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Adaptive Fuzzy Output Feedback Control for Robot Manipulators

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***Abstract*— In this paper, we propose an adaptive fuzzy output feedback control method for trajectory tracking control problem for robotic systems. Using Lyapunov method, we first develop a stable adaptive fuzzy state feedback control algorithm by assuming that the systems output and its derivatives are available for feedback control design. The algorithm combines fuzzy systems with robust adaptive controller. The fuzzy system approximates the *certainty equivalent* (CE)-based optimal controller while robustifying adaptive control term is used to cope with uncertainties that appeared from the effect of external disturbance, fuzzy approximation errors and other modeling errors. Then, an output feedback form of the position-velocity (state feedback) controller is proposed where unknown velocity signal is replaced by the output of model-free linear estimator. We show via asymptotic analysis that the tracking performance of the output feedback design can recover the performance achieved under the state feedback control design. Finally, the proposed method is implemented and evaluated on a 2-DOF robotic system to demonstrate the theoretical development for the real-time applications.**

KEY WORDS: Robotics, Adaptive Fuzzy Systems, Output Feedback.

I. Introduction

OVER the past few decades, there has been growing interest in many researches for developing an output feedback methods for nonlinear systems. Some of these results have been applied on uncertain nonlinear robotic systems, see for example [11]-[19]. The performance of above cited CE-based nonlinear observer-controller algorithms rely on the fact that there exists a known robot model dynamics. For a robotic manipulator, however, it is very difficult to obtain an exact system dynamics which ensures robust reconstruction of unknown states in order to guarantee asymptotic tracking error convergence. This is mainly because that robot dynamics is associated with many structured and unstructured uncertainties that cannot be exactly modeled. As a result, the output feedback design that is based on a nonlinear observer-controller [11]-[19] leads to a poor tracking performance. Another important assumption in CE-based nonlinear adaptive control design is that the uncertain model parameters are required to appear linearly with respect to known nonlinear functions of the system dynamics. The main practical problem of this linearity assumption is that the observer (high-speed)-controller gains increase with the increase of unknown parameters and initial conditions demanding very large control efforts. The requirement of exponentially fast observer dynamics makes the CE-based adaptive output feedback approach even more complex to realize in real-time applications as high observer gains amplifies the input and output disturbance causing high-frequency control chattering activity. As a consequence, model-based classical adaptive control design, either for the state or the output feedback design, might not be practically implementable or might be very expensive as, in practice, the control efforts in most nonlinear control systems are limited. If applicable, then high controller-observer gains may excite unmodeled high-frequency dynamics as well as amplify measurement noise associated with the joint position signals resulting poor tracking performance.

In this paper, we introduce direct adaptive fuzzy output feedback control approach for trajectory tracking control problem of robotic systems. The idea of using fuzzy system is to approximate CE-based classical nonlinear adaptive robot controller only using output measurements. Over the past decades, many fuzzy and neural networks based robot control technique have been reported in the literature, (see for example [1, 2, 3, 4, 5, 6, 7, 8 and references therein], to name a few). Most fuzzy and neural-network based designs for robotic systems are assumed that the position-velocity signals are available for feedback control design. To the best of our knoweldge, only results in the area of neuro-fuzzy output feedback for robotic systems can be found in [22]. The method proposed in [22] uses two similar networks to construct the obsever-controller closed loop model. One network is used for a controller design and one is required for an observer design. In contrust with [22], the proposed design is more efficient in terms of cost and hardware implementation as the design uses only one fuzzy system to approximate CE-based nonlinear adaptive controller. This is possible as the proposed observer dynamics is free from uncertain parameters and nonlinear system dynamics. To establish Lyapunov stability arguements, the common approach in fuzzy control system is to assume that fuzzy approximation and modeling errors are bounded by a small positive constant. As a matter of fact, the designer can only develop a fuzzy system that uses finite number of fuzzy rules and fuzzy membership functions as memory space is limited in most practical application. As a result, large fuzzy approximation errors may enter into the closed-loop system which may render unstable control system. To deal with bounded uncertainties that appeared due to the presence

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of the fuzzy approximation errors, external disturbance and other modeling errors, we combine robust adaptive control term with fuzzy systems. The proposed design has two steps. In the first step, we develop a direct adaptive fuzzy control system as a state feedback provided that the system states are available for feedback design. The control algorithm comprises two components as an adaptive fuzzy system to estimate CE-based classical nonlinear adaptive control law and a robustfying control term together with an adaptation mechanism to learn bounded uncertainities that represents fuzzy modelling errors and other external disturbance. Using Lyapunov analysis, we show that state feedback based design ensures the globally asymptotic stability of all the signals in the closed loop system. In the second part, for the given set of initial conditions, we estimate the region of ineterest and saturate the controller outside the estimated region of interest. Then, we replace the velocity signals in saturated control law by the output of the estimator to formulate an output feedback form of adaptive fuzzy control approach. The key feature of the proposed method is that the design is more effective with respect to cost and real-time implementation as one does not require an additional fuzzy system for observer dynamics as the observer dynamics is free from uncertain parameters and nonlinearities. The rest of the paper is organized as follows; In section II, we describe system model. Section III presents a new direct adaptive fuzzy control approach. The design is assumed that all the states are available for feedback. In section IV, we remove this assumption by replacing velocity signals by the output of the linear observer to develop fuzzy output feedback control method for trajectory tracking control of robotic systems. Secetion V shows an implementaion results of theoretical development on a 2-DOF robot manipulator to illustrate theoretical development of this paper. Finally, section VI concludes the paper.

