Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

A New Distributed Coverage Algorithm Based on Hexagonal Formation

A.R.Pourshoghi
Department of Electrical Engineering
Amirkabir University of Technology
Tehran, Iran
ahmad.pourshoghi@gmail.com

H.A.Talebi
Department of Electrical Engineering
Amirkabir University of Technology
Tehran, Iran
alit@aut.ac.ir

***Abstract*- A distributed coverage algorithm for a team of robots based on a hexagonal formation is presented in this paper for complete coverage of free space. The main property of hexagonal formation is the high fault tolerance due to the overlap coverage of the three agents in each direction. Moreover, a coverage degree of 3 is also guaranteed for almost all points of the area.**

Another important feature of the proposed approach is that it does not require the absolute positions of the agents. The formed cluster is then considered as a single robot; hence, the coverage algorithm developed for a single robot can be extended to solve the resulting coverage problem. A common objective, especially in a deterministic node deployment, is to use a minimum number of nodes (sensors). In this approach, however, the maximum coverage is guaranteed for a fixed number of agents.

***Keywords*—coverage, formation, decentralized, multiagent**

I. INTRODUCTION

Wireless Sensor Networks (WSN) are fast evolving topics in many applications. WSNs are used in a broad range of applications related to surveillance, national security, military, heath care and environmental monitoring, etc. Sensors can be deployed in a large field to observe concerned objects for surveillance or to gather information, such as temperature, humidity or to detect certain objects of interest [5]. Among different tasks for WSN, coverage task of free space is common to many applications. Automated de-mining, lawn-mowing, vacuum cleaning all require the robots to sense over a desired area. The coverage algorithms can be classified in terms of different criteria such as centralized or decentralized, static or dynamic coverage, random or fixed initial deployment, etc. A brief description of these issues is given below.

A. *Decentralized vs. centralized*

The coverage algorithms are either centralized or distributed and localized. By distributed and localized algorithms, we refer to a distributed decision process at each node that makes use of only neighbourhood information [1].

An observation is the communication/computation power usage ratio, which can be higher than 1000 [1], therefore local data processing, data fusion and data compression are highly desirable.

The presented algorithm in this paper is **local and distributed**. That is to say, each agent run its own algorithm and all the agents use the same algorithm. Also each agent just uses other agents' information which are in its communication range (the neighbouring set).

B. *Maximize network lifetime or minimum number of sensors*

According to [4], the coverage concept could be categorized in three types: Blanket coverage, barrier coverage, and sweep coverage. In the blanket coverage, the goal is to achieve a static arrangement of sensors that maximize the total detection area. In barrier coverage the goal is to achieve a static arrangement of nodes that minimizes the probability of undetected penetration through the barrier, whereas the sweep coverage is more or less equivalent to a moving barrier [1].

Clearly, in blanket coverage and barrier coverage, the network finally achieves a static arrangement. Algorithms such as Virtual force (VFA) [6], Distributed Self-Spreading Algorithm (DSSA) [7], and CLP algorithm [5] belong to this category. In this type of algorithms, the amount of coverage depends on the number of agents, hence the number of agents increase according to the area of interest. A clear objective, especially in a deterministic node deployment, is to use a minimum number of sensors [1].

In the proposed algorithm, a cluster is formed to sweep the area with the number of agents is fixed (n) and the amount of coverage is measured relative to time and 100% coverage is reachable for any areas with these n agents. Another important feature in WSNs would be the fault tolerance property.

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C. K-coverage and fault tolerance

It is assumed that a network has a coverage degree k (k-coverage) if every location is within the sensing range of at least k sensors. Networks with a higher coverage degree can obtain higher sensing accuracy and be more robust to sensor failure. It is shown that when the communication range R_c is at least twice the sensing range R_s,a k-covered network will result in a k-connected network [8].A k-connected network has the property that removing any k-1 nodes will still maintain the network connectivity.

Formation based coverage is a topic which is not considered in coverage problem as much as other approaches due to its sensitivity to the faults.

In this paper, we focus on the sensor coverage problem, i.e. to cover an area, with a fixed number of sensors. It is assumed that all sensors are mobile and each node knows its distance to other nodes located within its range. The presented algorithm has a coverage degree of 3 for almost all the points of plane and by assumption $R_c \geq 2R_s$ the network is 3-connected so even by removing any 2 agents of the system, the full coverage is still guaranteed (with coverage degree of 1). The rest of the paper is organized as follows: In Section II, a background material is given for formation. Section III presents our new proposed algorithm. Simulation results are given in Section IV and Section V concludes the paper.

