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Robust Control for Robot Manipulators By Using Only Joint Position Measurements

Shafiqul Islam and Peter X. Liu
Department of Systems and Computer Engineering
Carleton University, Ottawa, Ontario, Canada, K1S 5B6.

***Abstract*— In this work, we propose an output feedback sliding mode control (SMC) method for trajectory tracking of robotic manipulators. The design process has two steps. First, we design a stable SMC approach by assuming that all state variables are available for feedback. Then, an output feedback version of this SMC design is presented which incorporates a model-free linear observer to estimate unknown velocity signals. We then show that the tracking performance under output feedback design can asymptotically recover the performance achieved under state feedback based SMC design. A detailed stability analysis is given which shows semi-global uniform ultimately boundedness property of all the closed loop signals. The proposed method is implemented and evaluated on a 2-DOF robotic system to illustrate the effectiveness of the theoretical development.**

KEY WORDS: Robotics, Output Feedback, Adaptive Control.

I. Introduction

Over the past decades, the sliding mode robot control technique has been extensively studied by many researchers, (see for example [1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 23], to name a few). Most SMC designs reported in the literature, however, assume that velocity signals are available for feedback control design. In practice, it is very difficult to meet this assumption as advanced industrial robotic systems provide the user with only joint position measurements while the velocity sensors are removed from the system to reduce the cost and weight. In fact, the position measurements for manipulator joint can be obtained by means of encoders or resolvers. In contrast, the joint velocity measurements are obtained by means of tachometers or by differentiating the position measurements obtained with encoders or resolvers. The velocity signals are often contaminated by severe noise. The performance of the velocity-based SMC algorithms is limited as, in practice, the measurement noise amplifies with the increase of the values of the controller gains, see for example [8] and [12]. Specifically, due to the presence of the input and output disturbance, the SMC design generates excessive chattering and infinitely fast switching control causing poor tracking performance. As a matter of fact, such a large control effort based design may not be possible to realize in the real-time operation.

To deal with this practical problem, we propose to use observer to estimate the velocity signals in the SMC scheme. In contrast with the existing nonlinear observer for the *certainty equivalence* (CE) principle based adaptive output feedback design [13]-[17], the proposed estimator structure is very simple in the sense that it does not require system dynamics, uncertain parameters as well as the control inputs. The proof of stability of this method has two parts. In the first part, we derive the SMC scheme as a state feedback provided that all the state variables are available for feedback. Using the Lyapunov method, we show that the state feedback based SMC design achieves desired tracking property and guarantees that all the signals in the closed-loop system are bounded. This property is established by assuming that the parameters are unknown but belong to a known compact set which is relatively large. In the second part of the proof, we replace the unknown velocity signals by the output of the model-free estimator. For the given set of initial conditions, we first define the estimated region of interest for the state feedback based SMC design. Then, we saturate the controller outside the estimated region of interest ensuring that the output feedback controller remains bounded over the estimated region. In the face of large scale parametric uncertainty, the bounded control allows the designer to increase the speed of the observer dynamics without sacrificing the tracking performance. The proposed method can be used to formulate an output feedback form of any state feedback based SMC or adaptive SMC design reported in the literature. The rest of the paper is organized as follows. We first formulate SMC and its stability property by assuming that position-velocity signals are available for feedback design. We then propose an output feedback form of this state feedback based SMC design via replacing velocity signals by the output of the linear observer. A detailed analysis of the proposed method is given to show that the performance under SMC design can be recovered by the OFBSMC design. Adaptive version of the OFBSMC design is also given in this section. This section evaluates the stability condition on a 2-DOF robotic system that demonstrates the theoretical development of this paper. Finally, section III concludes the paper.

II. Output Feedback Sliding mode control

In this section, we first formulate the stability criterion for the SMC as a state feedback (position-velocity) control approach. Then, an output feedback version of this SMC is presented which incorporates a linear observer in order to remove the demand of the velocity signals from existing SMC design. Let us first consider the equation of motion for an n-link rigid robot [25] in error-state space form as

$$\dot{e}_1 = e_2, \dot{e}_2 = \phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau - \ddot{q}_d \qquad (1)$$

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where $e_1 = q - q_d$, $e_2 = \dot{q} - \dot{q}_d$, $\phi_1(e, q_d, \dot{q}_d) = -M^{-1}(q)\left[C(q,\dot{q})\dot{q} + G(q)\right]$ and $\phi_2(e_1, q_d) = M^{-1}(q)$, $q \in \Re^n$ are the joint position vectors, $\dot{q} \in \Re^n$ are the joint velocity vectors, $\ddot{q} \in \Re^n$ are the joint acceleration vectors, $\tau \in \Re^n$ is the input torque vector, $M(q) \in \Re^{n\times n}$ is the symmetric positive definite inertia matrix, $C(q,\dot{q})\dot{q} \in \Re^n$ is the coriolis and centrifugal loading vector and $G(q) \in \Re^n$ is the gravitational loading vector. We first assume that the desired trajectory $q_d(t)$ and its first and second derivatives are bounded as $Q_d \in \Re^{3n} = [q_d, \dot{q}_d, \ddot{q}_d]^T \in \Omega_d$ with compact set Ω_d. Let us consider the reference state as $\dot{q}_r = \dot{q}_d - \lambda e_1$ where $\lambda = diag[\lambda_1, \lambda_2, ..., \lambda_n]$ with $\lambda_n > 0$. Then, we define the sliding surface $S = e_2 + \lambda e_1$. The control objective is to drive the joint position $q(t)$ to the desired position $q_d(t)$. To obtain the control objective, we consider the following control law for the system (1)

