Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

A multiresolution flow-based multiphase image segmentation

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***Abstract*—In this work a variational model is proposed for simultaneous smoothing and multiphase image segmentation. By assuming that the pixel intensities are independent samples from a mixture of Gaussians, and by interpreting the phase fields as probabilities of pixels belonging to a certain phase, the model formulation is obtained by maximizing the mutual information between image features and phase fields. The proposed energy functional J_ϵ consists of three parts: the smoothing term for the reconstructed image, the regularization for the boundaries in hard segmentation, and a likelihood estimator based on the density function. The segmentation and image denoising are performed simultaneously through the flow equation obtained by minimizing the energy functional with respect to the mixture of Gaussian coefficients and variance. Some experimental results on segmenting synthetic and natural color images are presented to illustrate the effectiveness of the proposed model.**

***Index Terms*—multiphase segmentation, soft segmentation, variational approach.**

I. Introduction

The problem of segmenting an image into semantically significant components is one of the most important tasks to date in computer vision analysis and image processing and yet unsolved for low or middle-level vision problems. Despite the numerous segmentation techniques, image segmentation is still a subject of ongoing investigations, and it cannot be conclusively stated that the segmentation problem has been solved because of the diversity of the applications, such as simultaneous segmentation and recognition to reduce reliance on local features, geo-monitoring of land coverage for deforestation control, and fluid-like animation in still images. Also, image segmentation seems to be a prerequisite for further semantic analysis and storage or retrieval of image based multimedia data.

Its primal goal is to partition a given image domain Ω into K non-overlapping and connected components $\Omega_i, i = 1...K$ of $\Omega \ | \Gamma$, and a closed contour Γ so that the objects covered by each region share some specific properties. Each region path $I_i = I|\Gamma$ is denominated pattern, and the respective Ω_i is its support. The segmentation that partitions the image domain Ω along the contours and outputs non-overlapping pattern supports Ω_i is defined as "hard" Segmentation [18].

The standard segmentation methods can work reasonably well when the images are of high quality and have simple contents, but it can perform poorly on real images where simple thresholding or even adaptative thresholding may miss important features. There are several advanced approaches for image segmentation based on clustering, histogram, graph partition, region competition, region growing, and optimization procedures. In spite of other advanced segmentation procedures, this work focus on optimization based model where the variational principles have been used to design consistent frameworks for image segmentation. The main idea is to integrate shape priors into the functional to be minimized in a way that the expected image partition can be reached. Recently, region based statistics have been embedded into a variational formulation yielding several parametric and non-parametric approaches [4], [15], [8], [20].

The celebrated Mumford-Shah segmentation model [12] was one of the first models for variational image segmentation. The model treats the image denoising and the boundary preserving simultaneously. Once the denoising and segmentation procedures are two different aspects of the same problem, a good denoising process should distinguish between a set of significant regions and the edges between them. Minimizing the functional simultaneously with the dynamic variables results in a denoised image while preserving the boundaries.

The traditional "hard" segmentation approach, which is still very common in the computer vision community, suggests to start with a denoising process and to apply some sort of threshold afterwards. Once the image is successfully segmented, one can go to higher level problems such as motion analysis, classification, recognition, and 3D reconstruction. Soft segmentation allows each pixel to belong to multiple classes with varying degrees of membership. Soft segmentation can be converted to hard segmentation by using the maximum membership classification rule where each pixel is assigned to the class which has the highest membership value.

There are several techniques that use the soft segmentation concept such as the traditional k-means clustering [5], the split-and-link algorithm that computes overlapping segments in a pyramidal framework, and matting techniques, among others. However, the great majority of the matting techniques are not automatic, requiring some sort of user interaction to specify information such as: "*the foreground*", "*the background*", etc., in order to produce satisfactory matting results [19].

The research in soft segmentation has been motivated by

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natural images, medical imagery, and also in CT data where soft tissue structures are not well differentiated and the usual segmentation using thresholding is inadequate. Patterns in natural scenes , such as MRI data, do not have clear boundaries.

Recent work in soft segmentation has been made. In [10] a variational segmentation algorithm using Fuzzy Region Competition and Local Non-Parametric Probability Density Functions is designed to split an image in two regions based on their intensity distributions. A functional is proposed to integrate the unknown probability density functions of both regions within the optimization process.

In [19] the authors proposed an automatic approach to soft color segmentation optimizing a global objective function, exploiting the reliability given by global color statistics and local image composition flexibility. The model leads to an image model where the global color statistics of an image is represented by a Gaussian Mixture Model (GMM), and the color of a pixel are explained by a local color mixture model.

