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Robust Sideslip Estimation Using GPS Road Grade Sensing to Replace a Pitch Rate Sensor

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***Abstract*—This paper analyzes a promising method of road grade estimation for its potential use in aiding ground vehicle electronic stability control systems. The method in question incorporates GPS and motion sensors into a basic Kalman Filter model to achieve clean, high update estimates of the vertical and longitudinal velocity states from which the grade can be easily calculated. The knowledge of road grade is incorporated into a sideslip estimation scheme, replacing the pitch rate sensor, and the improvement in the sideslip estimate is evaluated. Simulated results are shown, as are results from using experimental data comparing this sideslip estimate with one obtained which utilizes the pitch rate gyro.**

***Index Terms*—Vehicle State Estimation, GPS/INS, Kalman Filtering, Sensor Reduction**

I. Introduction

Today's vehicle stability controls intend to achieve yawing, lateral, and rolling stabilization. The so-called electronic stability control (ESC) systems [1] were initially designed to directly attenuate the undesired vehicle yawing with limited lateral stability performance due to limited information on the vehicle's sideslip angle. The so-called roll stability control (RSC) system [2] adds a roll rate sensor and control algorithms to an ESC system to achieve vehicle roll stabilization. The extra roll rate sensor provides certain information about the total vehicle roll angle. Hence the vehicle side-sliding velocity can be estimated through the onboard lateral accelerometer which leads to a good sideslip angle estimation [2].

Adding a pitch rate sensor to the RSC sensor set [3] or using a 6 degree of freedom IMU sensor [4] leads to the further refinement of the vehicle roll and pitch angle computation leading to further refinement of robust sideslip angle estimation. For a vehicle implemented with a GPS receiver (e.g., for navigation or e-911 function), the vehicle pitch information can be readily obtained through GPS velocities, hence, the pitch rate sensor might be eliminated. Using GPS to compute road grade and aid sideslip angle is not new. Anderson uses kinematic sensor models to incorporate a single antenna GPS system with an INS into a Kalman filter to estimate sideslip [5]. In [6], Anderson expands this to use with a two antenna system and examines the effects of modeling errors in more detail. Bevly uses a sideslip estimate to generate a lateral velocity measurement which he incorporates into a lateral state Kalman filter with a lateral accelerometer. This could be generated from either a single or dual antenna GPS/INS Kalman filter. Roll rate information was not used to compensate the lateral accelerometer, thereby assuming a planar model [7]. Ryu uses a dual antenna GPS receiver with INS to estimate vehicle velocities, sideslip, roll, and road grade [8]. In [9], Bevly and Ryu present Kalman filtering methods for vehicle state estimation using both single and dual antenna GPS/INS systems. A planar model is no longer assumed, as the inertial measurements are now compensated for the roll effects [9].

These sideslip estimates can be used to estimate other important vehicle parameters, as shown by Bevly and Daily. In [10] and [11], Bevly and Daily use the lateral state estimates to determine tire parameters. In [12] Bevly applies knowledge of sideslip, obtained from an observer as before, to estimate the biases on low cost inertial sensors.

It is also possible to use magnetometers and magnetic sensors embedded into the roadway to estimate sideslip. In [13] Yang and Farrell demonstrate this by creating a vehicle state estimation system having three layers of redundancy which uses magnetometers, GPS, and INS to determine the vehicle states. The accuracies and observabilities of the different estimates are discussed regarding the availability of each of the sensors.

The above authors probe the limits of possibility afforded by the GPS/INS Kalman filter state estimation setup with regard to planar models and models that include road bank. However, dynamic maneuvers performed in the presence of a large road grade have not been explored. In urban areas the design limit for road grade can be up to 9% at speeds of 60 mph, and up to 12% for lower speeds around 30 mph. Rural roads can be up to 10% at design speeds of 40 mph and 8% for 60 mph design speeds [14]. Therefore knowledge of the road grade will be necessary to fully exploit these GPS/INS estimation schemes on steeper roadways.

