0 \end{pmatrix} \tilde{u},$$

$$q = (C_1(\theta) \quad D_{12} C_u \quad 0) \begin{pmatrix} X \\ X_u \\ X_y \end{pmatrix} + D_{11} w, \quad (35)$$

$$\tilde{y} = (0 \quad 0 \quad C_y) \begin{pmatrix} X \\ X_u \\ X_y \end{pmatrix}.$$

To simplify the above equation, (35) is rewritten as follows.

$$\begin{aligned} \dot{X}' &= A_f(\theta)X' + B_{f1}w + B_{f2}\tilde{u}, \\ q &= C_{f1}(\theta)X' + D_{f11}w, \\ \tilde{y} &= C_{f2}X'. \end{aligned} \quad (36)$$

The teleoperation system is expressed a convex combination of polytope as the parameter dependence of the teleoperation system is affine.

$$\begin{pmatrix} A_f(\theta) & B_{f1} & B_{f2} \\ C_{f1}(\theta) & D_{f11} & 0 \\ C_{f2} & 0 & 0 \end{pmatrix} \in P := Co \left\{ \begin{pmatrix} A_{fi} & B_{f1} & B_{f2} \\ C_{fi} & D_{f11} & 0 \\ C_{f2} & 0 & 0 \end{pmatrix}, i=1, \dots, 16 \right\} . \quad (37)$$

The gain-scheduling controller of the system is obtained if there is R and S satisfying LMIs given below.

$$\begin{pmatrix} N_R & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} A_{f1}R + RA_{f1}^T & RC_{f1}^T & B_{f1} \\ C_{f1}R & -\gamma I & D_{f1} \\ B_{f1}^T & D_{f1}^T & -\gamma I \end{pmatrix} \begin{pmatrix} N_R & 0 \\ 0 & I \end{pmatrix} < 0, \quad i = 1, \dots, 16,$$

$$\begin{pmatrix} N_S & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} A_{f2}S + SA_{f2}^T & SB_{f2} & C_{f2}^T \\ B_{f2}^T S & -\gamma I & D_{f2}^T \\ C_{f2} & D_{f2} & -\gamma I \end{pmatrix} \begin{pmatrix} N_S & 0 \\ 0 & I \end{pmatrix} < 0, \quad i = 1, \dots, 16, \quad (38)$$

$$\begin{pmatrix} R & I \\ I & S \end{pmatrix} \geq 0.$$

where N_R and N_S are denote bases of the null spaces of $(B_{f2}^T, 0)$ and $(C_{f2}, 0)$, respectively.

If there is no time delay between master and slave, the control scheme is perfectly matched with 4C control architecture. The relationships between gain-scheduling and 4C control schemes are as shown in below.

$$controller = \begin{bmatrix} -C_6 & C_m & -C_2 & -C_4 \\ C_3 & C_1 & -C_5 & -C_s \end{bmatrix}. \quad (39)$$

Although the control schemes are different because of the time delay, the relationships between inputs and outputs are same. Therefore, $C_m, C_s, C_1, \dots, C_6$ are used to formulate the transparency equation. The transparency is generally described with the transmitted impedance to the operator, Z_{to} . In this paper, Z_{to} and Z_e represent the relationship between F_m and x_m and between F_s and x_s , respectively. Then, F_m is formulated with transmitted impedance Z_{to} and sensitivity function to the disturbance S_{dist} .

$$F_m = Z_{to}x_m + S_{dist}F_{dist}. \quad (40)$$

$$Z_w = \frac{Z_m \{(1+C_3)Z_e + Z_{cs}\} + \{(C_m Z_{cs} - C_1 C_4) + Z_e(C_m + C_3 C_m - C_1 C_2)\}e^{-sT}}{\{(1+C_3)Z_e + Z_{cs}\} + \{(C_e Z_{cs} + C_3 C_4) + Z_e(C_6 + C_5 C_6 + C_2 C_3)\}e^{-sT}}. \quad (41)$$

$$S_{dist} = \frac{\{(1+C_3)C_4 - C_2 Z_{cs}\}e^{-sT_{d2}}}{\{(1+C_3)Z_e + Z_{cs}\} + \{(C_e Z_{cs} + C_3 C_4) + Z_e(C_6 + C_5 C_6 + C_2 C_3)\}e^{-sT}}. \quad (42)$$

The transparency of the system will be analyzed by comparison between Z_{to} and Z_e after controller synthesis.

V. SIMULATION

Simulation is conducted using MATLAB. The nominal model of the environment is given with $B_e(0)$ and $K_e(0)$. The real coefficients of the environment, however, are set as B_{env} and K_{env} . The differences between the coefficients are considered as the uncertainty of the environment. Therefore, the parameters of the environment are estimated and changed during the simulation in real-time. The time delays are given as functions of time t and the parameters are known at every moment. The gain-scheduling controller is computed with MATLAB LMI Control Toolbox [24]. After the controller synthesis, numerical simulations are conducted with MATLAB Simulink.

The master and slave devices are taken to be commercially available PHANTOM haptic devices. However, the extremely low damping coefficients of master and slave cause oscillatory behavior. It, therefore, needs more study, but the behavior is very similar with wave reflection as mentioned in [7]. A damping is added to master and slave to prevent the oscillation.

