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A new piecewise-linear stochastic resonance model

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***Abstract*—We put forward a new piecewise-linear stochastic resonance(SR) model for detecting the weak signal under the strong noise background for the large parameter case. Parameters of this piecewise-linear stochastic resonance system are less correlated than those of the traditional continuous non-linear bistable system, so that it is much easier to adjust the response characteristic of this system and generate the SR under the large parameter signals case. Here we show its description equations and character parameters. Through numerically simulating its performance under the noiseless, noisy, small parameter and large parameter cases respectively, we illustrate that this system is very helpful for detecting the weak signal under the strong noise background not only for the small parameters case, but also for the large parameters case.**

***Keywords*—stochastic resonance, piecewise-linear model, bistable system, noise**

I. INTRODUCTION

Stochastic Resonance(SR) was first proposed as an explanation of the observed periodicity in the ice ages on earth[1-4],but has since been observed in a large variety of physical, chemical and biological systems[5].In recent years, the weak signal detection under the strong noise background through the stochastic resonance (SR) has been proposed as a useful means in a wide variety of systems[6-13],including bistable system[6,8,14,15],excitable systems[16],threshold systems[17-21],and biological systems[7,13].

Here we put forward a piecewise-linear bistable model. Because its potential function curve is just like the letter "W", in the following we directly call it W-model for convenience. Compared with the continuous non-linear model, parameters in this model are much less correlated, easier to adjust, which allows us to realize the SR under the large parameters case more easily. In the following we give the potential description for this W-model at first, and then give its dynamic equation. Finally, through a numerical simulation we will demonstrate that by this model the weak signal can be detected even under the strong noise background.

II. W-MODEL

The potential function of the W-model is given below:

$$U(x)=\begin{cases}-\dfrac{c}{a-b}(x+a) & when \quad x<-b\\ \dfrac{c}{b}x & when \quad -b\le x<0\\ -\dfrac{c}{b}x & when \quad 0\le x<b\\ \dfrac{c}{a-b}(x-a) & when \quad b\le x\end{cases} \tag{1}$$

where *a, b, c* are real parameters and 0<*b*<*a*,, c>0.

This is a piecewise-linear function, which has two stable states $x=\pm b$ and one unstable state $x=0$. The barrier height of the potential is $\Delta U=c$. In Fig. 1 we demonstrate the curve of $U(x)$ while *a* = 2, *b* = 1, *c* = 1. To illustrate the properties of this model, we show the potential function of the classical Langevin equation in Fig. 2. Here the non-linear double stable system described by the Langevin equation is given by the following equation.

$$\dot{x}=\mu x-x^3+H(t)+\eta(t) \tag{2}$$

where the $H(t)$ is the input signal, $\eta(t)$ is the noise and $\mu>0$ is the system parameter. Its potential function can be given by

$$U_l(x)=-\frac{\mu}{2}x^2+\frac{1}{4}x^4 \tag{3}$$

This Langevin equation has two stable state $x=\pm\sqrt{\mu}$ and one unstable state $x=0$. The barrier height of the potential is $\Delta U=\mu^2/4$. From potential functions we can see that the parameters of the W model are less correlated than those of the Langevin system. For example, in the W model we can easily change the barrier height of the potential by simply altering the parameter c, meantime keep the position of the potential well. However, in the Langevin system, the barrier

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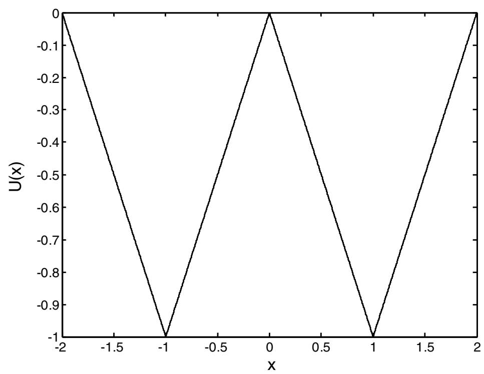