$$\tau(e, Q_d, \hat{\theta}) = \hat{M}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G}(q) - \mathcal{K}S - Ksgn(S) \quad (2)$$

where $\ddot{q}_r = \ddot{q}_d - \lambda e_2$, $\hat{M}$ $\hat{G}$ and $\hat{C}$ are the estimates of $M(q)$, G and C, $\mathcal{K} = diag[\mathcal{K}_1, \mathcal{K}_2, ..., \mathcal{K}_i,]$, $K = diag[K_1, K_2, ..., K_i]$, where $\mathcal{K}_i$ and K_i with $n = 1, 2, 3, 4...., n$ are positive constants. Now using (1) and (2), we simplify the closed-loop dynamics as $M\dot{S} + (C + \mathcal{K})S = \triangle\beta - Ksgn(S)$ with $\triangle\beta = (\hat{M} - M)\ddot{q}_r + (\hat{C} - C)\dot{q}_r + (\hat{G} - G) = \triangle M\ddot{q}_r + \triangle C\dot{q}_r$, where $\triangle M = \hat{M} - M$, $\triangle C = \hat{C} - C$ and $\triangle G = \hat{G} - G$. We now explore the convergence condition of the closed loop system in the Lyapunov-sense. To do that, we first define the following positive definite Lyapunov-like candidate function

$$V = \frac{1}{2}S^T MS \quad (3)$$

We take the derivative along the closed-loop trajectory, and then use the property 2 of [17] to obtain $\dot{V}$ as $\dot{V} = -S^T\mathcal{K}S - \sum_{i=1}^{n}\left(S_i\left[K_i sgn(|S_i|) - \triangle\beta_i\right]\right)$. We now consider that model parameters are unknown but belong to a known compact set as Ω. We also consider that the initial state errors $e_1(0)$ and $e_2(0)$ belong to another known compact set Ω_{co}. Using $\sum_{i=1}^{n}\left(S_i\left[K_i sgn(|S_i|) - |\triangle\beta_i|\right]\right) \geq 0$ with $K_i \geq |\triangle\beta_i|$, we can show that the derivative of the Lyapunov function satisfies the inequality $\dot{V} \leq -S^T\mathcal{K}S \leq 0$. This implies that the sliding surface satisfies the sufficient condition as $\frac{1}{2}\frac{d}{dt}S_i^2 \leq -\eta_i|S_i|$, where η_i is a positive constant [2] and [18]. Hence, the energy of S will be decaying as long as $S \neq 0$. To generate the chattering free control action, one estimates the switching function $sgn(.)$ by using a smooth bounded function [18].

A. Output feedback sliding mode control (OFBSMC) design

The analysis presented above is based on the strict assumption that all the state variables are available for feedback. To date, most existing SMC designs in the literature are based on this assumption. However, these algorithms cannot be applied directly on industrial manipulators. This is because advanced industrial manipulators (e.g., those from CRS Robotics Ltd, Applied AI Inc., etc.) do not have velocity sensors to reduce the weight and cost of the system. To obtain the velocity signal, the common practical approach is to differentiate the position measurements obtained from encoders or resolvers which are often contaminated by severe noise [8], [12] and [24]. As a consequence, the performance under state feedback based SMC approach is limited as, in practice, the measurement noise is amplified with the increase of the values of controller gains. In the face of the large parametric uncertainty, the demands of high control gains makes the design even more complex as high gains intensifies high-frequency control chattering activity. To deal with this practical problem, we propose an output feedback form of SMC (2) as $\tau(\hat{e}, Q_d, \hat{\theta}) = \hat{M}(q)\dot{\hat{\dot{q}}}_r + \hat{C}(q, \hat{e}_2 + \dot{q}_d)\hat{\dot{q}}_r + \hat{G}(q) - \mathcal{K}(\hat{e}_2 + \lambda e_1) - Ksat\left(\frac{\hat{S}}{\phi}\right)$, where $sat(.)$ is a bounded smooth saturation function, $\hat{\dot{q}}_r = \dot{q}_d - \lambda\hat{e}_1$, $\hat{\ddot{q}}_r = \ddot{q}_d - \lambda\hat{e}_2$, $\hat{S} = \hat{e}_2 + \lambda\hat{e}_1$ and the unknown velocity signals $\hat{e}_2$ is replaced by the output of the following linear estimator

$$\dot{\hat{e}}_1 = \hat{e}_2 + \frac{H_1}{\epsilon}\tilde{e}_1, \dot{\hat{e}}_2 = \frac{H_2}{\epsilon^2}\tilde{e}_1 \quad (4)$$