TABLE I. PARAMETERS USED IN SIMULATIONS

Parameter	Value	Parameter	Value
M_{op}	0.15 kg	M_s	0.072 kg
B_{op}	4.8 Ns/m	B_s	0.5 Ns/m
K_{op}	600 N/m	$B_e(0)$	20 Ns/m
M_m	0.072 kg	$K_e(0)$	100 N/m
B_m	0.5 Ns/m	B_{env}	25 Ns/m
		K_{env}	180 N/m

TABLE II. WEIGHTING FACTORS

Weighting factor	Value	Weighting factor	Value
W_p	20/(s+0.1)	W_m	1/50
W_f	0.04/(s+0.1)	W_s	1/50

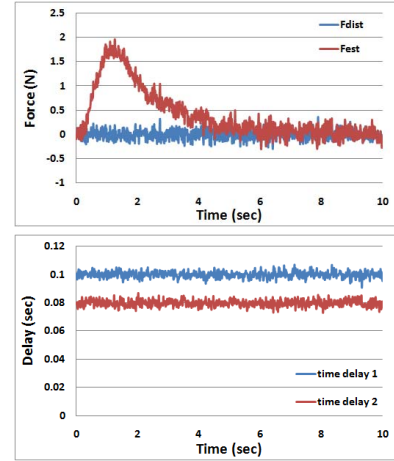


Figure 4. (a) Estimation error F_{est} and disturbance force F_{dist} . (b) Time delays between master and slave, $T_{d1}(t)$ and $T_{d2}(t)$.

Therefore, the damping coefficients of master and slave are increased from 0.005 Ns/m to 0.5 Ns/m. Table I show the parameters of human operator, master and slave which are obtained from [25] except the damping coefficients of master and slave. Time-varying parameters of the environment are also set as shown in Table I. The environment parameter variation ranges are chosen such that these sufficiently cover the parameter variation ranges of needle insertion cases in [23]. F_{est} , the estimation error, and F_{dist} , the disturbance force which is given as a Gaussian white noise, are obtained using simulations as shown in Fig. 4 (a). And F_{op} is modeled as 50t N till 0.5 sec and 25 N after 0.5 sec.

The time-varying delay between master and slave is represented by the sum of a constant and Gaussian white noise. $T_{d1}(t)$ and $T_{d2}(t)$ are randomly varied with the range from 0.06 sec to 0.12 sec as shown in Fig. 4 (b). Perfect tracking performance cannot be achieved because there is time delay between master and slave. However, tracking performance can be met in the low frequency range in which the time delay effect is negligible. The weighting factors of position and force tracking errors can be chosen as low frequency filters using this characteristic. The weighting factor for the penalty functions of high control inputs are set as the reciprocal of actuator

saturation value. The weighting factors used in the simulation are shown in Table II. The force and position tracking performances are shown in Fig. 5. Position and force tracking errors are very small although there are random time-varying delays, disturbances, and incorrect models of environment coefficients. The stability of the master and slave is maintained in the simulation.

To compute the Z_{to} and S_{dist} , approximation methods of the controllers are needed to reduce the orders of the controller. Routh model reduction technique is used in this paper [26]. Each controller is reduced to fifth order. The time delay between master and slave is modeled with third order Pade approximation. To draw the Bode plots of Z_{to} and Z_e according to the variation of K_{env} , the other parameters are set as constants. $T_{d1}(t)$, $T_{d2}(t)$ and B_{env} are set as 100ms, 80ms and 5Nsec/m, respectively. The effects of the environment estimation are omitted in this analysis. The Bode plots of Z_{to} and Z_e are obtained with various K_{env} . As shown in Fig. 6, the transmitted impedance and environment impedance are perfectly matched in low frequency ranges under about 0.4 Hz. However, the designed controller cannot achieve perfect transparency because of the time delay.

In this paper, tracking results of the transparency-optimized wave-based control scheme of [4] is compared with the results of the developed gain-scheduling control scheme. The comparison between different control schemes is difficult because the results are very sensitive to the controller being used and the guidelines of the control design are abstract in general. In this comparison, the controllers are same with [5]. However, b is chosen as 1 because M_s/M_m equals to one. f_{cut} is chosen as 4 Hz which shows best simulation results with previous selections of controllers. To show the adaptive nature of each control scheme, K_{env} suddenly changes 180 N/m to 500 N/m at five seconds after the start of the simulation. To observe only the effect of the control scheme, the environment estimation in gain-scheduling control scheme is omitted in this simulation. F_{dist} , F_{op} , $T_{d1}(t)$ and $T_{d2}(t)$ are same with previous simulation. As shown in Fig. 7, both control scheme show stable and transparent teleoperation. However, the tracking results of the gain-scheduling control scheme shows fast response.

VI. CONCLUSION

A design framework is developed for robust control of the teleoperation system with uncertainties. The time-varying transmission delays, disturbances, and time-varying properties or incorrect models of the environment are considered as the uncertainties of the system. The teleoperation system with uncertainties is modeled as a LPV plant to obtain the robust controller.