II. System Dynamics

The equation of motion for an n-link rigid robot is given by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + d \tag{1}$$

where $q \in \Re^n$ is the joint position vector, $\dot{q} \in \Re^n$ is the joint velocity vector, $\ddot{q} \in \Re^n$ is the joint acceleration vector, $\tau \in \Re^n$ is the control input torque, $M(q) \in \Re^{n\times n}$ is the symmetric and uniformly positive definite inertia matrix, $C(q,\dot{q})\dot{q} \in \Re^n$ is the coriolis and centrifugal loading vector, $G(q) \in \Re^n$ is the gravitational loading vector and d is the vector representing external disturbance and any other unmodel dynamical effects on the system. The model (1) can be decomposed into n dynamical equation as

$$\ddot{q} = \phi_f(q_1,q_2) + \phi_g(q_1)\tau + \phi_g(q_1)d \tag{2}$$

where $\phi_f(q_1,q_2) = -M(q_1)^{-1}\left[C(q_1,q_2)q_2 + G(q_1)\right]$ and $\phi_g(q_1) = M(q_1)^{-1}$ with $q_1 = q$ and $q_2 = \dot{q}$.

III. Adaptive Fuzzy controller: State Feedback approach

In this section, we propose a new direct adaptive fuzzy feedback control method for trajectory tracking control problem of robotic systems (1). The design is based on assumption that all the states are measurable from position-velocity sensors. Later, we modify state feedback based design via replacing the unknown state vectors by the output of the linear observer. Before start deriving this control method, let us introduce basic structure of fuzzy logic system that can approximate any well-defined nonlinear function over a given compact set with any degree of accuracy.

A. Fuzzy Logic system (FLS):

A FLS consists of four parts as the fuzzy rule base (knowledge base), the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier [1], [3] and references therein. The fuzzy rule base for the FLS comprises a collection of fuzzy if-then rules. The fuzzy inference engine utilizes the fuzzy if-then rules to perform a mapping from an input vector $x = [x_1, x_2, x_3, x_4, ..., x_n]^T \in \Omega_U \subset \Re^n$ to an output vector $y = [y_1, y_2, y_3, y_4, ..., y_m]^T \in \Omega_V \subset \Re^m$. Then the l-th *if-then* fuzzy rules are of the following form $R^l : If\ x_1\, is A_1^l\, and....and\ x_n is\ A_n^l\, then\ y\ is\ y^l$. Where $A_1^l,\ A_2^l,\ .\ .\ .\ ,\ A_n^l$ are fuzzy variables and y^l is the output of the fuzzy system. Now, using well-known product inference, center-average and singleton fuzzifier, we obtain the output of the fuzzy systems as

$$y(x) = \frac{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{A_i^l}(x_i)\right)\bar{y}^l}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{A_i^l}(x_i)\right)} = \Theta^T\xi(x) \tag{3}$$

where $\mu_{A_i^l}(x_i)$ is the membership function of the fuzzy variables x_i and Θ is the adjustable parameter vector as defined $\Theta = \left[\bar{y}_1^1\ \bar{y}_1^2\ ....\bar{y}_1^M, \bar{y}_2^1\ \bar{y}_2^2\ ....\bar{y}_2^M, ...\bar{y}_m^1\ \bar{y}_m^2....\bar{y}_m^M\right]^T$ and $\xi(x) = \left[\xi_1(x), \xi_2(x), \xi_3(x), ..., \xi_M(x)\right]^T$ is a set of fuzzy basis function defined as $\xi_l(x) = \frac{\prod_{i=1}^{n}\mu_{A_i^l}(x_i)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{A_i^l}(x_i)\right)}$.