II. BACKGROUND

Consider n number of agents, numbered from $z_1,...,z_n$ located in the plane. The position of i^{th} agent is given by (x_i, y_i), which can be represented in the complex plane as $z_i = x_i + jy_i$. Here we can consider the following control scheme for stabilizing the agents to an equilateral *polygon* [2]:

$$\dot{z}_i = u_i = \frac{1}{2}(z_{i+1} - z_i)\left(1 - \frac{b^2}{|z_{i+1} - z_i|^2}\right) + \frac{1}{2}(z_{i-1} - z_i)\left(1 - \frac{b^2}{|z_{i-1} - z_i|^2}\right), \quad i = 1,...,n, \quad (1)$$

where b is a positive constant represent the desired distance between the agents. To better understand the motivation behind this scheme in regard to [2], we can consider from the first term that if $|z_{i+1} - z_i| > b$ then $1 - b^2/|z_{i+1} - z_i|^2 > 0$ and thus the agent moves towards z_{i+1}, similarly if $|z_{i+1} - z_i| < b$ then $1 - b^2/|z_{i+1} - z_i|^2 < 0$ and the agent moves away from z_{i+1}. Therefore, the effect of this term is to stabilize z_i to a distance b from z_{i+1}. The second term in the equation guarantees preliminary that the centroid will remain stationary throughout the formation evolution.

To study the stability of this formation, [2] has introduced the notation $e_i = z_{i+1} - z_i$ and study the stability of the e dynamics with respect to the equilibrium $|e_i| = b$, $\forall i$.

Rewriting u in terms of e we have:

$$u_i = \frac{1}{2}e_i\left(1 - \frac{b^2}{|e_i|^2}\right) - \frac{1}{2}e_{i-1}\left(1 - \frac{b^2}{|e_{i-1}|^2}\right) \quad (2)$$

We can also write dynamics $\dot{e} = \dot{z}_{i+1} + \dot{z}_i$ as:

$$\dot{e}_i = \frac{1}{2}e_{i+1}\left(1 - \frac{b^2}{|e_{i+1}|^2}\right) - e_i\left(1 - \frac{b^2}{|e_i|^2}\right) + \frac{1}{2}e_{i-1}\left(1 - \frac{b^2}{|e_{i-1}|^2}\right), i = 1,...,n \quad (3)$$

Which are defined on

$$S := \{e \in C^n : |e_i| > 0, \forall i\} \quad (4)$$

Let

$$circ(a_0, a_1, ..., a_{n-1}) := \begin{bmatrix} a_0 & a_1 & \cdots & a_{n-1} \\ a_{n-1} & a_0 & \cdots & a_{n-2} \\ \vdots & \vdots & & \vdots \\ a_1 & a_2 & \cdots & a_0 \end{bmatrix}$$

denote a circulant matrix and $A_1 := circ(a_0, a_1, ..., a_{n-1})$. We have

$$e = A_1 z.$$

Introducing the function

$$\phi(s) = \frac{1}{2} s \left(1 - \frac{b^2}{|s|^2}\right) \quad (5)$$

We can rewrite (2), (3) as:

$$u_i = \phi(e_i) - \phi(e_{i-1}) \quad (6)$$

$$\dot{e}_i = \phi(e_{i+1}) - 2\phi(e_i) + \phi(e_{i-1}) \quad (7)$$

Using the fact that

$$-A_1 A_1^T = circ(-2, 1, 0, ..., 0, 1)$$

We can rewrite (7) as:

$$\dot{e} = -A_1 A_1^T \phi(e) \quad (8)$$

Applying LaSalle's theorem and posing some lemmas the following theorem is achieved:[2]

Theorem: Considering system (8), for any initial condition $e(0) \in S$, as $t \to \infty$,the solution e(t) approaches the equilibrium of the system (8) on the set S , given by :

$$E := \left\{ e \in S : \phi(e) \in \ker A_1^T \right\}$$
$$= \left\{ e \in S : \phi(e_i) = \phi(e_j), \forall i, j \in T \right\}$$

Also using the stability of system (8) it is shown that z_i converges to a stationary equilibrium and also in limit as $t \to \infty$; $|z_{i+1} - z_i| = b, \forall i$

Note that by using $(z_{i+1} - z_i)$, the control law (1) does not require the absolute positions of the agents.