Figure 1. Illustration of $U(x)$, *a*=2, *b*=1, *c*=1

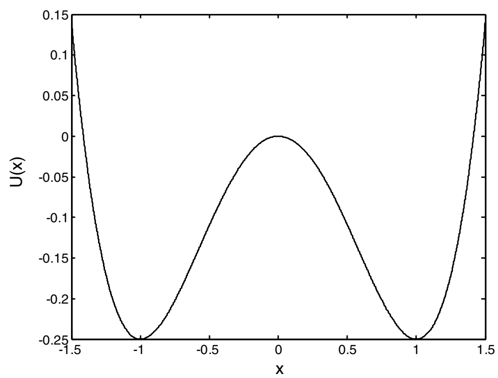


Figure 2. Illustration of $U_l(x)$, $\mu = 1$

height of the potential is largely correlated with its well position, which means if its height is changed, its potential well position will certainly be changed. Therefore the W model is very flexible to adjust.

Considering that the system is driven by the signal and the noise, the dynamic function of the W-type piecewise-linear model can be given by

$$\dot{x} = -\frac{dU(x)}{dx} + K(H(t) + \eta(t)) \tag{4}$$

where the $H(t)$ is the input signal and the $\eta(t)$ is the noise. *K* denotes the gain of the amplifier.

III. NUMERICAL SIMULATION

A. Simulation algorithm

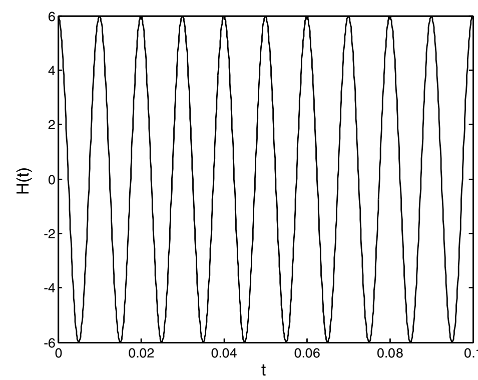
Since $U(x)$ is a piecewise-linear function, we use the Euler's method for the numerical simulation. So the Eq. (4) can be written as

$$x_{n+1} = x_n + h*[-\frac{dU(t_n)}{dx} + k(H(t_n) + \eta(t_n))] \tag{5}$$

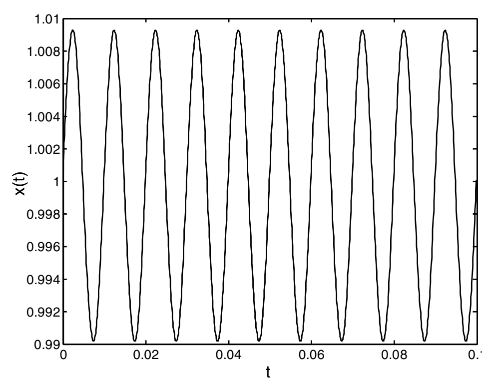
where *h* is the iteration step length.

B. The simulation under the condition that $\eta(t) = 0$

We set $\eta(t) = 0$ and $H(t) = A_0 \cos 2\pi ft$ in Eq. (5), where *f* is the frequency of the signal. Then we study the stable response of the W system under the period signal inputs. From the simulation we find that while the amplitude A_0 of the input signal is small and thus the output response cannot pass over the barrier, the system response will finally converge to a stable point, periodically vibrating around it. In Fig. 3 we give the simulation result under the condition that $A_0 = 6$, *f* = 100, *b* = 1, *a* = 2, *c* = 0.25 and *K* = 1. From the figure we can see that although the amplitude of the signal is between -6 and 6, the output signal can only periodically vibrate around the stable point with amplitude from 0.99 to 1.01 .If we increase the amplitude of the input signal A_0 , or the gain of the amplifier *K*, the amplitude of the output signal can be increased.