B. Adaptive fuzzy state feedback control

The objective is now to design a state feedback based adaptive fuzzy system such that the manipulator joint position $q(t)$ asymptotically tracks the bounded desired joint position $q_d(t)$. If the nonlinear functions are known and the plant dynamics (2) is free from external disturbance, then one can use the following well-known CE principle based controller to achieve desired tracking objective

$$\tau = \phi_g^{-1}(e_1, q_d)\left[-\phi_f(e) + \ddot{q}_d + K_D\dot{e} + K_P e\right] \tag{4}$$

where $e_1 = q_d - q$, $e_2 = \dot{q}_d - \dot{q}$, $K_P \in \Re^{n\times n}$ and $K_D \in \Re^{n\times n}$ are symmetric, positive definite, constant diagonal matrices. Then using (4) in (2), we have the following error model

$$\ddot{e} + K_D\dot{e} + K_P e = 0 \tag{5}$$

Then, the choice of control design parameter K_D and K_P ensures that $\lim_{t\to\infty} e(t) = 0$ as $t \to \infty$. This performance can be obtained when nonlinear functions and parameter uncertainty in the plant dynamics are exactly known. In addition, the disturbance free robot dynamics is also required to obtain such a global asymptotic stability property of the tracking error signals. However, in practice, the plant dynamics are highly uncertain and nonlinear that associated with many structured and unstructured uncertainties, so obtaining a control law (4) that ensures $\lim_{t\to\infty} e(t) = 0$ is not possible. In this work, our goal is to propose an adaptive fuzzy logic approach to approximate optimal controller (4) that ensures global asymptotic property of all the signals in closed loop error dynamics. The overall control law is defined as follows

$$\tau = \tau_d + \tau_s \tag{6}$$

where τ_d is the direct adaptive fuzzy controller to estimate τ of (4) and τ_s is an additional robustifying adaptive control term designed through Lyapunov analysis. The control term τ_s is used to update the bounded fuzzy approximation errors, external disturbances and other modeling error uncertainties. Then the error model can be re-defined under control (6) in the following state-space form

$$\dot{e} = Ae + B\phi_g(e_1)\left[\tau - \tau_d\right] - B\phi_g(e_1)d \\ -B\phi_g(e_1)\tau_s \tag{7}$$

where $A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -K_P & -K_D \end{bmatrix}$ and $B = \begin{bmatrix} 0_{n\times n} \\ I_{n\times n} \end{bmatrix}$. Let us consider that there exists a unique positive definite matrix, P, which satisfies the following Lyapunov equation $A^TP + PA = -Q$ where Q is a positive definite design matrix. Now our main focus is to develop an adaptive controller τ_d. To do that, let us first define the optimal parameter vector as $\theta_d^* = \arg\min_{\theta_d\in\Omega_d}\left[\sup_{e\in\Omega_c}\left(\tau_d(e|\theta_d) - \tau\right)\right]$ where Ω_d and Ω_c are the compact set. The optimal parameters θ_d^* is used to minimize the fuzzy approximation errors which is defined as $w_e = [\tau_d(e|\theta_d^*) - \tau] + d$. We then construct the error model as follows

$$\dot{e} = Ae + B\phi_g(e_1)\left[\tau_d(e|\theta_d^*) - \tau_d(e|\theta_d)\right] - B\phi_g(e_1)w_e \\ -B\phi_g(e_1)\tau_s \tag{8}$$

To adjust the parameter vector θ_d in the adaptive fuzzy control law $\tau_d(e \mid \theta)$, we use well-known T-S fuzzy system of the form (3) as $\tau_d(e|\theta_d) = \theta_d^T\xi(e)$. With these parameters, the optimal parameter controller is then defined by $\tau_d(e|\theta_d^*) = \theta_d^{*T}\xi(e)$ Then, the error model (8) has the following form

$$\dot{e} = Ae + B\phi_g(e_1)\tilde{\theta}_d^T\xi(e) - B\phi_g(e_1)w_e - B\phi_g(e_1)\tau_s \tag{9}$$

with $\tilde{\theta}_d = \theta_d^* - \theta_d$.

Assumption 1: The fuzzy approximation errors, external disturbances and other unmodel dynamics are bounded by a positive constant as $\|w_e\| \le \theta_s^*$.

Assumption 2: As the mass matrix $M(e_1)$ is a symmetric, bounded and positive definite then we can use bound on the function $\phi_g(e_1)$ as $\|\phi_g(e_1)\| \le \phi_{go}(e_1)$ with $\phi_{go}(e_1) > 0$.

Note that the assumption 2 is not restrictive as the bound depends on known position error measurements e_1 that are available from encoders. We now use Lyapunov second method to obtain stability condition for the closed loop system. To do that, let us consider the following Lyapunov function candidate

$$V = \frac{1}{2}e^TPe + \frac{1}{2\Gamma_d}\tilde{\theta}_d^T\tilde{\theta}_d + \frac{1}{2\Gamma_s}\tilde{\theta}_s^2 \tag{10}$$

where $\tilde{\theta}_s = (\theta_s^* - \theta_s)$. The parameter θ_s^* is a bounded constant defined in assumption 1. Differentiating (10) with respect to time along with the tracking trajectory of (9), we have

$$\dot{V} = e^TPAe + e^TPB\phi_{go}(e_1)\tilde{\theta}_d^T\xi(e) + \Gamma_s^{-1}\tilde{\theta}_s^T\dot{\tilde{\theta}}_s - \\ e^TPB\phi_{go}(e_1)\left(w_e + \tau_s\right) + \Gamma_d^{-1}\tilde{\theta}_d^T\dot{\tilde{\theta}}_d \tag{11}$$