Jansson estimates the road grade by combining GPS information with barometer and torque measurements into a Kalman filter [15]. Sahlholm and Johansson take a similar model based approach using driveline sensors and GPS; however they additionally present a method for recursively improving the grade estimate with new passes over the same road [16]. Lingman and Schmidtbauer also use a longitudinal vehicle model and Kalman filtering techniques to estimate both vehicle mass and road grade. This is done without any GPS information [17]. All of these are done under the context of

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longitudinal vehicle control, as opposed to lateral dynamic control applications. Bae and Ryu describe two methods for road grade estimation using GPS that are much more suitable for lateral estimation and control purposes [18]. These methods involve measuring total pitch directly with a dual antenna GPS receiver or taking the arctangent of the ratio of the vehicle up and forward velocities obtained from a single antenna [18]. This method is expanded in [19], where the up and forward velocities are estimated using a simple Kalman filter. Where prior work has vastly explored the potential of GPS/INS integration in a Kalman filter structure for lateral vehicle state estimation under planar assumptions and even extending this to banked roads, thereby including the effects of vehicle roll, this paper explores the sideslip angle estimation robust to the road grade by using GPS velocity based road grade computation. Such an estimation achieves roughly similar performance to the one using a pitch rate sensor. This work specifically applies the road grade knowledge to the single antenna kinematic observer presented in [7], [9]. It will be shown that using this observer most of the improvement resulting from accurate road grade knowledge comes through improving the roll angle estimate. That is, if a perfect roll angle were known, which is unrealistic, road grade knowledge would be of little use. This is intuitive, as the difference between the projection of the yaw rate of a vehicle that is on a hill onto the navigation frame and the actual measured yaw rate will be quite small, although on very steep hills this distinction can cause errors over moderate periods of integration. The road grade estimation method presented in [19] is the most suitable for laterally dynamic situations and therefore is used in this paper. Since pitch rate sensors are uncommon on most commercial vehicles, this road grade estimate affords the opportunity of replacing the pitch rate gyro with the increasingly common single antenna GPS unit. Therefore the primary contribution of this paper is to demonstrate that the pitch rate sensor can be removed from the sideslip estimation process by including a road grade estimate obtained from GPS velocities and accelerometer signals. This is shown both in simulation and on experimental data.

II. Vehicle State Estimation

A. Estimation Architecture

The goal of this research is an accurate, unbiased estimate of the sideslip of a vehicle which is robust to road bank and grade variations. The estimation strategy is largely directed by the sensors and corresponding measurements available. This work assumes the following sensors to be available: a five degree of freedom IMU (which can be found in a production vehicle with a roll stability control system or with a rollover curtain system without including a pitch rate sensor), a single antenna GPS receiver, the steer angle signal from the vehicle's CAN, and other sensor signals used in today's ESC systems such as wheel speed measurements.

In step one the grade is estimated using a Kalman filter setup to estimate the vertical and longitudinal velocities as described in [19], providing an accurate estimate of the road grade for use in the mechanization and rotation matrices. Next an estimate of sideslip is produced using the heading estimate obtained from the heading Kalman filter described in [7], [9]. The gyroscope signals are then rotated into the navigation frame using the IMU mechanization matrix. The Euler pitch angle used for rotation is the grade estimate. This neglects any extra body pitch in the suspension, leading to estimation errors if the vehicle is experiencing large suspension pitch. The Euler roll angle is estimated by integrating the Euler roll rate given by the mechanization, needing only an initial estimate of the roll angle. Now the estimates of road grade and total roll are used to rotate the accelerometers into the navigation frame. This cleans up any of the biases resulting from gravity in the accelerometer measurements. Estimation errors will result from these mechanization and rotation steps if the IMU is misaligned. The last step finally completes the sideslip estimation process with a simple Kalman filter comprised of the lateral velocity state and the bias on the lateral accelerometer. The accelerometer now has no gravity biases in it, and the final sideslip estimate is produced as in [7], [9].

B. Road Grade Estimation

The slope of the road in the longitudinal direction that a vehicle is traveling on is generally referred to as the road's grade. It can be presented in two forms: either as the actual slope of road or as the angle that the road makes with the horizon; where conversion between the two is a matter of simple trigonometry. Determination of this angle can be accomplished using the ratio of the vertical and horizontal speeds, assuming of course that the vehicle is moving. GPS receivers output a vertical speed and a speed-over-ground velocity vector, where the speed over ground is the vehicle's velocity vector in the local navigation frame. The magnitude of this vector can be taken to be the longitudinal speed of the vehicle in the navigation frame, assuming little to no sideslip. Therefore the arctangent of the two speeds can be taken to find the grade angle (it is preferred in angle form for its use in other calculations.) This method has been proven to produce high quality, unbiased estimates of the road grade, yet it should be noted that any bounce motions that the vehicle experiences will affect the grade estimate. However, this has been shown not to significantly diminish the estimator's performance [19].