III. THE PROPOSED DEPLOYMENT ALGORITHM

A. The Assumptions

The following assumptions are made to develop our proposed deployment algorithm:

a) Each node *i* could measure the distances with its neighbours z_{ij} (for example using a laser range finder)

B)Sensing ranges of a nodes are uniform disks(a circle centred at the location of the node and sensing radius of R_s)

c) The communication range R_c of a node is at least two times of its sensing range R_s .

d) All nodes have the same sensing and communication ranges.

B. Algorithm Definitions

The proposed algorithm given in this section is a modified version of (1). The modification is to keep the agents on a hexagonal formation. Towards this end, another term has been added to (1) to assure that all agents encircle the seed. $\lambda = 1$ unless the seed fails (This will be described in further details in Remark 2).The pair wise distance between 6 agents is set to $2R_s$ (n=6, b $= 2R_s$) so that the sensing range of all agents are tangent to each other and an agent will be on centroid (see Fig.1)

So the control law for each agent is:

$$\dot{z}_i = u_i = \frac{1}{2}(z_{i+1} - z_i)\left(1 - \frac{4R_s^2}{|z_{i+1} - z_i|^2}\right) + \frac{1}{2}(z_{i-1} - z_i)\left(1 - \frac{4R_s^2}{|z_{i-1} - z_i|^2}\right)$$
$$+ \lambda(z_i - z_0)\left(1 - \frac{4R_s^2}{|z_i - z_0|^2}\right), \quad i = 1, ..., 6, \quad (9)$$

The cluster starts from a corner of environment. For example in Fig .2 all the robots start from the lower left corner of the environment.

The only constraint on initial deployment of agents is that, they must be connected (from communication point of view). In other words, each agent should be in

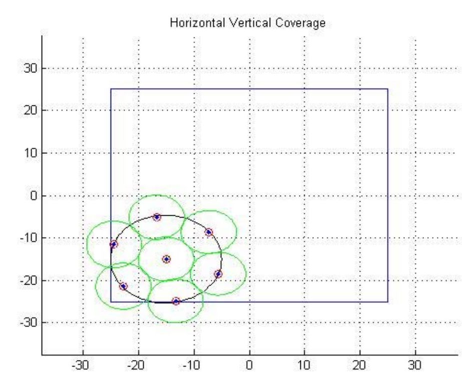

Figure1. Seven agents on a hexagonal formation

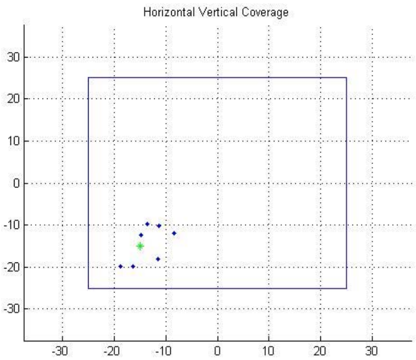

Figure2. Agents' initial deployment

communication range R_c of at least one other agent. Now, each agent computes its distance with its neighbouring agents and the one with minimum average distance with others is selected as seed agent and indexed as a_0 .

Once the seed agent is selected, it fixes its position and assumes a virtual circle with radius of $2R_s$ around itself and selects the nearest agent to this circle ($\min|z_{i0}-2R_s|$) and indexes it as a_1 . **This agent moves to decrease $|z_{10}-2R_s|$ to zero, hence it is on the circle now.** Then agent 1 selects nearest agent to itself and indexes it as a_2 and so on. Similarly, all agents will be indexed up to a_6 .

At this stage, each agent a_i could apply control law (9) just by computing the distance with its neighbouring agents and the seed. Hence, without having global positions of agents and just by using their relative distance with each other, the formation of Fig .1 will be achieved.

Note that by assumption $R_c \geq 2R_s$. ,the cluster is fully connected and all agents know each other indices .

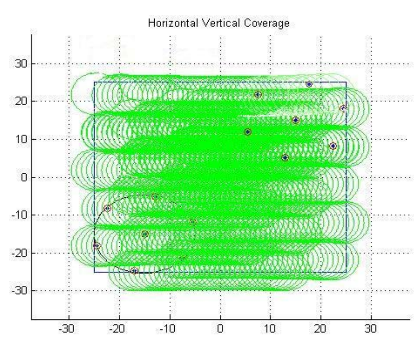

Figure3. Covered areas by agents. They start from lower left corner of area and end at the upper right of it

This assumption together with the constraint on the communication connectivity at initial deployment, guarantees that each agent can find another agent in its range for indexing. Using the above control scheme, we have a cluster of 7 agents on a hexagonal formation (an agent at the centroid and 6 at the vertices) and then start to coverage the environment using any single agent methods such as the one reported in [9]

In the simulations, we use a vertical horizontal algorithm to cover the area (in some literatures called boustrophedon).