To cope with uncertainty that entered into the closed loop system as a result of the fuzzy approximation errors, external disturbances and any other modeling uncertainties, we consider the following robust adaptive control law $\tau_s = -\tanh\left(\frac{e^TPB\phi_{go}(e_1)}{\epsilon_o}\right)\theta_s$ with the small value of ϵ_o and $\tanh\left(\frac{e^TPB\phi_{go}(e_1)}{\epsilon_o}\right) = \left[\tanh\left(\left(\frac{e^TPB\phi_{go}(e_1)}{\epsilon_o}\right)_1\right), \tanh\left(\left(\frac{e^TPB\phi_{go}(e_1)}{\epsilon_o}\right)_2\right), ...\right]^T$. To obtain $\dot{V} \le 0$, one needs to satisfy the following inequalities $\left[\left(\Gamma_d^{-1}\dot{\tilde{\theta}}_d + e^TPB\xi(e)\phi_{go}(e_1)\right)\right] \le 0$ and $\tilde{\theta}_s^T\left[\Gamma_s^{-1}\dot{\tilde{\theta}}_s - \tanh^T\left(\frac{e^TPB\phi_{go}(e_1)}{\epsilon_o}\right)\left(e^TPB\phi_{go}(e_1)\right)\right] \le 0$. Then, adaptation laws should be chosen to ensure $\dot{V} \le 0$ as $\dot{\theta}_d = \Gamma_d e^TPB\phi_{go}(e_1)\xi(e)$, $\dot{\theta}_s = -\Gamma_s\tanh^T\left(\frac{e^TPB\phi_{go}(e_1)}{\epsilon_o}\right)\left(e^TPB\phi_{go}(e_1)\right)$. Using these adaptation laws, one has the derivative of V as

$$\dot{V} \le -\frac{1}{2}e^TQe \le 0 \tag{12}$$

It is important to note that adaptation law defined above may exhibit discontinuous property which may result in unbounded parameter estimate. To ensures that the parameter θ_d and θ_s remains bounded over the compact sets as $\theta_d \in \Omega_d$ and $\theta_s \in \Omega_s$, let us introduce a projection algorithm to limits the parameter estimates. To proceed with that, let us now define the compact sets $\hat{\Omega}_d$ and $\hat{\Omega}_s$ such that $\hat{\Omega}_d \subset \Omega_d$ and $\hat{\Omega}_s \subset \Omega_s$. The convex hypercube Ω_d is defined as $\Omega_d = \{x_i \le \theta_{di} \le y_i\}$. Let $\Omega_{\delta d}$ as $\Omega_{\delta d} = \{x_i - \delta \le \theta_{di} \le y_i - \delta\}$ where δ is chosen such that $\hat{\Omega}_d \subset \Omega_{\delta d}$ and adaptation gain Γ_d to be positive constant. Then, the projection of the parameter adaptation rule can be written as $\dot{\theta}_d = Proj(\theta_d, \Phi_d)$ with

$$[Proj(\theta_d, \Phi_d)]_i = \begin{cases} \Gamma_d\Phi_{di} & \text{if } x_i \le \theta_{di} \le y_i \;\; or \\ & \text{if } \theta_{di} > y_i \;\; and \;\; \Phi_{di} \le 0 \;\; or \\ & \text{if } \theta_{di} < x_i \;\; and \;\; \Phi_{di} \ge 0 \\ \Gamma_d\bar{\Phi}_{di} & \text{if } \theta_{di} > y_i \;\; and \;\; \Phi_{di} > 0 \\ \Gamma_d\breve{\Phi}_{di} & \text{if } \theta_{di} < x_i \;\; and \;\; \Phi_{di} < 0 \end{cases}$$

where $\bar{\Phi}_{di} = \left[1 + \frac{y_i - \theta_{di}}{\delta}\right]\Phi_{di}$, $\breve{\Phi}_{di} = \left[1 + \frac{\theta_{di} - x_i}{\delta}\right]\Phi_{di}$ and Φ_{di} is the i-th element of $e^TPB\phi_{go}(e_1)\xi(e)$. Similarly, we can limit the parameter $\dot{\theta}_s$ using projection mechanism. To show that $\theta_d(t) \in \Omega_{d\delta}$ and $\theta_s(t) \in \Omega_{s\delta}$ $\forall t > 0$, we now apply above projection based adaptation mechanism to update θ_d and θ_s as $\dot{\theta}_d = Proj\left(\theta_d, \Gamma_d e^TPB\phi_{go}(e_1)\xi(e)\right)$ and $\dot{\theta}_s = Proj\left(\theta_s, -\Gamma_s\tanh^T\left(\frac{e^TPB\phi_{go}(e_1)}{\epsilon_o}\right)\left(e^TPB\phi_{go}(e_1)\right)\right)$. This impels that if the parameter estimates starts in the set $\theta_d(0) \in \Omega_d$ and $\theta_s(0) \in \Omega_s$ will remain in $\theta_d(t) \in \Omega_{\delta d}$ and $\theta_s(t) \in \Omega_{\delta_s}$ $\forall t \ge 0$. Based on our above analysis, let us state our main results for the position-velocity based adaptive control design.