Coupling the GPS system and the IMU together in a Kalman filter structure offers many advantages. The following equations, which relate the accelerometer measurements from the IMU to the true vehicle velocity, show how this is possible:

$$\begin{aligned} a_z &\approx \dot{V}_z + g + b_{az} + \omega_{accel} \\ a_x &\approx \dot{V}_x + g\sin\theta + b_{ax} + \omega_{accel} \end{aligned} \tag{1}$$

where a_z, a_x represent the accelerometer measurements in the z and x directions, $\dot{V}_z$ and $\dot{V}_x$ are the true accelerations, g is gravity, θ is the total Euler pitch, $b_{az,ax}$ is the inherent sensor bias, and ω is the sensor noise. This model for the accelerometer measurement assumes that the noise is Gaussian and that the only biases arise from the sensor's inherent random walk ($b_{az,ax}$) and from gravity. The model also assumes that no scale factor is present, or that it has already been accounted for. The accelerometer (and gyroscope) random walks are modeled as first order Markov processes [12], being driven by white noise ω_m and having a time constant T_m as follows:

$$\dot{b}_{az,ax}(t) = -\frac{1}{T_m} b(t)_{az,ax} + \frac{1}{T_m}\omega_m(t) \quad (2)$$

The Kalman filter now takes the following form, where the accelerometer measurement is the input and the GPS velocity is the measurement:

$$\begin{aligned}
\begin{bmatrix} \dot{\hat{V}}_z \\ \dot{\hat{b}}_{az} \end{bmatrix} &= \begin{bmatrix} 0 & -1 \\ 0 & -\frac{1}{T_m} \end{bmatrix} \begin{bmatrix} \hat{V}_z \\ \hat{b}_{az} \end{bmatrix} \dots \\
&\quad + \begin{bmatrix} 1 \\ 0 \end{bmatrix} (a_z - g) + \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{T_m} \end{bmatrix} \omega \\
y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{V}_z \\ \hat{b}_{az} \end{bmatrix} + \eta_z \\
\begin{bmatrix} \dot{\hat{V}}_x \\ \dot{\hat{b}}_{ax} \end{bmatrix} &= \begin{bmatrix} 0 & -1 \\ 0 & -\frac{1}{T_m} \end{bmatrix} \begin{bmatrix} \hat{V}_x \\ \hat{b}_{ax} \end{bmatrix} \dots \\
&\quad + \begin{bmatrix} 1 \\ 0 \end{bmatrix} (a_x - g\sin\theta) \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{T_m} \end{bmatrix} \omega \\
y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{V}_x \\ \hat{b}_{ax} \end{bmatrix} + \eta_x
\end{aligned} \quad (3)$$

where η and ω are the measurement noise and process noises, respectively, $\hat{V}_{z,x}$ represents the velocity estimates, and $\hat{b}_{az,ax}$ represent the bias estimates. Both of these estimator models are observable. The road grade estimate is the arctangent of the two speed estimates.

$$\hat{\theta} = -\arctan\frac{\hat{V}_z}{\hat{V}_x} \quad (4)$$

Since the input to the Kalman filter is an accelerometer, the process noise is taken to be noise from the accelerometer where the noise driving the bias Markov process is included as well. Both are taken to be Gaussian white noise. This gives

$$\begin{aligned}
\omega &= \begin{bmatrix} \omega_{accel} \\ \omega_m \end{bmatrix} \\
\omega_{accel} &\sim N(0, \sigma_{accel}^2) \\
\omega_m &\sim N(0, \sigma_m^2) \\
E\{\omega\omega^T\} &= Q = \begin{bmatrix} \sigma_{accel}^2 & 0 \\ 0 & \sigma_m^2 \end{bmatrix}
\end{aligned} \quad (5)$$

The measurement noise is the noise on the GPS velocity signal, which is also assumed to be Gaussian white noise. Due to satellite orientations the noise is generally higher on the vertical speed measurement than on the speed over ground measurement. The measurement noise is taken to be

$$\begin{aligned}
\eta_x &\sim N(0, \sigma_{GPSx}^2) \\
\eta_z &\sim N(0, \sigma_{GPSz}^2) \\
E\{\eta_x\eta_x^T\} &= R_x = \sigma_{GPSx}^2 \\
E\{\eta_z\eta_z^T\} &= R_z = \sigma_{GPSz}^2
\end{aligned} \quad (6)$$