where $\tilde{e}_1 = e_1 - \hat{e}_1$, $\tilde{e}_2 = e_2 - \hat{e}_2$, and H_1 and H_2 are the positive constant matrices. In contrast with CE-based nonlinear adaptive output feedback design given in [13]-[17] and [20], the observer structure (4) is independent of the system dynamics, uncertain model parameters and control inputs. The observer dynamics (4) can be viewed as a simple chain of integrator plus correction terms injected by the output error term. To analyze the convergence rate for the OFBSMC design, we use singular perturbation method. To begin with that, we first define an observer error dynamics. Using OFBSMC, one obtains $\dot{e}_2$ as $\dot{e}_2 = \phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau(\hat{e}, Q_d, \hat{\theta}) - \ddot{q}_d$. Then, the observer error becomes $\dot{\tilde{e}}_1 = \tilde{e}_2 - \frac{H_1}{\epsilon}\tilde{e}_1$ and $\dot{\tilde{e}}_2 = -\ddot{q}_d + \phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau(\hat{e}, Q_d, \hat{\theta}) - \frac{H_2}{\epsilon^2}\tilde{e}_1$. We now replace the observer errors by the scaled estimation error to form a singularly perturbed system as $\eta_1 = \frac{e_1 - \hat{e}_1}{\epsilon} = \frac{\tilde{e}_1}{\epsilon} \Rightarrow \epsilon\eta_1 = \tilde{e}_1$ and $\eta_2 = e_2 - \hat{e}_2 = \tilde{e}_2$ with a small positive parameter ϵ. Using this scaled estimation error, one gets $\epsilon\dot{\eta} = B\epsilon[-\ddot{q}_d + \phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau(e - \zeta(\epsilon)\eta, Q_d, \hat{\theta})] + A_o\eta$, where $A_o = \begin{bmatrix} -H_1 & I \\ -H_2 & 0_{n\times n} \end{bmatrix}$ and $\zeta(\epsilon) = \begin{bmatrix} \epsilon I_{n\times n} & 0_{n\times n} \\ 0 & I_{n\times n} \end{bmatrix}$. The positive constants H_1 and H_2 are chosen such that the matrix A_o is Hurwitz. We now show that the performance achieved under

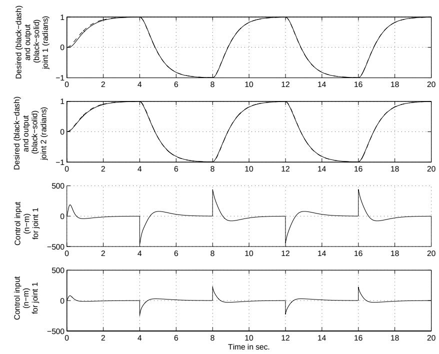

Fig. 1. The desired (black-dash), output tracking (radians) (black-solid for $\tilde{\theta} = 10$ and control input (newton-meters) for joints 1 & 2 under state feedback based ASMC.

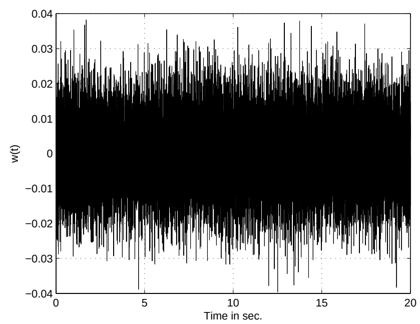

Fig. 2. The disturbance level $w(t)$ for the input $\tau(t)$ and output $q(t)$

state feedback based SMC (2) can be recovered asymptotically by the OFBSMC design. This performance recovery analysis has been shown by using singular perturbation method, where the closed-loop system has the following standard form

$$\dot{e} = B\left[\phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau((e - \zeta(\epsilon)\eta), Q_d, \hat{\theta}) - \ddot{q}_d\right] + Ae \quad (5)$$

$$\epsilon\dot{\eta} = B\epsilon\left[-\ddot{q}_d + \phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau(e - \zeta(\epsilon)\eta, Q_d, \hat{\theta})\right] + A_o\eta \quad (6)$$

