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Robust Tracking Using Hybrid Control System

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***Abstract*— In this paper, a multi-model based hybrid sliding mode control (HSMC) system is proposed for trajectory tracking control problem of robotic systems. The idea of introducing multi-model/controller based HSMC design is to reduce the level of parametric uncertainty in order to reduce the controller gains that reduces the control effort. The key idea is to allow the parameter estimate of classical sliding mode control (SMC)design to be reset into a model that best approximates the plant among a finite set of candidate models. For this purpose, we uniformly distribute the compact set of unknown parameters into a finite number of smaller compact subsets. Then we design a family of candidate controllers for each of these smaller subsets. The derivative of the Lyapunov function candidate is used as a resetting criterion to identify a candidate model that closely approximates the plant at each instant of time. The proposed method is evaluated on a 2-DOF robot manipulator to demonstrate the effectiveness of the theoretical development.**

I. INTRODUCTION

The single model (SM) classical sliding mode control for robotic systems has been extensively studied by many researchers, (see [1-13], to name a few). Classical SMC is a powerful approach for uncertain nonlinear system. To ensure the closed-loop stability under SMC scheme, the design, however, demands very large control gain. The crucial practical problem is that the control gain has to increase with the increase of the parametric uncertainty in order to achieve good transient tracking performance. The well-known problem of having high control gain is that it amplifies the input and output disturbance causing high-frequency chattering and infinitely fast switching control phenomenon. As a consequence, the existing design might not be practically implementable or might be very expensive as, in practice, the control effort in most nonlinear control systems are limited. If applicable, then high controller gain may excite unmodeled high-frequency dynamics as well as amplify disturbance associated with the input and output resulting poor tracking performance. To reduce the control gains, we introduce multi-model based HSMC scheme for robust tracking control for robotic systems. The idea of this approach is to extend the classical SM-based SMC approach by allowing the parameter estimate to be changed into a family of candidate parameters model. First, we propose a pre-routed switching-logic technique, where an inequality for the derivative of the Lyapunov function is used as a resetting criterion. Results in this direction for single-input single-output (SISO) systems can be found [17]-[19] and references therein. It shows that the pre-routed switching nature may cause an undesirable transient tracking in the presence of large number of candidate controllers. This is mainly because if the number of candidate controllers become large then the switching has to travel through a large number of candidate controllers before converging to the candidate that guarantees the resetting condition. To improve the transient tracking performance in the pre-routed switching-logic, we allow the parameter estimates to be reset instantaneously so that the control system can improve overall tracking performance. This idea can be described as follows. First, we subdivide the compact set of uncertain parameters into a smaller compact subsets, and construct a family of candidate controllers corresponding to each of these smaller parameter subsets. Then, at each instant of time, we compare candidate controllers to see which control generates largest guaranteed decrease in the value of the Lyapunov inequality. If the controller that currently acting in the loop satisfies the Lyapunov inequality (pre-specified resetting inequality) then we keep it otherwise switch to the controller corresponding to the candidate model that best approximates the plant.

The rest of the paper is organized as follows: In section II, we formulate the problem associated with the SM-based SMC approach that motivates this works. In section III, multi-model based HSMC design is introduced to improve the transient tracking response as well as to reduce the control chattering phenomenon from classical SMC approach. This section IV presents implementation results to demonstrate the theoretical development of this paper. Finally, section V concludes the paper.

II. CLASSICAL SMC DESIGN

In this section, we illustrate the problem associated with the SM-based classical SMC approach. For this purpose, we analyze the stability property of the classical SMC design for an n-link rigid robot manipulators [14]-[15]. The error-state space model of this system dynamics can be written as

$$\dot{e}_1 = e_2, \dot{e}_2 = \phi_1(e, q_d, \dot{q}_d) + \phi_2(e_1, q_d)\tau - \ddot{q}_d \qquad (1)$$

where $e_1 = q - q_d$, $e_2 = \dot{q} - \dot{q}_d$, $\phi_1(e, q_d, \dot{q}_d) = -M^{-1}(q)\left[C(q,\dot{q})\dot{q} + G(q)\right]$, $\phi_2(e_1, q_d) = M^{-1}(q)$, $q \in \Re^n$ is the joint position vector, $\tau \in \Re^n$ is the input torque, $M(q) \in \Re^{n\times n}$ is the symmetric and uniformly positive definite inertia matrix, $C(q,\dot{q})\dot{q} \in \Re^n$ is the coriolis and centrifugal loading vector, and $G(q) \in \Re^n$ is the gravitational loading vector. Let us define the reference state as $\dot{q}_r = \dot{q}_d - \lambda e_1$, where $\lambda = diag[\lambda_1, \lambda_2, ..., \lambda_n]$ with positive constants λ_1, λ_2,.....,λ_n. Then, we define the sliding surface $S = e_2 + \lambda e_1$. The control objective is to drive the joint position $q(t)$ to the desired position $q_d(t)$. This objective can be achieved via selecting an input τ such that the sliding surface satisfies the sufficient condition as $\frac{1}{2}\frac{d}{dt}S_i^2 \leq -\eta_i|S_i|$ [4] and [11], where η_i is a