All noise characteristics match those used in [12]. This filter structure combines the advantages of the high accuracy, unbiased GPS velocity measurements with the high-update rate signals from the IMU, thereby offering high quality velocity estimates (and therefore a high quality road grade estimate) at a frequency suitable for control signals. The GPS measurement enables correction for any accelerometer biases, while the accelerometer allows for continued velocity tracking during GPS outages over short time intervals [19]. The simulation results are shown in III and display the achievable performance of this grade estimation method.

III. Results

A. Simulation

Carsim© was used to generate simulation data for initial results and analysis. The simulated run involved an SUV driving downhill on a constant -10% grade on a road with several turns. Simulated noise matching the sensor characteristics in [12] was added to the sensor outputs. Sideslip was estimated first by including the knowledge of road grade obtained from the road grade estimator. Sideslip was then estimated without the knowledge of road grade, resulting in a large deterioration in performance. Plots of the sideslip and roll estimates suggest confirmation of the initial hypothesis that the grade error propagates primarily through the roll estimate. The grade knowledge is used to rotate the yaw gyro into the navigation frame, but as stated before this yields only a marginal improvement. It should be noted that in general, random walk from the integration of the yaw gyro will always lead to errors in the generated lateral velocity measurement. A Monte Carlo simulation was performed to deal with the random walk by offering a statistical comparison of the performance with and without grade knowledge.

Figs. 1 and 2 show the estimates of the vertical states, while Fig. 3 shows the vertical velocity estimation error converging to within the predicted error bounds. Fig. 4 shows the standard deviation of the vertical velocity estimation error to be within the mean of the predicted bounds. The longitudinal velocity estimation error shows similar performance. The results shown in these plots indicate successful, accurate estimation of the vertical velocity and road grade using the means described in II-B. Fig. 5 shows the vehicle path in the horizontal plane. Figs. 6 and 7 show sideslip and roll estimates which incorporate the road grade estimate. These results show that the road grade estimate can be used to aid the sideslip estimation methods presented in [7] and [9], allowing for accurate sideslip esitmation on roads with large grades. Figs. 8 and 9 show examples of the degredation in performance caused by a lack grade knowledge when in the presence of steep grades. Specifically, Fig. 9 shows this error in the roll estimate, which becomes the primary error source in the sideslip estimate. This error arises because no knowledge of the road grade is incorporated into the IMU mechanization, resulting in an inaccurate value of the roll rate. It assumes a zero Euler pitch angle which is not true because of the road grade. This error will grow over time as the roll rate is integrated. Tab. I lists the

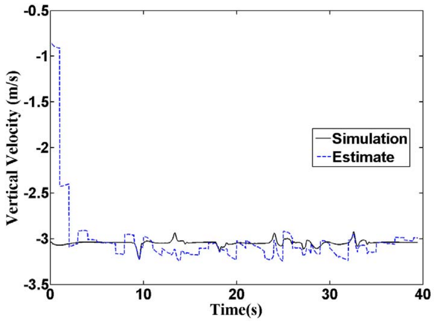

Fig. 1. Vertical velocity estimate.

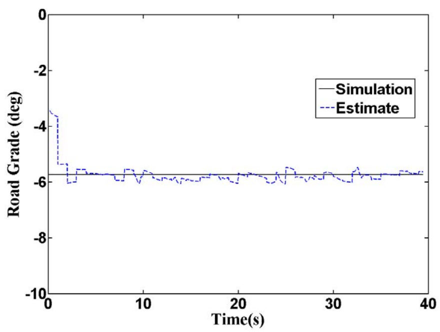

Fig. 2. Road grade estimate.

empirical statistical characteristics of the maximum errors in each run over 1000 iterations of the Monte Carlo simulation. It offers a comparison between state estimates when the grade knowledge from the road grade estimate is used and when it is not. The error growth over each run is due to random walk, and therefore is different for each run. The maximum error for each run is taken, and the mean and standard deviation of these maximums are listed in Tab. I. It should be noted that this simulated maneuver lead to a continuous integration of the yaw sensors used in sideslip estimation for 30 seconds, while the roll sensors used in the roll estimation were integrated over the entire 40 seconds.