where $A = \begin{bmatrix} 0 & I_{n\times n} \\ 0_{n\times n} & 0 \end{bmatrix}$. To begin with the recovery analysis, we first assume that the system output and its derivatives are available for feedback. Then, design a SMC as a state feedback control law (2) such that the design meets the desired tracking objectives. We then replace the unknown velocity state vectors in SMC by the output of the observer (4). If we increase the initial state estimates as well as the initial parameter estimates then the observer speed will also require to increase for robust reconstruction of unknown states. The problem with high speed observer is that it causes large control action during transient phase. To protect the plant from transient control effect, we saturate the control input within the estimated region of interest of the state feedback based SMC scheme. This saturation function will only be active during the transient period. To obtain such a bounded control, let us consider that $\Omega_c = \{e \mid e^T Q_{sm} e \leq c\}$, $c > 0$, be an estimate of the region of attraction of the state feedback based SMC design, $\tau(e, Q_d, \hat{\theta})$, where $Q_{sm} = 0.5\begin{bmatrix} \lambda^2 M & \lambda M \\ I & M \end{bmatrix}$. Since $\tau(e, Q_d, \hat{\theta})$ is a continuous function over e, Q_d and $\hat{\theta}$, then there always exists a maximum control $\tau_i max = max|\tau_i(e, Q_d, \hat{\theta})|$, $1 \leq i \leq n$, such that the input can be saturated over the compact set Ω_{cr} that satisfies $\tau_i^s(e, Q_d, \hat{\theta}) = \tau_i max\ sat\left(\frac{\tau_i(e, Q_d, \hat{\theta})}{\tau_i max}\right) = \tau_i(e, Q_d, \hat{\theta})$, $\forall e(0) \in \Omega_{co} = \{e(0) \mid e(0)^T Q_{sm} e(0) \leq c_4\}$ where $c_4 < c$, $\theta \in \Omega$, $\hat{\theta} \in \Omega$, $Q_d \in \Omega_d$ and $\tau_i max$ is taken over for all $e \in \Omega_{c_r}$ with $c_r > c$ and $sat(.)$ is a smooth bounded saturation function. Then, $\forall e(t) \in \Omega_c$ and $\hat{\theta}(t) \in \Omega$, one has $|\tau_i(e, Q_d, \hat{\theta})| \leq \tau_i max$ $\forall t \geq 0$ and $\tau_i^s(e, Q_d, \hat{\theta}) = \tau_i(e, Q_d, \hat{\theta})$. Hence, the saturation function will not be effective after peaking period is over and the control input $\tau_i(e, Q_d, \hat{\theta})$ is globally bounded [19]. We now replace the state vectors e in the control law by state estimator $\hat{e}$. Then, the bounded OFBSMC can also be achieved via saturating the outside of the region of interest Ω_c as $\tau_i^s(\hat{e}, Q_d, \hat{\theta}) = \tau_i max\ sat\left(\frac{\tau_i(\hat{e}, Q_d, \hat{\theta})}{\tau_i max}\right) = \tau_i(\hat{e}, Q_d, \hat{\theta})$ $\forall \hat{e}(0) \in \Omega_{co}$, $\forall \hat{e} \in \Omega_c$, $\forall e(0) \in \Omega_{co}$, $\forall e \in \Omega_c$, $\forall \hat{\theta} \in \Omega$, $\forall \theta \in \Omega$ and $Q_d \in \Omega_d$. Note that the bounded output feedback control will be required when unknown parameters and initial conditions belong to large compact set. We now summerize our main results for the OFBSMC design.

Theorem 1 : *Consider the closed-loop system (5) and (6). Then, for any given compact set of $e(0) \in \Omega_{co}$, $\hat{e}(0) \in \Omega_{co}$ and $\hat{\theta}(0) \in \Omega$ there exists a small ϵ_1^* such that for all $0 < \epsilon < \epsilon_1^*$, all the state variables of the closed-loop systems are bounded by a bound that can be made arbitrary small by using small value of ϵ_1^*.*

Proof: The proof of the above Theorem 1 consists of two parts. In the first part, we show that there exists a short transient period $T_1(\epsilon) \in [0, T_2]$ during which the fast variable η approaches a function of the order $O(\epsilon)$, while the slow variables $(e, \hat{\theta})$ remain in a subset of the domain of attraction. In the second part, we establish the boundedness of the signal $e(t)$ for all $t \in [T_1(\epsilon), T_3]$, where $T_1(\epsilon) \in (0, \frac{T_2}{2}]$ and $T_3 \geq T_2$ is the first time $(e(t), \hat{\theta}(t))$ exists from the set Ω_c. This part shows that the state variables $(e(t), \hat{\theta}(t))$ remain bounded for $t \geq 0$.

Part 1: We first show that there exists a finite time T_2, independent of ϵ, such that for all $t \in [0, T_2]$ the slow variable remains bounded in the set Ω_c. To show that, we define the positive definite Lyapunov like function candidate $V = \frac{1}{2}S^T MS$. We also consider that all initial conditions are bounded. We choose $e(0) \in \Omega_{co} \subseteq \Omega_c$, that includes $e_1(0) \in \Omega_{co_1}$ and $e_2(0) \in \Omega_{co_2}$, $\theta(0) \in \Omega$, where Ω_c is the domain of attraction, Ω_{co} is the compact set chosen to cover any bounded initial condition. Then, for the given initial set of e_1 and e_2, we have $c_4 = \max_{e_1 \in \Omega_{co1}, e_2 \in \Omega_{co2}} \frac{1}{2}S^T MS$, where $c_4 > 0$. Then define the compact set as $e(t) \in \Omega_c$ with $c > c_4$. Now, our aim is to prove that the energy function V remains bounded by a constant c. To verify that we first take the derivative of the Lyapunov function (3). Then we use the property 2 of [17] and the bounded output-feedback law on the set Ω_{cr} as $\tau^s(\hat{e}, Q_d, \hat{\theta}) = \left[\tau^s(e, Q_d, \hat{\theta}) - \tau^s(\zeta(\epsilon)\eta, Q_d, \hat{\theta})\right]$ with $\tau(e, Q_d, \hat{\theta}) = \tau^s(e, Q_d, \hat{\theta})$ to simplify the derivative of V as

$$\dot{V} = -S^T \mathcal{K} S - \sum_{i=1}^{n}\left(S_i\left[K_i sat\left(\frac{|S_i|}{\phi_i}\right) - \triangle\hat{\beta}_i\right]\right) - S^T\tau^s(\zeta(\epsilon)\eta, Q_d, \hat{\theta}) \quad (7)$$