Shen et al. proposed, in [18], a stochastic-variational model for soft (or fuzzy) Mumford-Shah segmentation of mixture image patterns. Unlike the classical hard Mumford-Shah segmentation, Shen's model allows each pixel to belong to each image pattern with some probability.

In this paper, we propose a variational model for simultaneous smoothing and multiphase image segmentation by assuming that the pixel intensities are independent samples from a mixture of Gaussians and interpreting the phase fields as probabilities of pixels belonging to a certain phase. Then, the model formulation is obtained by maximizing the mutual information between image features and the phase fields.

The solution of the proposed energy functional J_ϵ converges to the solution of Shen's model [18] when ϵ goes to zero.

II. Gaussian Mixture

Consider the following simplex constraint

$$P : \Omega \rightarrow \Delta_{K-1}$$

the probability $(K-1)-simplex$, i.e., $p_i >= 0, i : 1 = K$ and $\sum_{i=1}^{K} p_i = 1$. This constraint induces coupling among different channels. For small ϵ, each ownership p_i tends to polarize to 0 or 1, however they have to cooperate with each other according to the simplex constraint. Let $u_i(x) \in H^1(\Omega)$ be a smooth function associated with each pattern label $l = i$, and $U(x) = (u_1(x), u_2(x), ..., u_K(x))$. The soft segmentation problem consists in estimating the optimal vectorial pair of ownerships and patterns given I(x),

$$(P*, U*) = argmax_{(P,U)} Prob(P, U|I),$$

where, by the Bayesian formula, the posterior is obtained by

$$Prob(P, U|I) = Prob(I|P, U)Prob(P)Prob(U)/Prob(I),$$

assuming that the mixture patterns U and the mixture rules P are independent.

Here, it is assumed that the patterns are all Gaussians with mean fields $u_1, u_2, ..., u_K$ and share the same variance . Then, at any given pixel $x \in \Omega$,

$$(I|l(x) = i) \quad N(u_i(x), \sigma^2), i = 1 : K.$$

The Gaussian probability density function (p.d.f.) is given by:

$$g(I|\mu, \sigma) = \frac{1}{(\sqrt{2}\pi\sigma)} exp(-\frac{(I-\mu)^2}{2\sigma^2}).$$

The probability density function of the mixture image I at any pixel x is given by

$$Prob(I(x)|P(x), U(x)) = \sum_{i=1}^{K} Prob(I|l(x) = i) = Prob(l(x) = i) = \sum_{i=1}^{K} g(I|u_i(x), \sigma)p_i(x). \quad (1)$$

Because $(I(x)|P, U)$ is independent of $(I(y)|P, U)$ for any two distinct pixels x and y considering two fields P and U, the energy is given by:

$$E[I|P, U] = -\lambda \int_\Omega log\left(\sum_{i=1}^{K} g(I|u_i(x), s)p_i(x)\right),$$

for some scalar $\lambda > 0$.

The following Modica-Mortola phase-field energy model [11] is used for the ownership distribution $\mathbf{P}(x) = (p_1(x), p_2(x), ..., p_K(x))$:

$$E_\epsilon[\mathbf{P}] = \sum_{i=1}^{K} \int_\Omega \left(9\epsilon|\nabla p_i|^2 + \frac{(p_i\ (1-p_i))^2}{\varepsilon}\right) \quad (2)$$

III. The Proposed Method

Inspired by the Modica-Mortola phase field model for ownership energy and the models given by Mumford and Shah [12] and Shen [18] for image segmentation, we propose in this work the following minimization functional:

$$\min J_\epsilon[P, U \mid I] = \frac{\lambda}{1+\epsilon} \sum_{i=1}^{K} \int_\Omega (u_i - I)^2 p_i + \alpha(1+\epsilon) \sum_{i=1}^{K} \int_\Omega |\nabla u_i|^2 + \sum_{i=1}^{K} \int_\Omega \left(9\epsilon|\nabla p_i|^2 + \frac{(p_i\ (1-p_i))^2}{\epsilon}\right) \quad (3)$$

Using the $\Gamma - Convergence$ theory, the proposed functional is closely related to the original Mumford-Shah energy functional as well as the Shen's functional when $\epsilon \rightarrow 0^+$. Parameter ϵ's decreasing behavior allows for good performance of the flow equation which minimizes the functional when dealing with soft segmentation. Since parameter ϵ is not constant, the model can deal with several difficulties in segmenting an image.