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positive constant. This condition implies that the energy of S will be decaying as long as $S \neq 0$. To obtain the control objective, we consider the following control law for the robot system (1)

$$\tau(e, Q_d, \hat{\theta}) = \hat{M}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G}(q) - \mathcal{K}S - Ksgn(S) \tag{2}$$

where $\ddot{q}_r = \ddot{q}_d - \lambda e_2$, $\hat{M}$, $\hat{G}$ and $\hat{C}$ are the estimate of M, G and C, $\mathcal{K} = diag[\mathcal{K}_1, \mathcal{K}_2, ..., \mathcal{K}_n,]$, $K = diag[K_1, K_2, ..., K_n]$ with $\mathcal{K}_n > 0$ and $K_n > 0$. Now using (1) and (2), we simplify the closed-loop dynamics as $M\dot{S} + (C + \mathcal{K})S = \triangle\beta - Ksgn(S)$, where $\triangle\beta = (\hat{M} - M)\ddot{q}_r + (\hat{C} - C)\dot{q}_r + (\hat{G} - G) = \triangle M\ddot{q}_r + \triangle C\dot{q}_r + \triangle G$, where $\triangle M = \hat{M} - M$, $\triangle C = \hat{C} - C$ and $\triangle G = \hat{G} - G$. We now use the following Lyapunov-like function candidate to establish the convergence condition for the closed-loop model

$$V = \frac{1}{2}S^T MS \tag{3}$$

where M is symmetric and positive definite matrix. As M is symmetric and positive definite then $V > 0$ for $S \neq 0$. The function V can be considered as energy indicator for S. Let us now show that the energy V decays as long as $S \neq 0$. To do that, we take the derivative (3) along the closed-loop trajectory and then use the property 2 of [16] to obtain $\dot{V}$ as $\dot{V} = -S^T\mathcal{K}S - \sum_{i=1}^{n} (S_i [K_i sgn(|S_i|) - \triangle\beta_i])$. From $\dot{V}$, we can see that $\dot{V} \leq 0$ holds only if $\sum_{i=1}^{n} (S_i [K_i sgn(|S_i|) - |\triangle\beta_i|]) \geq 0$. This can only be shown when $K_i \geq |\triangle\beta_i|_{max}$ with upper bound $|\triangle\beta_i|_{max}$ that satisfies $|\triangle\beta_i|_{max} > |\triangle\beta_i|$. If $S_i > 0$ and $K_i \geq |\triangle\beta_i|_{max}$, then we have $\triangle\beta_i - K_i sgn(|S_i|) = \triangle\beta_i + K_i \leq 0$. This implies that $S_i [\triangle\beta_i - K_i sgn(|S_i|)] \leq 0$. Similarly, if $S_i < 0$ and $K_i \geq |\triangle\beta_i|_{max}$ then we can write $\triangle\beta_i - K_i sgn(|S_i|) = \triangle\beta_i - K_i \geq 0$ such that $S_i [\triangle\beta_i - K_i sgn(|S_i|)] \leq 0$. Hence $\sum_{i=1}^{n} [S_i (\triangle\beta_i - K_i sgn(|S_i|))] \leq 0$. As $\mathcal{K}$ is a positive definite matrix then first term of $\dot{V}$ can be written as $-S^T\mathcal{K}S \leq 0$. Based on our above analysis, we can write $\dot{V}$ as $\dot{V} = \sum_{i=1}^{n} (S_i [\triangle\beta_i - K_i sgn(|S_i|)]) - S^T\mathcal{K}S \leq -S^T\mathcal{K}S \leq 0$. Then equation (3) can be viewed as an energy indicator for S. This implies the decay of the energy of S as long as $S \neq 0$. Thus, the sufficient condition $\frac{1}{2}\frac{d}{dt}S_i^2 \leq -\eta_i|S_i|$ is satisfied. To reduce the control chattering activity, we estimate the switching function $sgn(.)$ by using a smooth bounded saturation function $sat(.)$.