where $\hat{\triangle}\beta$ is defined as $\triangle\hat{\beta} = \triangle\beta + [(\hat{M} - M)\lambda\eta_2 + (\hat{C} - C)\lambda\epsilon\eta_1 + (\hat{G} - G)]$ with $\hat{S} = S - (\eta_2 + \lambda\epsilon\eta_1)$, $\ddot{\hat{q}}_r = \ddot{q}_r + \lambda\eta_2$ and $\dot{\hat{q}}_r = \dot{q}_r + \lambda\epsilon\eta_1$. Since ϵ is a small positive constant and the fast variable η enters into the slow dynamics via bounded function $\tau^s((e - \zeta(\epsilon)\eta), Q_d, \hat{\theta})$, $\forall e \in \Omega_c$, $\forall Q_d \in \Omega_d$, $\forall \theta \in \Omega$ and $\forall \hat{\theta} \in \Omega$, then, in view of the dynamical property of 1 and 3 of [17], the second part of $\hat{\triangle}\beta$ can be simplified as $\|(\hat{M} - M)\lambda\eta_2 + (\hat{C} -$

$C)\lambda\epsilon\eta_1 + (\hat{G} - G)\| \leq k_{sm}$ for $k_{sm} > 0$. Then, we can write $\dot{V}$ as $\dot{V} \leq -\sum_{i=1}^{n}\left(S_i\left[K_i sat\left(\frac{|S|}{\phi_i}\right) - \triangle\beta_i - k_{sm_i}\right]\right) - S^T\mathcal{K}S - S^T\tau^s(\zeta(\epsilon)\eta, Q_d, \hat{\theta})$. Now, for $|K_i| \geq |\triangle\beta_i|$, we have $\left[K_i sat\left(\frac{|S|}{\phi_i}\right) - \triangle\beta_i\right] \geq 0$. As η enters into slow subsystem via bounded function then $\dot{V}$ can be simplified further as $\dot{V} \leq -\varpi_o V + \alpha_o$ $\forall e \in \Omega_c$, $\forall\hat{\theta} \in \Omega$ and $\forall\theta \in \Omega$, where $\alpha_o = \gamma_s k_{sm} + \gamma_s\alpha_1$, $\varpi_o = \frac{\lambda_{min.}(\mathcal{K})}{\lambda_{max.}(Q_{sm})}$, γ_s and α_1 are the bound for $\|S\|$ and $\|\eta\|$ over the set Ω_c, respectively. Then, the solution for $\dot{V}$ can be derived as $V(t) \leq V(0)e^{-\varpi_o t} + \frac{\alpha_o}{\varpi_o}(1 - e^{-\varpi_o t})$. As $V(0) \leq c_4 < c$, then we conclude that there always exists a finite time T_2, independent of ϵ, such that for all $t \in [0, T_2]$ $V(t) \leq c$. We now prove that over the time interval $[0, T_2]$ the fast variable η converge to a very small value. To show that, let us consider the following Lyapunov function candidate for the fast observer-error model (6)

$$W(\eta) = \eta^T P\eta \quad (8)$$

where $P = P^T > 0$ is the solution of the Lyapunov equation $PA_o + A_o^T P = -I$. Applying $PA_o + A_o^T P = -I$, one can simplify the derivative of (8) along the trajectory (6) as $\dot{W}(\eta) \leq -\frac{1}{\epsilon}\|\eta\|^2 + 2\eta^T PB[-\ddot{q}_d + \phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau^s((e - \zeta(\epsilon)\eta), Q_d, \hat{\theta})]$. Now, for any given $e(0) \in \Omega_{co}$, $\hat{e}(0) \in \Omega_{co}$, $\theta(0) \in \Omega$ and $\hat{\theta}(0) \in \Omega$, we have the following bound

$$\left\| \left[-\ddot{q}_d + \phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau^s((e - \zeta(\epsilon)\eta), Q_d, \hat{\theta})\right] \right\| \leq k_1 \quad (9)$$

for $k_1 > 0$. We omit the remaining proof of the part 1 for brevity.

Part 2: Let us now study the slow subsystems over the time interval $[T_1(\epsilon), T_3]$. That is to study the property of the slow variable $(e, \hat{\theta})$ when $\|\eta\|$ converges closed to the origin. For the time interval $[T_1(\epsilon), T_3]$, we choose $e(0) \in \Omega_{co}$ and $\hat{\theta}(0) \in \Omega$. Then we can write the tracking error model as

$$\dot{e}_2 = \left[\phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau^s(e, Q_d, \hat{\theta}) - \ddot{q}_d\right] + \left[\phi_2(e_1, q_d)\tau^s(\hat{e}, Q_d, \hat{\theta}) - \phi_2(e_1, q_d)\tau^s(e, Q_d, \hat{\theta})\right] \quad (10)$$

This can be viewed as a perturbed closed-loop model under state feedback over the time interval $[T_1(\epsilon), T_3]$. From part 1, we already know that the perturbation term $\|\eta\|$ decays to a level where $\|\eta\|$ is of order $O(\epsilon)$ and $W(\eta(t)) \leq \epsilon^2\beta$. Due to space limitation, we remove the remaining proof of the part 2. •