The cluster starts from lower left corner of the environment and sweep the area in the right direction until its frontier agent reaches the boundary (Note that each point in sweep direction is sensed with three agents).Then the cluster move upward to reach to the next slice and start sweeping in the left direction. Algorithm will go on until the upper agent of the cluster sense the upper bound of the environment. It means that this slice is the last one and the coverage is complete at the end of this slice. Fig3 shows the sensing ranges of agent during coverage.

Remark 1: The proposed method depends on the number of agents being 7. In most cases, the number of the agents is not an imposed issue and can be selected by the designers according to the field area and the required performance. However, in situations where scaling is required for any reason (e.g. to cover a large area in a short time), the number of the agents can be selected as $7\times n$ to form n clusters. Then, each cluster can be considered as a single robot and use any multi robot algorithms to cover the area such as those used in [10], [11], [12].

Remark 2: If an agent fails, there are two possibilities. First, if it is a seed agent, the λ factor in (9) become zero and other agents can keep their formation on hexagonal form and continue covering with a coverage degree of 2. Second, if the failed agent is not a seed, the around agents start to form a pentagonal formation and keep circulating around the seed during the coverage.

IV. SIMULATION RESULTS

To verify the proposed coverage algorithm, a set of simulations were performed on a 50 by 50 unknown area with 7 agents deployed initially at the lower left corner of the area. All agents have the same sensing range R_s = 5 and communication range R_c = 12 .After the initial deployment they start to make hexagonal formation with a seed inside at the centroid (Fig. 4.a). Then the cluster starts to sweep the area horizontally till

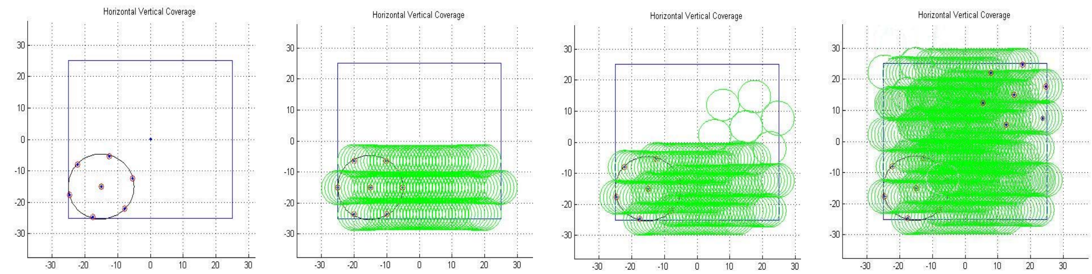

Figure4. (a) Making hexagonal formation with a seed inside at the centroid (b) sweeping the area horizontally till the frontier agent sense boundary (c) moving vertically up to next slice (d) The coverage is complete.

the frontier agent senses the right boundary (Fig. 4.b). Now, the cluster moves vertically $4R_s$ up to the next slice (Fig. 4.c) and again starts to sweep horizontally up to the left boundary. The final vertical move is determinate when the upper agent senses distance to boundary less than $4R_s$. In this case, the cluster does its final horizontal sweep and the coverage is finished at the end of this move (Fig. 4.d)

V. CONCLUSIONS

In this paper, a new distributed multi-robot coverage algorithm was proposed. This approach is based upon hexagonal formation in order to guarantee a coverage degree of 3 for most points of the area and high fault tolerance. The control laws prove a stable formation during the coverage. Another important feature of the proposed approach is that it does not require the absolute positions of the agents. A single robot algorithm was used to maintain the coverage by the formed cluster. Assumption $R_c \geq 2R_s$. guarantee that the network remain connected during the process. In the proposed algorithm, we formed a cluster and sweep the area so that the number of agents is fixed (n) and the amount of coverage is measured relative to time and 100% coverage is reachable for any areas with these n agents.

Future works could use good property of hexagonal formation with a seed inside to divide cluster into sub clusters against obstacle and merge it again into one. Another interesting approach is to use multi clusters with different initial deployment and merging them as they meet.

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