The level of uncertainty in classical SMC design can be reduced by adding an adaptation term. For this purpose, we propose to introduce an estimation law to develop an adaptive sliding mode control (ASMC) algorithm as $\tau(e, Q_d, \hat{\theta}) = Y(e, \dot{q}_r, \ddot{q}_r)\hat{\theta} - \mathcal{K}S - Ksat\left(\frac{S}{\phi}\right)$ with $\dot{\hat{\theta}} = -\Gamma Y^T(e, \dot{q}_r, \ddot{q}_r)S$, where $Y(e, \dot{q}_r, \ddot{q}_r)\hat{\theta} = \hat{M}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G}(q)$ [15], $\hat{\theta}$ is the estimate of the robot dynamics and its operating environments such as link masses and payloads, $\Gamma = diag[\Gamma_1, \Gamma_2, ..., \Gamma_n]$ with constant diagonal elements $\Gamma_n > 0$ and $Y(e, \dot{q}_r, \ddot{q}_r)$ is the regressor matrix. The parameter estimates $\hat{\theta}$ can be adjusted with the smooth parameter projection mechanism [20] as $\dot{\hat{\theta}}_i = [Proj(\hat{\theta}, \Psi)]_i$ for $\theta \in \Omega = \{\theta \mid a_i \leq \theta_i \leq b_i\}, 1 \leq i \leq p\}$, where Ψ_i is the i-the element of the column vector $-Y^T(e, \dot{q}_r, \ddot{q}_r)S$, γ_{ii} is the i the element of Γ and $\delta > 0$ is chosen such that $\Omega \subset \Omega_\delta$ with $\Omega_\delta = \{\theta \mid a_i - \delta \leq \theta_i \leq b_i + \delta\}, 1 \leq i \leq p\}$. The idea of introducing projection mechanism is to ensure that $\hat{\theta}(t)$ remains bounded on the set Ω_δ for all $t > 0$. Then, the closed-loop error model can be written as $M\dot{S} = Y(e, \dot{q}_r, \ddot{q}_r)\tilde{\theta} - (C + \mathcal{K})S - Ksat\left(\frac{S}{\phi}\right)$. The proposed adaptive control law is designed by using the control Lyapunov function as $V(e, \tilde{\theta}) = \frac{1}{2}S^T MS + \frac{1}{2}\tilde{\theta}\Gamma^{-1}\tilde{\theta}$, where $\tilde{\theta} = (\theta - \hat{\theta})$. Using the property 2 of [16], the time derivative V along the closed-loop error trajectories can be simplified as $\dot{V}(e, \tilde{\theta}) \leq -\lambda_{min}(\mathcal{K})\|S\|^2 - K\|S\| \leq 0$ $\forall e \in \Omega_c$, $\forall\hat{\theta}(0) \in \Omega$, $\forall\theta(0) \in \Omega$ and $\hat{\theta}(t) \in \Omega_\delta$.

A. Motivating example

In this subsection, we examine the developed Lyapunov stability condition for classical SMC scheme. More specifically, our interest is to investigate the effect of the parametric uncertainty on the control gain as well as on the control system performance. To begin with this analysis, we first consider that we know the uncertain parameter θ. Then the controller is a simple feedback-linearizing regulator as $\tau(e, Q_d, \theta) = M(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + G(q) - \mathcal{K}S - K\left(\frac{S}{\phi}\right)$ with positive constant K and ϕ. The origin of the closed-loop system under this control law can be shown globally exponentially stable provided that the value of positive constant K and ϕ are chosen via using well known pole-placement technique. Now, if we consider that parameter θ, representing the manipulator dynamics and its payloads, is unknown then we can estimate the parameter $\hat{\theta}$ of θ. Then, the continuous sliding mode control can be written as $\tau(e, Q_d, \hat{\theta}) = \hat{M}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G}(q) - \mathcal{K}S - Ksat\left(\frac{S}{\phi}\right)$ where $sat(.)$ is a bounded saturation function that satisfies $sat(y) = \begin{cases} -1 \text{ if } y < -1 \\ \;\; y \;\;\text{ if } |y| \leq 1 \\ \;\; 1 \;\;\text{ if } y > 1 \end{cases}$. It is intuitively clear that if $|\triangle\beta_i|$ is small then $|K_i|$ is small and if $|\triangle\beta_i|$ is large then $|K_i|$ is large. But, if $|\triangle\beta_i| \approx 0$, that is, $(\theta - \hat{\theta}) \approx 0$ with $|S| \leq \phi$ then this control law can recover the performance achieve under the control law $\tau(e, Q_d, \theta)$. However, in the face of large parametric uncertainties, the design demands very large values of K and $\frac{1}{\phi}$ to meet the desired tracking objective. To illustrate that, let us now consider a 2-link robotic system [16]. The dynamic equations for this system can be defined as

$$\begin{bmatrix} m_{11} \, m_{12} \\ m_{21} \, m_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} c_{11} \, c_{12} \\ c_{21} \, c_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{4}$$