Theorem 1: Let us consider the closed loop system (9) composed of the system (2) and the control input (6) along with *the updating law using the parameter projection is bounded and the tracking errors converge to zero as the time goes to infinity.*

Proof: From (12), we have $\dot{V} \leq -\frac{1}{2}e^TQe \leq 0$. If $e \neq 0$, then we conclude that $\dot{V}$ is negative in e space. This implies that $(V, e, \theta_d, \theta_s) \in L_\infty$. Since all the variables in the right hand side of (9) are bounded then we can also conclude that $\dot{e} \in L_\infty$. Hence, e is uniformly continuous and bounded. Now take integral (12) from 0 to T, we have $V(T) - V(0) \leq -\int_0^T \frac{\lambda_{min.}(Q)}{2}\|e\|^2$. Using Lyapunov equation (9), we can write $\int_0^T \frac{\lambda_{min.}(Q)}{2}\|e\|^2 \leq \frac{1}{2}e(0)^TQe(0) + \frac{1}{2\Gamma_d}\tilde{\theta}_d(0)^T\tilde{\theta}_d(0) + \frac{1}{2\Gamma_s}\tilde{\theta}_s^2(0)$ which implies that $e \in L_2$. Since e is uniformly continuous over the interval $[0, \infty)$ with $T = \infty$ then, using Barbalat's lemma [9], we can conclude that $\lim_{t\to\infty} \dot{V} = 0$ and $\lim_{t\to\infty} e = 0$.

IV. Adaptive fuzzy output feedback design

The stable state feedback algorithm developed in previous section is based on using the strict assumption that the output and its derivatives are available for feedback. This assumption may become an important issue in the real-time operation. This is because advanced robotic systems only equipped with encoder that provides joint position measurement. The velocity sensors are removed to reduce weight and cost of the systems [20]-[21] and [24]. In this section, our aim is to remove this assumption from algorithm (6) by incorporating a linear observer to estimate unknown velocity states as

$$\dot{\hat{e}}_1 = \hat{e}_2 + \frac{H_1}{\epsilon}\tilde{e}_1, \dot{\hat{e}}_2 = \frac{H_2}{\epsilon^2}\tilde{e}_1 \tag{13}$$

where $\tilde{e}_1 = (e_1 - \hat{e}_1)$ and ϵ is a design constant needs to be specified. The observer dynamics is independent of the model uncertainty, nonlinear dynamics and any other auxiliary dynamical nonlinearities. The observer dynamics can be re-written in the following compact form as

$$\dot{\hat{e}} = A_c\hat{e} + LD(\epsilon)\tilde{e}_1 \tag{14}$$

where $A_c = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$, $L = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}$ and $D(\epsilon) = \begin{bmatrix} \frac{1}{\epsilon}I_{n\times n} \\ \frac{1}{\epsilon^2}I_{n\times n} \end{bmatrix}$ with zero and identity matrix of 0 and I. Now our goal is to show that the performance achieved under adaptive fuzzy design (6) can be recovered asymptotically by the fuzzy output controller in which velocity signals are replaced by the output of the observer (13). This performance recovery analysis is shown via using singular perturbation theory. To proceed with that, we first develop singularly perturbed observer-controller closed loop system where the observer error model has the following compact form

$$\dot{\tilde{e}} = A_o\tilde{e} + B\left[\phi_{go}(e_1)\tilde{\theta}_d^T\xi(e) - \phi_{go}(e_1)\tau_s - \phi_{go}(e_1)w_e - Ke\right]$$

with $A_o = [A_c - \bar{L}C]$, $\bar{L} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}$, $K = [0_{n\times n}\ 0_{n\times n}; K_P\ K_D]$ and $C = [I_{n\times n}\ 0_{n\times n}]$. Now we redefine the observer error dynamics by scaled estimation error to construct singularly perturbed system. For this purpose, we introduce new variables as $\eta_1 = \frac{e_1 - \hat{e}_1}{\epsilon} = \frac{\tilde{e}_1}{\epsilon} \Rightarrow \epsilon\eta_1 = \tilde{e}_1$ and $\eta_2 = e_2 - \hat{e}_2 = \tilde{e}_2$ with a small positive parameter ϵ. Using this scaled estimation error, we construct the standard singularly perturbed systems as follows

$$\epsilon\dot{\eta} = A_o\eta + \epsilon B\left[\phi_{go}(e_1)\left(\tilde{\theta}_d^T\xi(e) - w_e - \tau_s\right) - Ke\right] \tag{15}$$