TABLE I
MONTE CARLO SIMULATION MAX ESTIMATION ERROR RESULTS (DEG)

	With Grade Est.		Without Grade Est.	
	Sideslip	*Roll*	*Sideslip*	*Roll*
Mean	1.057	0.915	3.685	6.198
1σ	0.282	0.263	0.454	0.292

B. Experimental Results

It is desirable to determine whether or not the road grade estimate can serve as a suitable substitute for a pitch rate gyro when estimating sideslip in the presence of road grade, since pitch rate gyros would be extra sensors on many vehicles that may already have GPS. In order to evaluate this potential, the

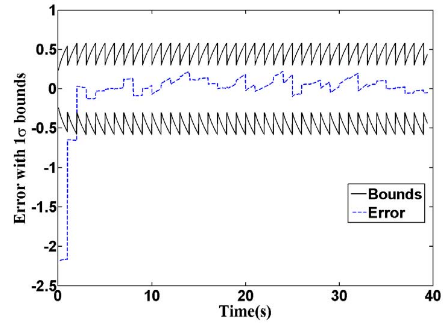

Fig. 3. Vertical velocity estimation error (m/s).

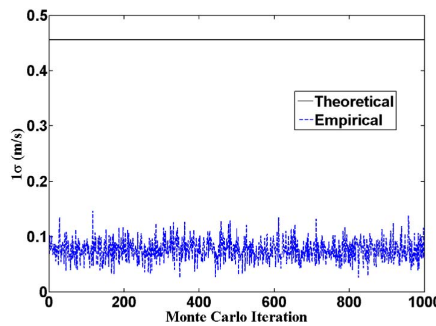

Fig. 4. Vertical velocity estimation error standard deviation.

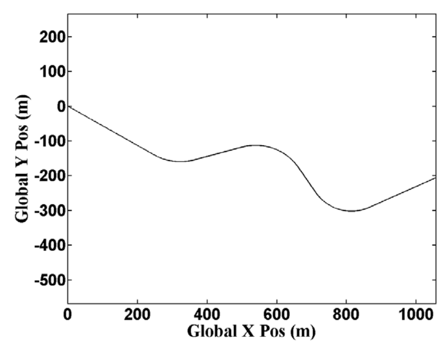

Fig. 5. Simulated vehicle path.

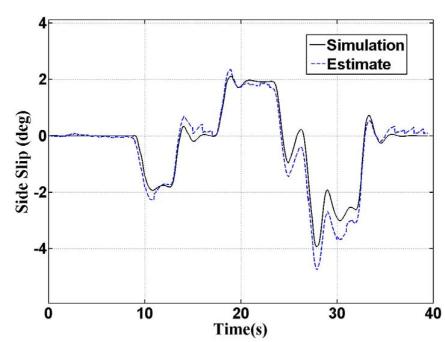

Fig. 6. Sideslip angle estimate when using road grade estimate.

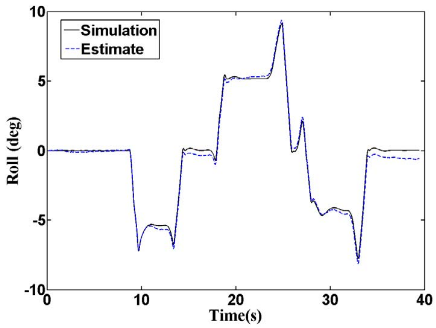

Fig. 7. Roll angle estimate when using road grade estimate.

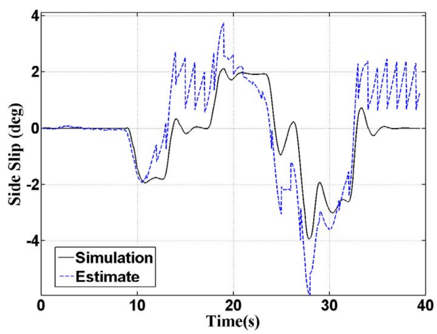

Fig. 8. Sideslip angle estimate when *not* using road grade estimate.