with $m_{11} = (\theta_1 + 2\theta_2 + 2\theta_2 \cos q_2)$, $m_{12} = (\theta_2 + \theta_2 \cos q_2)$, $m_{21} = (\theta_2 + \theta_2 \cos q_2)$, $m_{22} = \theta_2$, $c_{11} = -2\dot{q}_2\theta_2 \sin q_2$, $c_{12} = -\dot{q}_2\theta_2 \sin q_2$, $c_{21} = \dot{q}_1\theta_2 \sin q_2$, $c_{22} = 0$, $\theta_1 = m_1 l^2$, $\theta_2 = m_2 l^2$ and l is the link lengths and m_1 and m_2 are the masses of link 1 and link 2, respectively,. The mass of the link 1 and link 2 assumed to be changed when the manipulator tracks the desired task with various payloads. Let us now generate the reference trajectory, $q_d(t)$, for the given robot model to follow, a square wave with a period of 8 seconds and an amplitude of ± 1 radians is pre filtered with a critically damped 2nd-order linear filter using a bandwidth of $\omega_n = 2.0$

rad/sec. The parameter $\theta \in \Re^2$ is assumed to be unknown but belongs to a known set as $\theta \in \Omega = [-10, 10]$. Let us define the initial values of the position and velocity error vectors as $|e(0)| = 2$. We now define the value of λ as $\lambda_1 = \lambda_2 = 2$. The value of K and ϕ can be chosen to ensure that the closed-loop trajectories converge to an invariant set where $|S| \leq \phi$. This implies to guarantee $K \geq |\triangle\beta|_{max}$ with $\triangle G = 0$ as given system operating in the horizontal plane. We now show convergence property of the closed loop trajectory that satisfies the Lyapunov-stability property as $\dot{V} + \sigma V \leq 0$ with $\sigma = \frac{\lambda_{min}\mathcal{K}}{\lambda_{max}(M)}$. To show $\dot{V} + \sigma V \leq 0$, the value of K and ϕ are required to meet the inequality as $\frac{K}{\phi} \geq 771.1321$ with $\Omega_o \subset \{S(0)^T M(0) S(0) \leq 3357.2\}$. If $|S| > \phi$ then the control gain K has to satisfy the inequality as $K \geq 771.1321$. Notice that if we consider that there is no uncertainty, that is, $|\hat{\theta}_1 - \theta_1| = 0$ and $|\hat{\theta}_2 - \theta_2| = 0$, then one has $|\triangle\beta| = 0$. This implies $K = 0$ and $\dot{V} \leq -g_0 V$. This means that the design requires to satisfy less conservative condition. The value of the feedback controller gain $\mathcal{K}$ and λ can be chosen such that they ensures an acceptable transient as well as steady state tracking performance of the closed loop system [21]. Note that the value of $\frac{K}{\phi}$ will increase with the increase of the parametric uncertainty $\triangle\beta_i$. However, small value of ϕ and high value for the discontinuous gain K resulting poor tracking performance in the presence of high-frequency unmodeled dynamics as well as the input and output disturbance. To reduce control gains, we introduce an alternative method in the following section that allows the designer to keep smaller value of K and higher value of ϕ.

III. Hybrid Sliding Mode Control

We have examined the stability property of the classical SMC technique on robotic manipulators. It shows that the control gain increases with the increase of the parametric uncertainity in order to achieve good transient tracking performance. The main reason for showing poor transient tracking behavior is because of the assumption that the nonlinear functions to be linearly parameterized with respect to unknown parameters. To improve the transient tracking performance, one may increase the controller gains. The problem, however, is that the high-control gain require to increase with the increase of the parametric uncertainty resulting very large control effort. In particular, when the level of uncertainty is large, three parameters of the observer-controller (Γ, K and $\frac{1}{\phi}$) require to be very high to ensure good transient tracking performance. However, the use of high-gain is not a practical solution as high gain causes the control system to deteriorate as it increases the steady-state noise sensitivity causing control chattering activity. In practice, such a large control effort based design may not be realizable as control inputs in most practical system designs are restricted. To reduce the control gain, we introduce multi-model based SMC technique that allows the designer to keep smaller value of Γ and K and higher value of ϕ. The main goal of this idea is to reduce the level of uncertainty via resetting the parameter estimate of classical SMC into a model which best approximates the plant at each instant of time. This implies to identify a control vector, $\tau(e, Q_d, \theta)$, corresponding to a model θ that closely approximates the parameters of the manipulator and its loads that operating in the workspace. To choose most closest possible model from a family of candidates, we introduce on-line estimation of the derivative of the Lyapunov-function candidate. The proposed idea can be designed as follows. We first consider that the unknown plant parameters, θ, belongs to a known but comparatively large compact set Ω, where θ denotes the inertial parameters of the robot arms and its load operating in the work space. Then, we equally distribute the parameter set Ω into a finite number of smaller compact subsets such that $\theta_i \in \Omega_i$ with $\Omega = \bigcup_{i=1}^{N} \Omega_i$ and $\theta \in \Omega_i$. Then, for a given compact set of the initial condition of interest $e(0) \in \Omega_{co}$, we design a HSMC, bounded in e via saturating outside the region of interest Ω_c where $\Omega_c = \begin{bmatrix} 0.5\lambda^2 M & 0.5\lambda M \\ 0.5I & 0.5M \end{bmatrix}$ corresponds to each of these smaller parameter subsets as