B. Adaptive sliding mode control (ASMC)

The level of uncertainty in classical SMC and OFBSMC design can be reduced by adding an adaptation term. For this purpose, we first propose to introduce an estimation algorithm to develop an adaptive SMC algorithm as a state feedback as

$$\tau(e, Q_d, \hat{\theta}) = Y(e, \dot{q}_r, \ddot{q}_r)\hat{\theta} - \mathcal{K}S - Ksat\left(\frac{S}{\phi}\right) \quad (11)$$

with $\dot{\hat{\theta}} = -\Gamma Y^T(e, \dot{q}_r, \ddot{q}_r)S$, where $(e, \dot{q}_r, \ddot{q}_r) = \hat{M}(q)\ddot{q}_d + \hat{C}(q, \dot{q}_r)\dot{q}_d + \hat{G}(q)$, $\hat{\theta}$ is the estimate of the robot dynamics and its operating environments such as link masses and payloads, $\Gamma = diag[\Gamma_1, \Gamma_2, ..., \Gamma_n]$ with constant diagonal elements $\Gamma_n > 0$ and $Y(e, \dot{q}_r, \ddot{q}_r)$ is the regressor matrix. The parameter estimates $\hat{\theta}$ can be adjusted with the smooth parameter projection scheme [22] as $\dot{\hat{\theta}}_i = [Proj(\hat{\theta}, \Psi)]_i$ for $\theta \in \Omega = \{\theta \mid a_i \leq \theta_i \leq b_i\}, 1 \leq i \leq p\}$, where Ψ_i is the i-the element of the column vector $-Y^T(e, \dot{q}_r, \ddot{q}_r)S$, γ_{ii} is the i the element of Γ and $\delta > 0$ is chosen such that $\Omega \subset \Omega_\delta$ with $\Omega_\delta = \{\theta \mid a_i - \delta \leq \theta_i \leq b_i + \delta\}, 1 \leq i \leq p\}$. The idea of introducing projection mechanism is to ensure that $\hat{\theta}(t)$ remains bounded on the set Ω_δ for all $t > 0$. Then the closed-loop model error can be written as $M\dot{S} = Y(e, \dot{q}_r, \ddot{q}_r)\tilde{\theta} - (C + \mathcal{K})S - Ksat\left(\frac{S}{\phi}\right)$. The proposed adaptive control law is designed by using the control Lyapunov function as $V = \frac{1}{2}S^T MS + \frac{1}{2}\tilde{\theta}\Gamma^{-1}\tilde{\theta}$, where $\tilde{\theta} = (\theta - \hat{\theta})$. Using the property 2 of [17], the time derivative V along the closed-loop error trajectories can be simplified as $\dot{V}(e, \tilde{\theta}) \leq -\lambda_{min}(\mathcal{K})\|S\|^2 - K\|S\| \leq 0$ $\forall e \in \Omega_c$, $\forall\hat{\theta}(0) \in \Omega$, $\forall\theta(0) \in \Omega$ and $\hat{\theta}(t) \in \Omega_\delta$.

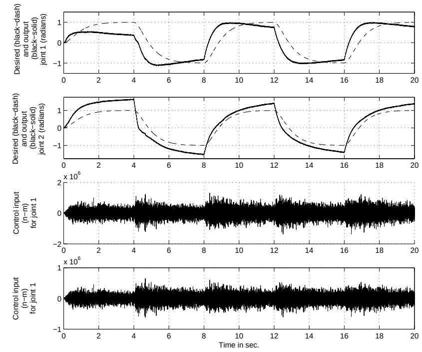

Fig. 3. The desired (black-dash), output tracking (radians) (black-solid for $\tilde{\theta} = 10$ and control input (newton-meters) for joints 1 & 2 under state feedback based ASMC.

C. Adaptive output feedback sliding mode control (AOFBSMC)

The above design is implementable only when all the process states are measurable. To relax this strict assumption, we now replace the unknown state vectors in (11) by the output of the estimator (4) as $\tau(\hat{e}, Q_d, \hat{\theta}) = Y(\hat{e}, \dot{\hat{q}}_r, \ddot{\hat{q}}_r)\hat{\theta} - \mathcal{K}\hat{S} - Ksat\left(\frac{\hat{S}}{\phi}\right)$ with $\dot{\hat{\theta}} = \Gamma Y^T(\hat{e}, \dot{\hat{q}}_r, \ddot{\hat{q}}_r)S$. To smooth the parameter estimates, we may use parameter projection scheme, where the state estimates replaced by the output of the observer (4). Then the closed-loop model under AOFBSMC approach has the following form

$$\dot{e} = B\left[\phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau((e - \zeta(\epsilon)\eta), Q_d, \hat{\theta}) - \ddot{q}_d\right] + Ae \quad (12)$$

$$\epsilon\dot{\eta} = B\epsilon\left[-\ddot{q}_d + \phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau(e - \zeta(\epsilon)\eta, Q_d, \hat{\theta})\right] + A_o\eta \quad (13)$$

eith $\dot{\hat{\theta}} = Proj\left(\hat{\theta}, \Psi(e - \zeta(\epsilon))\eta, Q_d, \hat{\theta})\right)$. Then, we consider that for any given $e(0) \in \Omega_{co}$, $\hat{e}(0) \in \Omega_{co}$, $\theta(0) \in \Omega$ and