New control scheme is developed to employ the gain-scheduling control framework. The control scheme employs an online estimation of the environment parameters. The robust stability is maintained in spite of the uncertainty of the environment and disturbances. Simulation results show that the position and force tracking errors are very small and the system shows stable behavior. The transparency of the system is also analyzed in the frequency domain. It is, however, necessary to explore the oscillatory behavior with small damping

coefficients in the master and the slave. Experiments in the real environment will be also included in the future work.

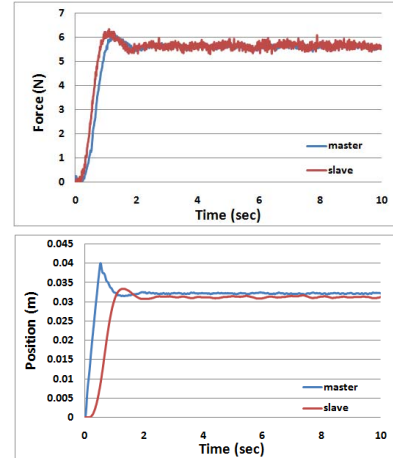


Figure 5. (a) Force tracking performance. (b) Position tracking performance.

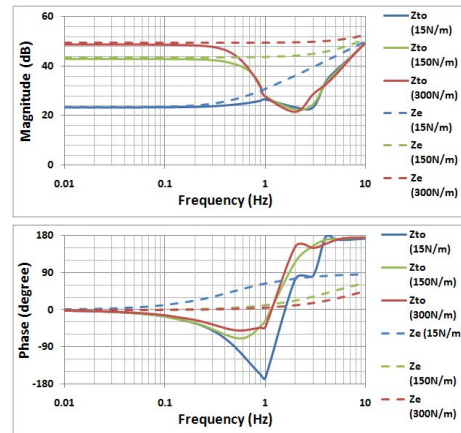


Figure 6. Bode plots of transmitted impedance and environment with various stiffness coefficients.

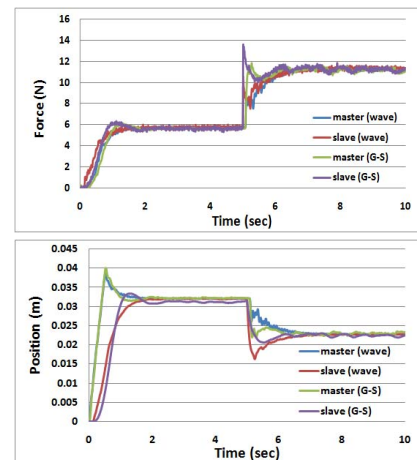


Figure 7. (a) Force tracking performance. (b) Position tracking performance.

APPENDIX

$$A(\theta) = \begin{bmatrix} \frac{B_m + B_{op}}{M_m + M_{op}} & \frac{K_{op}}{M_m + M_{op}} & 0 & 0 & 0 & 0 & -\frac{4\theta_4}{M_m + M_{op}} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{B_s + \theta_1}{M_s} & \frac{\theta_2}{M_s} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{M_{op}B_m - M_mB_{op}}{M_m + M_{op}} & -\frac{M_mK_{op}}{M_m + M_{op}} & 0 & 0 & -2\theta_3 & 0 & \frac{4M_{op}\theta_4}{M_m + M_{op}} \\ 0 & 1 & 0 & 0 & 0 & -2\theta_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2\theta_4 \end{bmatrix},$$

$$B_1^T = \begin{bmatrix} \frac{1}{M_m + M_{op}} & 0 & 0 & 0 & \frac{M_m}{M_m + M_{op}} & 0 & 0 \\ 0 & 0 & -\frac{1}{M_s} & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_2^T = \begin{bmatrix} \frac{1}{M_m + M_{op}} & 0 & 0 & 0 & -\frac{M_m}{M_m + M_{op}} & 0 & 1 \\ 0 & 0 & \frac{1}{M_s} & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$C_1(\theta) = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ \frac{M_{op}B_m - M_mB_{op}}{M_m + M_{op}} & -\frac{M_mK_{op}}{M_m + M_{op}} & -\theta_1 & -\theta_2 & 0 & 0 & \frac{4M_{op}\theta_4}{M_m + M_{op}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$D_{11}^T = \begin{bmatrix} \frac{M_m}{M_m + M_{op}} & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, D_{12}^T = \begin{bmatrix} 0 & -\frac{M_m}{M_m + M_{op}} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$C_2(\theta) = \begin{bmatrix} \frac{M_{op}B_m - M_mB_{op}}{M_m + M_{op}} & \frac{M_mK_{op}}{M_m + M_{op}} & 0 & 0 & 4\theta_3 & 0 & 0 & -\frac{4M_{op}\theta_4}{M_m + M_{op}} \\ 0 & -1 & 0 & 0 & 0 & 4\theta_3 & 0 & 0 \\ 0 & 0 & \theta_1 & \theta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$D_{21}^T = \begin{bmatrix} -\frac{M_m}{M_m + M_{op}} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, D_{22}^T = \begin{bmatrix} \frac{M_m}{M_m + M_{op}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

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