$$\tau^i(e, Q_d, \theta_i) = Y(e, \dot{q}_r, \ddot{q}_r)\theta_i - \mathcal{K}S - K_i sat\left(\frac{S}{\phi_i}\right) \quad (5)$$

with $(\theta, \theta_i) \in \Omega_i$, such that for every $\theta \in \Omega_i$ all the signals in the closed-loop system under (5) started inside the sets Ω_{co} are bounded and the output tracking error trajectories converge to zero. The value of λ and constant diagonal elements of the positive definite matrices $\mathcal{K}$ are chosen in such a way that they ensures an acceptable transient performance of the closed-loop system, The discontinuous control gains K_i and $\frac{1}{\phi_i}$ are selected such that $(\theta, \theta_i) \in \Omega_i$. The control term $\mathcal{K}S$ is common to all the N candidate controllers. The regressors model $Y(e, \dot{q}_r, \ddot{q}_r)$ [15] is also common to all the candidate controllers.

The distribution of the model sets Ω is based on *a priori* known bound of the robot dynamics and its operating environments. We consider nominal parameter model for each smaller compact subsets, Ω_i,. The parameter sets can also be split into non-uniform and non-overlapping regions as long as it covers the entire parameter space. However, for non-overlapping and non-uniform case, one requires to use strict assumption such that there exists a single model that guarantees asymptotic tracking property. Our main focus is now how to identify a suitable model/control pair, $\tau(e, Q_d, \theta)$, from a finite set of candidate pairs, $\tau^i(e, Q_d, \theta_i)$,. To supervise which candidate model will be used to generate the final control vector, a logic is required to devlop in such a way which guarantees that all the signals in the closed-loop systems are bounded, and the error trajectories converge to zero. We consider that there exists a small time constant, say t_d, such that the solution of the closed loop system is well defined. The candidate controllers are saturated outside of the region of interest Ω_c to protect the plant from switching affect.

First, we introduce a pre-routed switching-logic to identify a controller corresponding to the parameter model θ_i*, with $i = i*$ and $i* \in \mathcal{M}(i*)$, from a finite set of candidate models using on-line estimation of the Lyapunov-inequality. A pre-routed logic can be found in [17]-[19] for a class of SISO systems, where a quadratic performance index based model identification errors is employed as a resetting condition. The logic that we proposed in Algorithm 1 is based on using

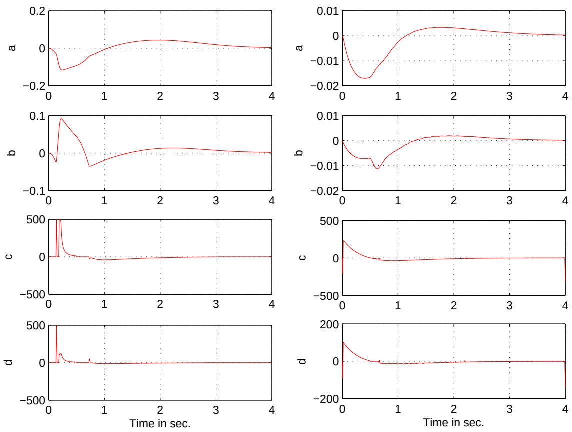

Fig. 1. The implementation results with Theorem 1 and Theorem 2 under $\tilde{\theta} = 8$: Left column is for Theorem 1 and right column is for Theorem 2, where a: output tracking errors (radians) (red) for joint 1, b: output tracking errors (radians) (red) for joint 2, c: control input for joint 1, d: control input for joint 2.

the inequality for the derivative of the Lyapunov-function candidate.