$\hat{\theta}(0) \in \Omega$, the following inequality holds

$$\|[-\ddot{q}_d+\phi_1(e, q_d, \dot{q}_d)+\phi_2(e_1, q_d)\tau^s((e-\zeta(\epsilon)\eta), Q_d, \hat{\theta})]\| \leq k_{2o} \quad (14)$$

for $k_{2o} > 0$. We now state the main results for the AOFBSMC design in the following Theorem 2.

Theorem 2 : *Let us consider the closed-loop system (12)-(13). Suppose that $\hat{\theta}(0) \in \Omega$ and $(e(0), \hat{e}(0)) \in \Omega_{co}$ is bounded. Then, there exists ϵ_1^* such that for all $0 < \epsilon < \epsilon_1^*$, all the signals in the closed-loop system will be bounded by a bound that can be made arbitrarily small for small value of the observer design constant ϵ.*

Proof: The proof of stability of Theorem 2 can be shown along the line of Theorem 1. So we removed the stability analysis for brevity.•

Remark 1: It is worth noting that the bounded inequality (9) and (14) does not impose any restriction either in theory or in practice. Note also that these inequalities do not enforce growth condition on systems nonlinearities. The constant k_1 and k_{2o} (that used in observer error analysis) are used to establish the semi-global stability property of theorems 1 and 2. For the given set of initial condition of interest, the designer can calculate the value of k_1 and k_{2o} over the domain of attraction Ω_c. The value of k_1 and k_{2o} can be calculated as follows. For the given $e(0) \in \Omega_{co}$, $\hat{\theta}(0) \in \Omega$ and $\theta(0) \in \Omega$, we calculate the terms $\phi_1(e, q_d, \dot{q}_d)$ and $\phi_2(e_1, q_d)$. Then, for given $\ddot{q}_d$ and initial conditions of interest, we define the saturation level τ_{max} (maximum bound on $\tau(e, Q_d, \hat{\theta})$). For a small value of ϵ and initial error estimates $\hat{e}(0) \in \Omega_{co}$, we then calculate the bound on output feedback $\tau^s((e-\zeta(\epsilon)\eta), Q_d, \hat{\theta})$ as well as the bound on the term $-\ddot{q}_d + \phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau^s((e-\zeta(\epsilon)\eta), Q_d, \hat{\theta})$.

Design and Implementation process: In this part, we analyze the property of the Lyapunov stability argument established in Theorem 2 on a 2-DOF serial link manipulator, which has the following dynamical model [17]: $\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$ with $m_{11} = (\theta_1 + 2\theta_2 + 2\theta_2 \cos q_2)$, $m_{12} = (\theta_2 + \theta_2 \cos q_2)$, $m_{21} = (\theta_2 + \theta_2 \cos q_2)$, $m_{22} = \theta_2$, $c_{11} = -2\dot{q}_2\theta_2 \sin q_2$, $c_{12} = -\dot{q}_2\theta_2 \sin q_2$, $c_{21} = \dot{q}_1\theta_2 \sin q_2$ and $c_{22} = 0$, where $\theta_1 = m_1 l^2$, $\theta_2 = m_2 l^2$ and l is the link lengths and m_1 and m_2 are the masses of link 1 and link 2, respectively. Due to the space limit, we have removed the evaluation of Theorem 1. We assume that the parameters θ_1 and θ_2 of the given robot dynamics are unknown but belong to comparatively large compact sets as $\theta \in \Re^2 \in \Omega = \{-10 < \theta_1 < 10, -10 < \theta_2 < 10]$. Suppose that the initial states $|e(0)| \leq 2$, $|\hat{e}(0)| \leq 2$ and the initial estimated parameters as $\hat{\theta}(0) = 0$. In our evaluation, we will examine the tracking convergence property of ASMC and AOFBSMC approach with respect to estimation errors as $\tilde{\theta} = 10$. To generate smooth control input, we use bounded function $sat(.)$ instead of $sign(.)$ function. The value of λ and $\mathcal{K}$ are chosen such that they ensures an acceptable transient and steady state tracking error objectives [26]. For our evaluation, we consider $\mathcal{K}$ as $\mathcal{K}_1 = 125$ and $\mathcal{K}_2 = 125$. The bandwidth of the controller is chosen as $\lambda_1 = \lambda_2 = 2$. The value of Γ, K and $\frac{1}{\phi_i}$ are selected

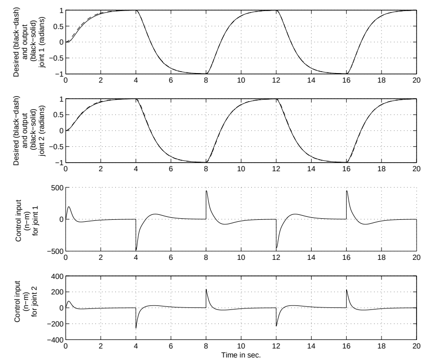

Fig. 4. The desired (black-dash), output tracking (radians) (black-solid for $\tilde{\theta} = 10$ and control input (newton-meters) for joints 1 & 2 under theorem 2.