(a) Input signal



(b) Output signal

Figure 3. Illustration of the input and output signals under the condition that $\eta(t) = 0$, $H(t) = A_0 \cos 2\pi ft$, $A_0 = 6$, *f*=100 and *K*=1

In Fig.4 we show the simulation result under the condition of K = 100 and the other parameters remaining the same. Under this case, the output $x(t)$ can break through the potential barrier and move between two potential wells.

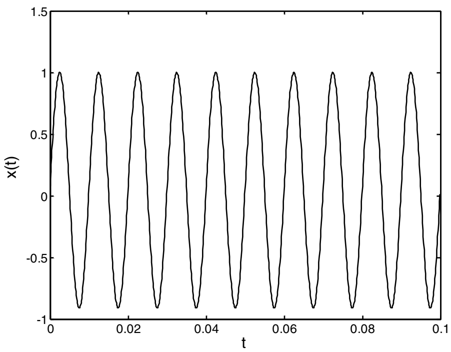


Figure 4. The simulation result under the condition of K=100, and other parameters remains the same as that in Fig. 3.

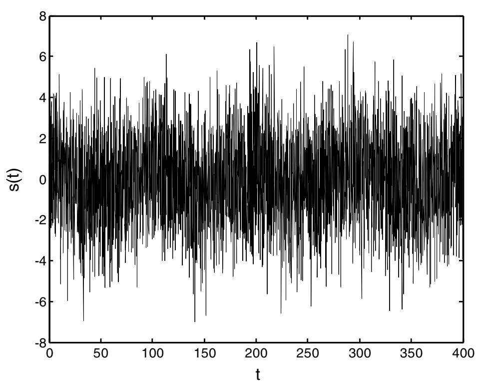
C. The simulation for small parameters and $\eta(t) \neq 0$

We set parameters in Eq.(5) as $\eta(t) \neq 0$, $H(t) = A_0 \cos 2\pi f t$, and $s(t) = H(t) + \eta(t)$, then discuss the stable response of the W-type bistable system. We assume that the $\eta(t)$ is the Gaussian white noise with zero mean, whose self-correlation function is

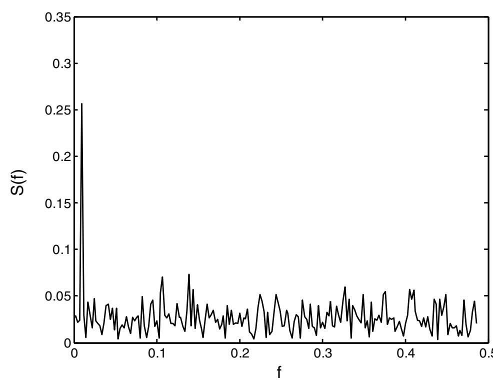
$$E[\eta(t)\eta(t+\tau)] = 2D\delta(t-\tau) \tag{6}$$

where D is the intensity of the noise.

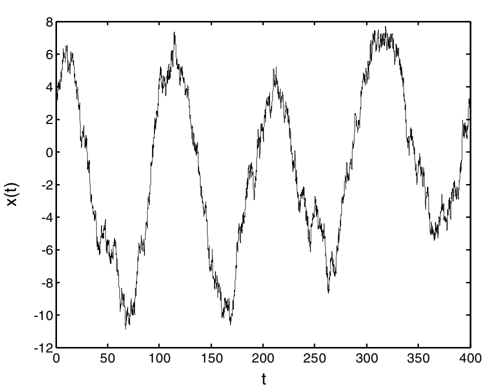
Under the small parameter case, i.e. $A_0 << 1$ and $f << 1$, the simulation result shows that the W-system can generate the SR and the intensity of the noise, D, can be larger than 1. In Fig. 5 and Fig. 6 we give the simulation results for f = 0.01 and f = 0.02 respectively.