Algorithm 1: *Suppose that the controller index $i \in \mathcal{M}$ is acting in the loop at time t. Then we follow the following pre-routed switching-logic to identify a controller that satisfies the pre-specified Lyapunov inequality*

- **A.** *Assuming that the initial time $t_o = 0$, controller index $i \in \mathcal{M} = \{1, 2, 3, ..., N\}$ and a dwell time constant $t_d > t_o$.*
- **B.** *Then, we put the classical SMC scheme, $\tau(e, Q_d, \hat{\theta})$, in the loop and dwell it for a short period of time $t \in [t_o, t_o + t_d]$.*
- **C.** *For $t \geq t_o + t_d$, we check the pre-specified resetting inequality using with the derivative of the Lyapunov-function candidate $\dot{V}(t) \leq 0$. If the inequality satisfies then we keep this classical SMC controller in the loop. If not then we put the first candidate controller, $\tau^i(e, Q_d, \theta_i)$, with $i = 1$.*
- **D.** *We again dwell this controller for small time t_d and monitor the inequality for derivative of the Lyapunov function to see whether or not the function decreasing sufficiently fast to switch to the next candidate controller. If the controller does not satisfy the inequality then we switch again to the next candidate controller, $\tau^i(e, Q_d, \theta_i)$, with $i = 2$. We repeat the search until we find a controller that satisfies the derivative of the Lyapunov inequality.*

Based on our above analysis, let us state our main results in the following Theorem 1.

Theorem 1 : Consider the closed loop system formulated by (1) and (5) under the switching-logic defined by Algorithm 1. Then there exists a time such that the controller according to the logic stated in Algorithm 1 is tuned to the plant that ensures $\dot{V}(t) \leq 0$. Then, the sliding mode control system with the estimator resetting condition ensures that all the signals in the closed-loop system are bounded and tracking errors converge to zero.

As the parameter sets Ω_i overlap then the candidate control that satisfies the resetting condition does not require to be unique.

Proof: The proof of Theorem 1 can be shown along the line of the switching-logic defined in Algorithm 1. Due to space limitation, we remove the details proof of Theorem 1.

The problem with the pre-routed supervisory logic is that when the number of candidate controllers become large then the long switching search may produce unacceptable transient tracking performance. This is mainly because, in the presence of large number of candidate controllers, the switching has to travel through a large number of candidate controllers before converging to the one that satisfies the Lyapunov inequality. In addition, if the parameter changes after switching events due to fault then the logic stated in Algorithm 1 will be insensitive to the parameter change which may result large transient tracking performance.

To avoid an unacceptable transient tracking from pre-routed switching-logic, we now allow the parameter estimates to be reset instantaneously using with the following switching Algorithm 2.

Algorithm 2: *Suppose that the controllers $i \in \mathcal{M} = \{1, 2, 3, ..., N\}$ and the resetting inequality are available at any time. Then, we follow the following logic to identify controller corresponding to a model that best approximates the plant at any instant of time*

- **A.** *Assuming that the initial time $t_o = 0$, the controller index $i \in \mathcal{M} = \{1, 2, 3, ..., N\}$ and the small positive dwell time constant $t_d > t_o$.*
- **B.** *We first apply classical controller $\tau(e, Q_d, \hat{\theta})$ and dwell it for some time, $t \in [t_o, t_o + t_d]$.*
- **C.** *At $t \geq t_o + t_d$, we check the derivative of the fixed Lyapunov inequality $\dot{V}(t) \leq 0$ to see which candidate controller satisfies the resetting condition. If the classical controller satisfies the resetting inequality then we stay with that controller until the moment of time the Lyapunov inequality violated. But, if the classical SMC controller does not satisfy the inequality, $\dot{V}(t) \leq 0$, then, at $t \geq t_o + t_d$, we reset to the candidate controller that satisfies the resetting condition.*
- **D.** *If the resetting inequality $\dot{V}(t) \leq 0$ never violated then there will not be any switching. Then, the output trajectory tracks the desired one, e.i., $q(t) \rightarrow q_d(t)$,.*
- **E.** *If at some time, say $t_i \geq t_o + t_d$ with $t_o = t_i$, tuned controller does not satisfy the resetting criterion then another candidate that guarantees $\dot{V}(t) \leq 0$ will be put into the system as there always exists a guaranteed minimum value of $\dot{V}(t)$ at that instant of time.*

Theorem 2 : Consider the closed-loop system formulated by using (1) and control input (5) under the switching-logic defined in Algorithm 2. Then, there exists a time such that the classical controller is reset into a family of candidate controllers, $\tau^i(e, Q_d, \theta_i)$ with $i \in \mathcal{M}$, that satisfies the Lyapunov inequality $\dot{V}(t) \leq 0$. Then, the closed-loop system under SMC along with the estimator resetting condition ensures that all the signals in the closed-loop system are bounded.