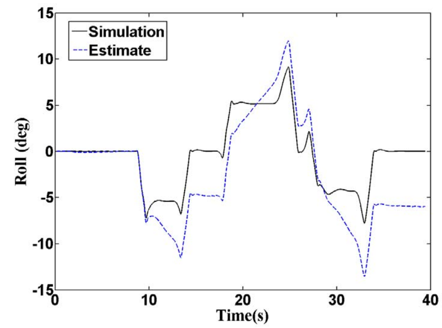

Fig. 9. Roll angle estimate when *not* using road grade estimate.

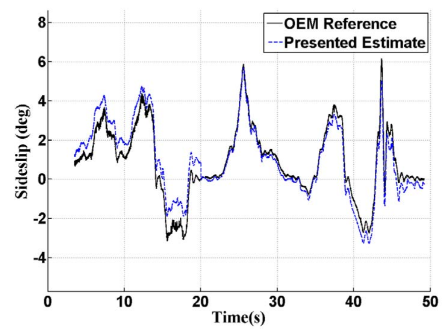

Fig. 10. Comparison of the pitch-rate based estimate and the estimate presented in this work.

state estimation methods used here are applied to experimental data provided by Ford motor company using a research sideslip angle estimate receiving roll and pitch angles computed using algorithms similar to those in [3]. The resulting estimates are compared with such experimental data which incorporates road grade information via the pitch rate gyro. A sensor mounting offset of 1.5 degrees pitch was determined off-line and accounted for by adding the offset to the grade estimate. Figs. 10 - 12 show the comparison of the pitch-rate sensor based computations including the sideslip estimate with the presented GPS pitch based computations. It can be seen that the sideslip estimate aided by road grade information from GPS closely matches the estimate aided by the pitch rate gyro, demonstrating that GPS information can be used to replace the pitch rate sensor while achieving the desired sideslip angle estimation performance. The closely matching grade estimates in Fig. 11 and the estimation residuals in Fig. 12 verify that the grade estimator is indeed functioning properly with the experimental data. Note that the 1.5 degree offset is accounted for in these plots. Fig. 13 shows the current performance achieved when attempting to estimate the mounting offset online. Currently this is not as accurate as desired; future work will further develop the capability to estimate this offset online.

IV. Conclusion

A GPS/INS based method of road grade estimation was used to aid in using a production RSC sensor set in computing the vehicle sideslip angle. Such a strategy aids a kinematic sideslip estimation method which is robust to road grade variation without the need of a pitch rate sensor. Estimation was performed using Carsim© simulated data and vehicle experimental data and yields promising results. This research shows the potential of using this road grade estimate to replace the pitch rate gyro in estimating a robust sideslip angle. Future work will investigate errors arising from large body pitch in the suspension or sensor misalignment, and how these might be accommodated.

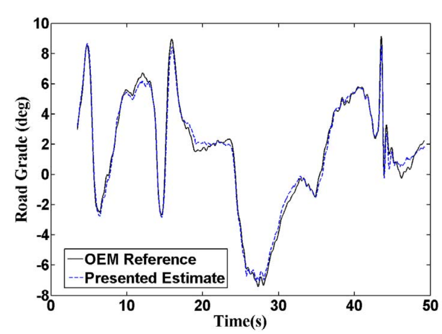

Fig. 11. Comparison of the road grade measurement to the estimate presented by this work.

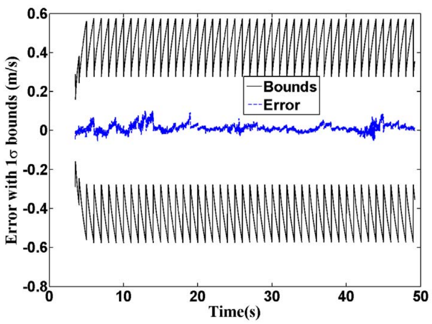

Fig. 12. Convergence of the residual of the vertical velocity state.

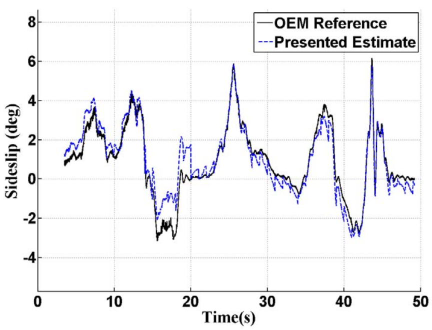

Fig. 13. Sideslip estimate when $1.5°$ offset is accounted for by online bias estimation.

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