As we can see from the above singularly perturbed error model that if the uncertainty level increase, then the second term will also be increased. To reduce the influence of the uncertainties that is to minimize the effect of the second term, we have to use small ϵ to make the term close to zero which may cause large control action in the face of large scale uncertainties. To protect the plant from large transient control effort, we introduce saturated control technique where the input is bounded over the domain of interest provided by the state feedback controller (6). To do that, we use the Lyapunov analysis as follows. We first assume that all the initial conditions are bounded by the compact set Ω_b. Then we choose the initial sets as $\theta_d^*(0) \in \Omega_d$, $\theta_s^*(0) \in \Omega_s$ and $e(0) \in \Omega_e$. Using the universal approximation theorem, we also consider that the optimal parameter controller is bounded $\forall e \in \Omega_c$, that is, $\theta_d^* \in \Omega_d$. As we know from our state feedback design that if $\theta_d(0) \in \Omega_d$ and $\theta_s(0) \in \Omega_s$, then projection adaptive law ensures $\theta_s(t) \in \Omega_{\delta s}$ and $\theta_d(t) \in \Omega_{\delta d}$ $\forall t \geq 0$. Then, we can choose $c_{1d} = \max\limits_{e\in\Omega_e} \frac{1}{2}e^TPe$, $c_{2d} = \max\limits_{\theta_d^*\in\Omega_d, \theta_d\in\Omega_{\delta d}} \frac{1}{2\Gamma_d}(\theta_d^* - \theta_d)^T(\theta_d^* - \theta_d)$ and $c_{3d} = \max\limits_{\theta_s^*\in\Omega_s, \theta_s\in\Omega_{\delta s}} \frac{1}{2\Gamma_s}(\theta_s - \theta_s^*)^2$ where $c_{1d} > 0$, $c_{2d} > 0$ and $c_{3d} > 0$. Then we choose a positive constant c such that $c > (b = c_{1d} + c_{2d} + c_{3d})$. Note that the set $e(0) \in \Omega_e$ is selected to cover any given bounded initial conditions, but once they are chosen the controller will be designed based on them. Now we define the compact set Ω_c such that $e(t) \in \Omega_c = \{e \mid e^TPe \leq c\}$ $\forall t \geq 0$ as the domain of interest for the state feedback based control design (6). Since the input τ_d and τ_s are continuous function with respect to e, Q_d, θ_d and θ_s then they are bounded on the compact set of these variables as $\Omega_c \times \Omega_d \times \Omega_{\delta d} \times \Omega_{\delta s}$ with $Q_d \in \Re^{3n} = [q_d,\ \dot{q}_d,\ \ddot{q}_d]^T \in \Omega_d$. Now we define the maximum bound for the control input τ as $S = (S_1 + S_2)$ provided that τ_d and τ_s have maximum bound as $S_1 = max|\tau_d(e, Q_d, \theta_d)|$ and $S_2 = max|\tau_s(e, \theta_s)|$. Then, during transient peaking period, the control input saturate outside the set Ω_{cr} with $c_r > c$ and we have $\tau^s = \tau$ where $\tau = \theta^T\xi(e)$ with $\theta = [\theta_d,\ \theta_s]^T$ for all $e(0) \in \Omega_e$, $\theta_s(0) \in \Omega_s$, $\theta_d(0) \in \Omega_d$, $e(t) \in \Omega_c$, $\theta_s(t) \in \Omega_{s\delta}$ and $\theta_d(t) \in \Omega_{d\delta}$. If we now replace the state vectors e in the control law by the output of the state estimator $\hat{e}$ then the bounded adaptive output feedback controller can also be achieved via saturated function. Then the closed loop system in state space form can be re-written in the following standard perturbed model

$$\epsilon\dot{\eta} = A_o\eta + \epsilon B\left[\phi_{go}(e_1)\left(\tilde{\theta}_d^T\xi(e) - \tau_s - w_e\right) - Ke\right] \tag{16}$$

$$\dot{e} = Ae + B\phi_{go}(e_1)\left[\tilde{\theta}_d^T\xi(e) - \tau_s - w_e\right] \tag{17}$$

with $\dot{\theta}_d = Proj(\theta_d, \phi_d((e - \zeta(\epsilon))\eta, Q_d, \theta_d))$ and $\dot{\theta}_s = Proj(\theta_s, \phi_s((e - \zeta(\epsilon)\eta), \theta_s))$. We now state the main result for the output feedback design:

Theorem 2: Consider the closed-loop control system defined by (16)-(17) composed of the plant (2) and the output feedback control law along with the projection parameter mechanism.

If we take the initial conditions $e(0) \in \Omega_e \subseteq \Omega_c$ and $\theta(0) \in \Omega$ then there exists $\epsilon^* > 0$ such that $\forall \epsilon \in [0, \epsilon^*]$, all the state variables of the closed loop system are bounded by a bound that can be made airbitrarily small with the small value of ϵ.