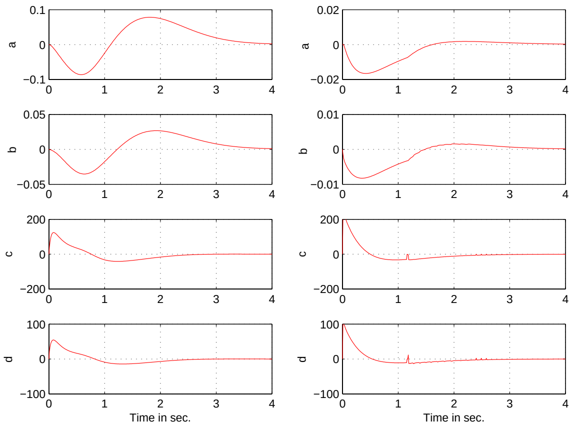

Fig. 2. The implementation results with Theorem 1 and SM-based SMC under $\tilde{\theta} = 8$: Left column is for classical SMC algorithm and right column is for theorem 2, where a: output tracking errors (radians) (red) for joint 1, b: output tracking errors (radians) (red) for joint 2, c: control input for joint 1, d: control input for joint 2.

Proof: The proof of Theorem 2 has two parts.

In the first part, we show that the parameter estimate is reset only when the control Lyapunov-function under multi-model $V_i(e, \tilde{\theta}_i)$ with $\tilde{\theta}_i = (\theta_i - \theta)$ is nonincreasing sequence with respect to i. Then we can conclude that $V_i(e, \tilde{\theta}_i)$ is bounded when $V_0(e, \tilde{\theta})$ with $\tilde{\theta} = (\hat{\theta} - \theta)$ is bounded.

In the second part, we prove that $V_0(e, \tilde{\theta})$ is bounded. Due to space limit, we omit the remaining proof and its available from the authors.

IV. Design and implementation results

This subsection presents the design and implementation process of the HSMC strategy on 2-DOF robotic system (4). To begin with that, we first consider that the plant parameter $\theta \in \Re^2$ of (4) belong to a known but comparatively large compact sets as $\Omega \in [-10,\ 10]$. We define the initial states as $|e(0)| \leq 2$, $|\hat{e}(0)| \leq 2$ and $\hat{\theta}(0) = 0$. Then we split the parameter set Ω equally into a finite number of smaller compact subsets as $\theta_i \in \Omega_i$ with $\Omega = \bigcup_{i=1}^{41}\{\Omega_i\}$, that is, $\Omega = \bigcup_{i=1}^{41}\{\theta_i\} = \{-10, -9.5, ., ., ., ., ., ., ., ., 9.5, 10\} \times \{-10, 10\}$. Note that it is important to reduce the number of the compact subsets for computational simplicity. This can be done by increasing the subinterval between two compact subsets without sacrificing tracking performance. This is possible as the manipulator parameters and the masses of the working loads are known to be within a specified range. The control design parameters λ_1, λ_2, $\mathcal{K}_1$ and $\mathcal{K}_2$ are chosen as $\lambda_1 = 2$, $\lambda_2 = 2$, $\mathcal{K}_1 = 15$ and $\mathcal{K}_2 = 15$. Note that the control gains λ_1, λ_2, $\mathcal{K}_1$ and $\mathcal{K}_2$ are common to all $i = 41$ candidate controllers. The value of λ_1, λ_2, $\mathcal{K}_1$ and $\mathcal{K}_2$ are required to choose in such a way that they ensures acceptable transient tracking performance. From the practical point of view, λ_1, λ_2, $\mathcal{K}_1$ and $\mathcal{K}_2$ can be designed according to well established design rules for PD control, for example, such that the closed loop would be critically damped and the response would be fast enough [21]. The discontinuous control gain K and $\frac{1}{\phi}$ as well as the learning gain Γ are chosen such that $(\theta, \theta_i) \in \Omega_i$. For our evaluation, these design constants are chosen as $K_{i1} = 15$, $K_{i2} = 15$, $\phi_{i1} = 0.7$, $\phi_{i2} = 0.7$ and $\Gamma = [\Gamma_1\ \ 0; 0\ \ \Gamma_2]$ with $\Gamma_1 = 10$ and $\Gamma_2 = 10$. Note that for the given set of initial conditions of interest, we can increase the value of ϕ and reduce the value of K. On the other hand, one may use boundary layer technique to avoid control chattering activity [11]. Then, we define the dwell time $t_d = 0.03$. Applying with the above design parameter sets, let us now construct a family of candidate controllers as a state feedback $\tau^i(e, Q_d, \theta_i) = Sat\left[Y(e, \dot{q}_r, \ddot{q}_r)\theta_i - \mathcal{K}S - K_i sat(\frac{S}{\phi_i})\right]$ with $i = 41$. Then, we implement Theorem 1 and Theorem 2 on the given model to examine the tracking performance of the logic defined in Algorithm 1 and Algorithm 2. The implemented results are given in Figure 1 under $\tilde{\theta} = 8$, that is, $i* = 37$,. The left column of the Figure 1 is for Theorem 1 and the right column of the Figure 1 is for Theorem 2. By comparing left and right column of the Figure 1, one can observe that the tracking errors under Theorem 1 of switching Algorithm 1 are larger than the errors under Theorem 2 of switching Algorithm 2. We also notice from our results is that the transient tracking errors under pre-routed switching-logic of Theorem 1 increase with the increase of i^*. This is mainly because the logic has to travel through a number of candidate controllers before converging to the one that satisfies the resetting inequality. As a result, relatively large control effort and transient tracking errors can be seen during transient phase under Theorem 1. Such undesirable transient behavior of pre-routed switching-logic can be reduced by using instantaneous switching-logic proposed in Algorithm 2 of theorem 2.