IV. The Euler Lagrange Equations

The Euler-Lagrange equation for the functional $E[P, U \mid I]$ can be implemented by the following gradient descend equations:

$$-\alpha(1+\epsilon)\Delta u_i - \frac{\lambda}{2(1+\epsilon)} p_i (I - u_i) = 0, \tag{4}$$

$$-18\epsilon\Delta p_i + 2\epsilon^{-1} p_i(1-p_i)(1-2p_i) = \langle V\rangle + \frac{\lambda}{1+\epsilon}(I-u_i)^2 \tag{5}$$

where $\langle V\rangle$ is defined as in J.Shen's paper,$\langle V\rangle = \frac{1}{K}\sum_{i=1}^{K} V_i$;

$$V_i = \frac{\lambda}{1+\epsilon}(I-u_i)^2 - 18\epsilon\Delta p_i + 2\epsilon^{-1} p_i(1-p_i)(1-2p_i) \tag{6}$$

V. Numerical Procedures

Considering that

$$p_i(1-p_i)(1-2p_i) = p_i(1-p_i)^2 - p_i^2(1-p_i)$$

and since $\sum_{i=1}^{K} p_i = 1$ and $\Delta\left(\sum_{i=1}^{K} p_i\right) = 0$, the multiresolution flow based multiphase image segmentation can be performed by solving the Euler-Lagrange equations using the following iterative system:

$$\begin{aligned} u_i^{n+1} &= u_i^n + dt_u L_u(I, u_i^n, p_i^n) \\ p_i^{n+1} &= p_i^n + dt_p L_p(I, u_i^n, p_i^n); \end{aligned} \tag{7}$$

where

$$L_u(I, u_i, p_i) = \alpha(1+\epsilon)\Delta u_i + \frac{\lambda}{2(1+\epsilon)}(I-u_i)^2$$

$$\begin{aligned} L_p(I, u_i, p_i) = {} & 18\epsilon\Delta p_i - 2\epsilon^{-1} p_i(1-p_i)(1-2p_i) + \\ & \frac{\lambda}{K(1+\epsilon)}\sum_{i=1}^{K}(I-u_i)^2 - \frac{\lambda}{(1+\epsilon)}(I-u_i)^2 + \\ & \frac{2}{\epsilon K}\sum_{i=1}^{K}(2p_i^3 - 3p_i^2) + \frac{2}{\epsilon K}. \end{aligned}$$

A. Initialization Procedure:

To start the iterative process we need to define some input values, such as, the initial values for the ownership distributions p_i, the initial u_i, α, λ, and ϵ. In this work we consider in equation 7, ϵ as a function of n such that $\epsilon(n) \rightarrow 0$ when $n \rightarrow \infty$. In the beginning, this procedure performs stronger smoothing allowing better results when the image is contaminated by noise or contains fine textures.

The adopted procedure is: given $p_i(x)$, $i = 1, 2, ..., K$, we consider $u_i(x, y) = I(x, y)$ if $p_i(x, y) = 1$, and $u_i(x, y) = 0$ if $p_i(x, y) = 0$.

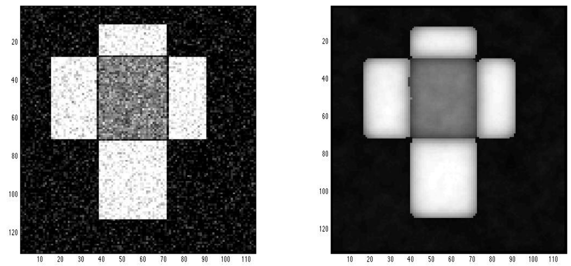

Fig. 1. The segmentation of a noisy image. The initial noisy image and the segmented image using the Proposed Model.

VI. Experimental Results

To show the effectiveness of the proposed multiphase segmentation model, three experiments are reported. Different from other related works [16] and [18], parameter ϵ is not constant. Here, the value of parameter ϵ starts equal to 10 and decreases in each iteration, since $\epsilon \rightarrow 0^+$. The used parameters were set to α=3; λ=1; and dt_p=dt_u=0.005. Figures 1, 2, and 3 illustrate the performance of the proposed model applied on three images: a piecewise image, a natural landscape image (beach), and a color image of horses, respectively.

Figure 1 illustrates the proposed model's ability to reconstruct a piecewise image with additive Gaussian noise. Figure 1 presents the results obtained using the proposed model for 100 iterations.

Figure 2 presents the initial color image, the reconstructed image, and its four ownership distributions: $p_1(x)$, $p_2(x)$, $p_3(x)$, and $p_4(x)$ obtained from the proposed model after 50 iterations. This example shows that the proposed model emulates how humans perceive natural images that do not have clear-cut hard boundaries between two different parts. The softly transiting ownerships p_i's, in spite of hard segmentation, is able to distinguish the sky, the water, the sand, and the vegetation parts.