Proof: The proof of theorem 2 has two steps. In the first step, we show that there exists a short transient period during which the fast variable η decays to a small level as $O(\epsilon)$, that is, $\|\eta\| \leq k\epsilon$ with $k > 0$ and small $\epsilon > 0$. During this time period, the slow variables (e, θ) remain within a bounded subset of the domain of interest Ω_c. In the second step, we analyze the stability property of slow variables $(e(t), \theta(t))$ during time interval $t \in [T_1, T_3]$, where T_3 is the first time when $(e(t), \theta(t))$ exist from the set Ω_E. During this time interval, fast variable η approaches to the order $O(\epsilon)$ as $W(\eta(t)) \leq \epsilon^2\beta$, $\forall e(0) \in \Omega_e$ and $\forall \theta(0) \in \Omega$ $\forall t \in [T_1, T_3]$. Using this fact, we now prove that there is a constant such that $\forall \epsilon \in (0, \epsilon_1^*)$ the output feedback controller guarantees the stability of the singularly perturbed closed loop system $\forall t \in [T_1, T_3]$. Due to space limitation, we remove details proof and can be obtained from authors.

Remark 1: Notice from our design is that the adaptation law depends on the upper bound of $\phi_{go}(e_1)$. Since, the mass matrix $M(e_1)$ is a symmetric, bounded and positive definite then we can use upper bound on $\phi_g(e_1)$ as $\|\phi_g(e_1)\| \leq \phi_{go}(e_1)$. Moreover, one can choose $\phi_{go}(e_1) = 1$ to make the proposed design free from plant information accept the known sign of $\phi_{go}(e_1)$.

Remark 2: The proposed design uses only one fuzzy system, which is an important feature for practical application as it reduces the cost and computational efforts by saving the memory space.

Remark 3: The Lyapunov-stability analysis in most fuzzy control system reported in the literature relies on the fact that the fuzzy approximation errors, external disturbance as well as modeling error uncertainties are bounded by a small positive constant. In view of universal fuzzy approximation theorem, one may find a fuzzy system using with large number of fuzzy membership function to estimate any given real continuous function with a small fuzzy approximation errors. However, in real-time application, the designer can only develop a fuzzy system that uses finite number of fuzzy rules and fuzzy membership functions as memory space is limited in most practical application. As a result, large fuzzy approximation errors may cause unstable closed-loop control system. To deal with that, we introduce robust adaptive control term to learn and compensate bounded uncertainties of the fuzzy approximation errors, unmodeled dynamics and other unknown external disturbance.

Remark 4: The proposed observer-controller algorithm does not use robot dynamics and uncertain model parameters while they should be exactly known in CE-based nonlinear adaptive output feedback design [11]-[19]. In contrast with [11]-[19], the proposed design does not require nonlinear function to be linearly parameterized with respect to uncertain model parameters. Unlike model based design [11]-[19], theorem 2 does not need to compute tedious nonlinear-regressor model dynamics.

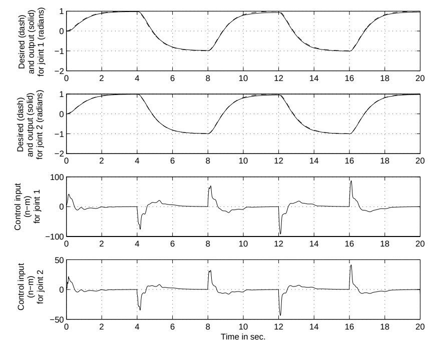

Fig. 1. The desired (black-dash), output feedback tracking (radians) (black-solid) and control input (newton-meters) for joints 1 & 2 for Theorem 1.