in such a way that guarantees fast parameter learning. As the results are semi-global, then the designer can select the control gains Γ, K and $\frac{1}{\phi_i}$ provided that $\theta \in \Omega$. For our evaluation, we choose the value of discontinuous and learning gains Γ, K and $\frac{1}{\phi}$ as $\Gamma = 10I_{2\times2}$, $K = 15I_{2\times2}$ and $\phi = 0.7$. Note that, for the given set, the designer can increase the value of ϕ and decrease the value of K. With these control design parameters, let us apply ASMC design (11) on the given model. The tested results are given in Figure 1 with the chosen parameter estimator errors as $\tilde{\theta} = 10$ (black-solid). It is worth noting that the value of Γ, K and $\frac{1}{\phi_i}$ will increase with the increase of the parametric uncertainties, that is, Ω and Ω_{co}. The main practical problem is that high gains are mainly responsible for amplifying the output and input disturbances in state feedback based SMC approach. In practice, high control gain may excite hidden unmodeled dynamics resulting poor tracking performance. This is because high-gain intensifies the control chattering activity as it amplifies the disturbances associated with the output (position-velocity measurements), input and disturbance such as backlash, friction, external disturbance, entering into the closed system [8], [12] and [25]. To illustrate this argument, we now examine the tracking performance of ASMC algorithms under non ideal situation where we add a band limited white disturbance noise, $w(t)$, into the output $q(t)$ (position measurement) input $\tau(t)$ to the system. For our evaluation, the level $w(t)$ for the output $q(t)$ and input $\tau(t)$ are considered as given in Figure 2. Then, we apply ASMC design (11) on the given system with the estimation errors as $\tilde{\theta} = 10$. The tested results under $\tilde{\theta} = 10$ are depicted in Figure 3. We notice from our implementation that the tracking errors increase with the increase of the parametric uncertainty. More specifically, the control effort becomes very large that may not be possible to realize as available control inputs are restricted in real-world operation. This large control action is mainly because of the derivative action of the noisy position signals that used in position-velocity based ASMC law (11). Now our aim is to show that the large control effort under position-velocity based ASMC design can be reduced by using AOFBSMC. We keep the same controller design parameters as used for the evaluation of the position-velocity

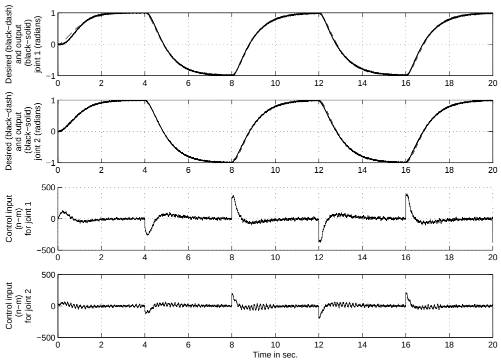

Fig. 5. The desired (black-dash), output tracking (radians) (black-solid) and control input (newton-meters) for joints 1 & 2 under AOFBSMC of theorem 2 with $\tilde{\theta} = 10$.

based ASMC algorithm. Then, we choose observer design constants of H_1 and H_2 as $H_1 = I_{2\times2}$, $H_2 = I_{2\times2}$ and $\epsilon = 0.05$ that guarantees fast recovery of unknown velocity signal. With these set up, we now apply the AOFBSMC on the given system. The results are given in Figure 4. We can see from our results that the performance under AOFBSMC design is similar to the performance under ASMC design (11). We now add a band limited white noise, $w(t)$, into the output measurement $q(t)$ and input $\tau(t)$ of the system as depicted in figure 3. Then, using the following observer-controller design parameters as $\mathcal{K}_1 = 125$, $\mathcal{K}_2 = 125$, $\Gamma = 10I_{2\times2}$, $K_1 = 15$, $K_2 = 15$, $\phi_1 = 0.7$, $\phi_2 = 0.7$, $\epsilon = 0.05$, $H_1 = 5I_{2\times2}$ and $H_2 = 5I_{2\times2}$, we apply AOFBSMC on the given robot system. The tested resuts are presented in figure 5 with $\tilde{\theta} = 10$. By comparing figures 3 (ASMC) and 5 (AOFBSMC), we can observe that the tracking performance under AOFBSMC is almost closed to zero while relatively large tracking errors can be seen under ASMC design. Notice also from these results that the required control effort under AOFBSMC (see figure 5, for $\tilde{\theta} = 10$) is much smaller than the control effort demanded under ASMC (see figure 3, for $\tilde{\theta} = 10$) design.

III. Conclusion

In this paper, an output feedback sliding mode robot control method has been proposed to deal with the problem associated with the velocity based SMC design. The Lyapunov method has been utilized to establish the semi-global stability condition of all the closed-loop signals. This property has been shown via using parameter projection and control saturation technique. The tested results validate the Lyapunov stability arguments of the proposed method. The evaluation on a 2-DOF robotic system has been shown to demonstrate the theoretical development for the real-time application.

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