Figure 3 shows the obtained results when dealing with color images using three ownership distributions $p_1(x)$, $p_2(x)$, and $p_3(x)$. The first row shows an image containing horses and the extracted parts (horses, grass, and sky) using the ownership distributions p_i's after 300 iterations of the Proposed Model. The second row of Figure 2 illustrates the performance of the proposed model when the number of iteration grows. The picture shows the same part extracted (horses) using the final phase field after 30, 200, and 250 iterations. It can be seen that the horses, sky, and grass areas are reasonably well extracted.

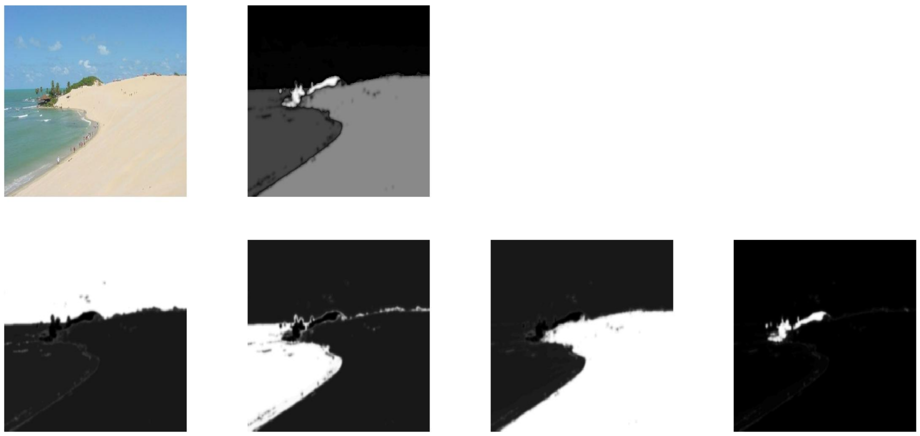

Fig. 2. Example of a four phase soft segmentation. First row: The initial given image and the reconstructed segmented image. Second row: The four ownership distribution $p_1(x)$, $p_2(x)$, $p_3(x)$, and $p_4(x)$ obtained using the Proposed model.

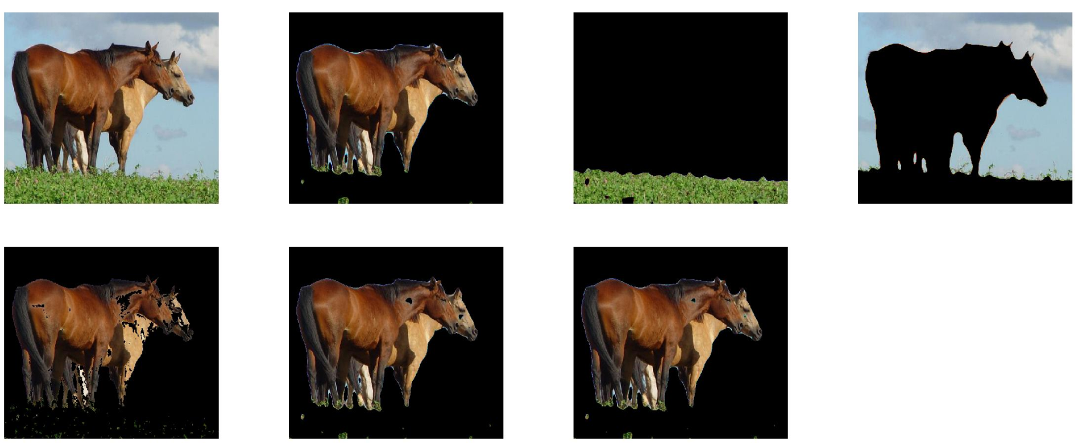

Fig. 3. Segmentation of the horses, grass, and sky regions from a color image . First row: The initial color image, the extracted horses, grass, and sky using the final phase fields after 300 iterations. Second row: three horse-results using the phase fields with 30, 200, and 250 iterations

VII. Conclusion

A multiphase model for variational image segmentation was presented in this paper. The model deals with phase fields, which are interpreted as the probabilities of pixels belonging to regions.

The statistical nature of the proposed technique ensures the soft transition of the ownerships p_i's instead of hard segmentation. The model formulation is derived by maximizing the mutual information between the phase fields and a probability density function based on image features. The model is general and can easily deal with multiple different regions. The parameter ϵ allows a multiscale balance between the smoothing process and the forcing term $(I - u)$. The smoothing decreases as ϵ goes to 0^+. In order to show the wide applicability of the model, the algorithm was tested on several images with the same parameter settings. The results showed that the phase fields can be utilized in order to obtain fuzzy boundaries. The proposed algorithm was successfully applied to challenging synthetic and real life images with robustness to noise.

Acknowledgment

The first author acknowledges the financial support of CNPq (Proc.308798/2006-1 and 151083/2008-8).

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