V. Design synthesis and implementation results

In this section, we present the design process of the proposed method on a 2-DOF robotic system for the real-time implementation. The dynamic equations for this robot system can be defined as [17] $[m_{11}\ m_{12}; m_{21}\ m_{22}][\ddot{q}_1; \ddot{q}_2] + [c_{11}\ c_{12}; c_{21}\ c_{22}][\dot{q}_1; \dot{q}_2] = [\tau_1; \tau_2]$ where $m_{11} = (\theta_1 + 2\theta_2 + 2\theta_2 \cos q_2)$, $m_{12} = (\theta_2 + \theta_2 \cos q_2)$, $m_{21} = (\theta_2 + \theta_2 \cos q_2)$, $m_{22} = \theta_2$, $c_{11} = -2\dot{q}_2\theta_2 \sin q_2$, $c_{12} = -\dot{q}_2\theta_2 \sin q_2$, $c_{21} = \dot{q}_1\theta_2 \sin q_2$ and $c_{22} = 0$. To generate the reference trajectory for the given robot model to follow, a square wave with a period of 8 seconds and an amplitude of ± 1 radians is pre filtered with a critically damped 2nd-order linear filter using a bandwidth of $\omega_n = 2.0$ rad/sec. Now we assume that the compact set as: $\Omega = \{-10 \leq \theta_d \leq 10, -10 \leq \theta_s \leq 10\}$ and take $\delta = 0.1$. The plant parameters are used for our evaluation as $\theta_1 = 4$ and $\theta_2 = 4$. Let us define the initial values of position and velocity error vectors as $e(0) = 0$. We first examine the property of state feedback based design of Theorem 1 by assuming that all the states are available. To develop the fuzzy system, we define the input variables as $\{q_1, q_2, \dot{q}_1, \dot{q}_2\}$. To formulate the fuzzy bassis function, let us choose five fuzzy sets for each input variables. Then membership functions for each inputs $q_j (j = 1, 2)$ are chosen as $\mu_{A_j^1}(q_j) = \frac{1}{1+\left[\exp\left(\frac{(q_j+0.75)}{0.1}\right)\right]^2}$, $\mu_{A_j^2}(q_j) = \exp\left(-\left(\frac{(q_j+0.5)}{0.3}\right)^2\right)$, $\mu_{A_j^3}(q_j) = \exp\left(-\left(\frac{q_j}{0.3}\right)^2\right)$, $\mu_{A_j^4}(q_j) = \exp\left(-\left(\frac{(q_j-0.5)}{0.3}\right)^2\right)$ and $\mu_{A_j^5}(q_j) = \frac{1}{1+\left[\exp\left(-\left(\frac{(q_j-0.75)}{0.1}\right)\right)\right]^2}$. The membership functions for each inputs $\dot{q}_j (j = 1, 2)$ are also defined as $\mu_{A_j^1}(\dot{q}_j) = \frac{1}{1+\left[\exp\left(\frac{(\dot{q}_j+1.13)}{0.2}\right)\right]^2}$, $\mu_{A_j^2}(\dot{q}_j) = \exp\left(-\left(\frac{(\dot{q}_j+0.75)}{0.45}\right)^2\right)$, $\mu_{A_j^3}(\dot{q}_j) = \exp\left(-\left(\frac{\dot{q}_j}{0.45}\right)^2\right)$, $\mu_{A_j^4}(\dot{q}_j) = \exp\left(-\left(\frac{(\dot{q}_j-0.75)}{0.45}\right)^2\right)$ and $\mu_{A_j^5}(\dot{q}_j) = \frac{1}{1+\left[\exp\left(-\left(\frac{(\dot{q}_j-1.13)}{0.2}\right)\right)\right]^2}$. Then we pick the controller

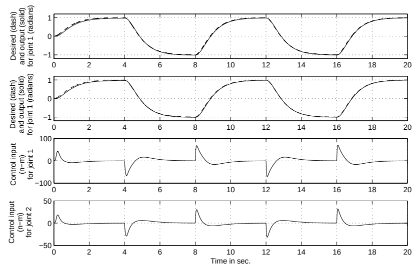

Fig. 2. The desired (black-dash), output feedback tracking (radians) (black-solid) and control input (newton-meters) for joints 1 & 2 for Theorem 2.

gains as $K_P = diag(150, 150)$ and $K_D = diag(120, 120)$. Now, we choose identity matrices of Q to solve the Lyapunov-equation $A^T P + P^T A = -Q$. The design constants K_P and K_D are chosen such that the closed loop system provides an acceptable transient tracking performance. For the given set of the initial conditions of interest, we choose the learning gains $\Gamma_d = 3000$ and $\Gamma_s = 3000$. To make the design free from system dynamics, we use $\phi_{go} = 1$. With these design constants, we apply theorem 1 on the given system. The tested results for Theorem 1 is depicted in Figure 1. Our aim is to show that performance achieved under Theorem 1 can be recovered by Theorem 2. To show that, let us replace the unknown velocity signals by the linear estimator defined by (13). Then we choose the observer-controller design parameters as $H_1 = I_{2\times 2}$, $H_2 = I_{2\times 2}$, $K_P = diag(150, 150)$, $K_D = diag(120, 120)$ and small value of observer design constant $\epsilon = 0.01$ that ensures exponentially fast response of the observer model. From the practical point of view, the design parameters (proportional and derivative gains) K_P, K_D, H_1, H_2 and ϵ With these design parameters, let us implement Theorem 2 on the given robotic system. The results are depicted in Figure 2. In view of Figures 1 and 2, we can see that the control performance under output feedback recovers the performance achieved under state feedback approach.

VI. Conclusion

In this work, we have presented a direct adaptive fuzzy output feedback controller for trajectory tracking control problem of robotic manipulators. A detailed convergence analysis has been given to show the boundedness of all the signals in the observer-controller closed-loop system. A robust adaptive control term has also been used to deal with the fuzzy approximation errors, external disturbances and other modeling uncertainties. The proposed design can be viewed as an alternative approach to CE-based nonlinear adaptive output feedback method. Unlike nonlinear adaptive output feedback design, the proposed method is free from model dynamics and uncertain model parameters. The implementation results of the proposed method on a 2-DOF robotic system have been given to demonstrate the theoretical development for the real-world applications. We have shown that controller-observer design parameters can be tuned to achieve semi-global stability property of all the closed loop signals.

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