Let us now compare the tracking performance of Theorem 2 with the SM-based classical SMC algorithm. The implementation results are depicted in Figure 2 with $\tilde{\theta} = 8$ (right column is for Theorem 2 and left column is for the SM-based SMC design). The tests are conducted under the same set of the controller design parameters as considered for our last evaluation. By comparing left and right column of the Figure 2, we can observe that the tracking errors under Lyapunov-based switching almost closed to zero but relatively large tracking errors can be seen under single model based SMC algorithm. Note that the learning gains (the value of Γ) under classical SMC design is ten times faster than multi-model based SMC scheme of Theorem 2.

To examine the robustness of Theorem 2, we now introduce a more complicated situation. In this case, we assume that the plant parameter changes instantaneously over the compact set $\theta \in \Omega$. we consider that the plant is initially operating under the parameter $\theta_1 = 8$ and $\theta_2 = 8$. Then, at $t = 12$ sec., the parameter θ is changed from $\theta_1 = 8$ and $\theta_2 = 8$ to $\theta_1 = 4$ and $\theta_2 = 4$. Again, at $t = 30$ sec., the parameter θ is changed from $\theta_1 = 4$ and $\theta_2 = 4$ to $\theta_1 = 2$ and $\theta_2 = 2$. The controller design parameters for this experiment are selected as $K_{i1} = 125$, $K_{i2} = 125$, $\phi_{i1} = 0.7$ and $\phi_{i2} = 0.7$ with $i = 41$ and $\Gamma = [\Gamma_1\ \ 0; 0\ \ \Gamma_2]$ with $\Gamma_1 = 10$ and $\Gamma_2 = 10$. With these sets up, we then implement Algorithm 2 on the given system. The implemented results are presented in Figure

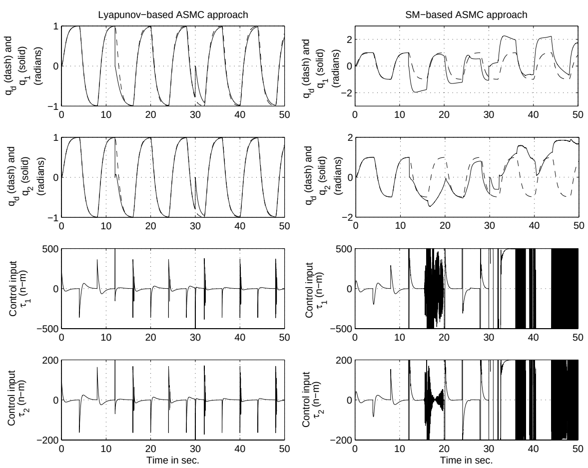

Fig. 3. The desired ($q_{d1} = q_{d2} = q_d$) (black-dash), output tracking (radians) (q_1 for joint 1, q_2 for joint 2) (black-solid) and control input (newton-meters) for joints 1 (τ_1) & 2 (τ_2) (left column is for Theorem 2, right column is for SM-based classical ASMC design).

3 (left column of Figure 3 is for Theorem 2). To compare this results with the SM-based classical SMC scheme, we now apply classical ASMC design on the same system with the same design constants of Theorem 2. The results are given in the right column of the Figure 3. By comparing left and right column of the Figure 3, one can observe that the output tracking trajectories under Theorem 2 converge to the desired one (see left column) even in the face of the modeling error uncertainties. However, the output tracking errors under SM-based ASMC approach (see right column) increases with the increase of parametric uncertainty. The effect of uncertainties can also be observed from the control action, where high frequency switching and control chattering activity increase with the increase of the parametric uncertainties.

V. CONCLUSION

We have shown that multi-model based HSMC can be used to improve the transient tracking performance for the trajectory tracking control problem of robotic systems. The key feature of the design is to allow the parameter estimate of the classical SMC to be changed into a model which closely approximates the manipulator dynamics. The proposed logic can be used to remove undesirable transient tracking control problem from existing pre-routed switching logic. The purpose of hybrid dynamical model is introduced to avoid of using chattering and infinitely fast switching control problem in robust control strategy. The evaluation on a 2-DOF robotic system clearly illustrates the demand of HSMC design.

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