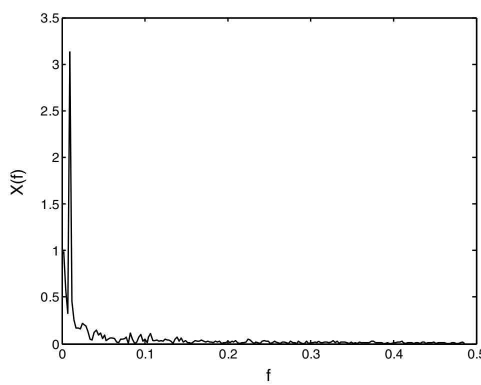
(a) Input signal



(b) The spectrum of the input signal

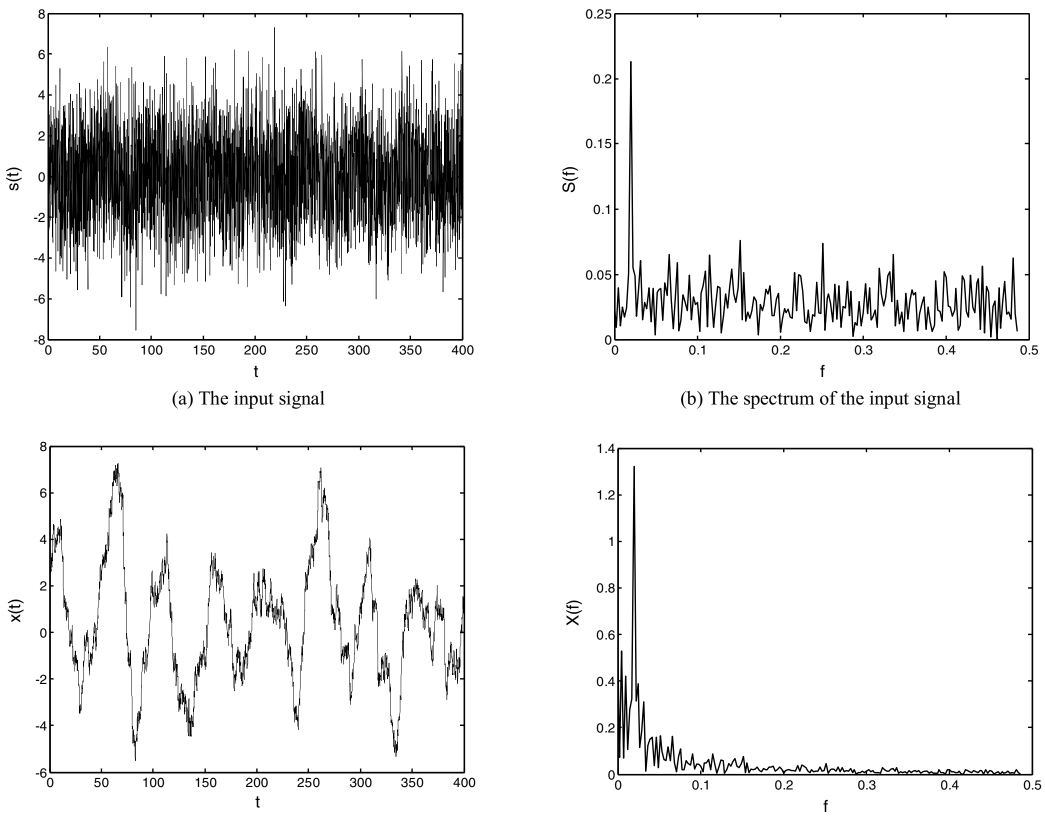


(c) The output signal



(d) The spectrum of the output signal

Figure 5. The SR of the W system under the small parameter case, while f=0.01, D=2, A_0 =0.5, and f_s =10, where f_s denotes the sample frequency



(a) The input signal

(b) The spectrum of the input signal

(c) The output signal

(d) The spectrum of the output signal

Figure 6. The SR of the W system under the small system parameters case, while f=0.02, D=2, A_0 =0.5, and f_s =10

D. *The simulation for large parameters and* $\eta(t) \neq 0$

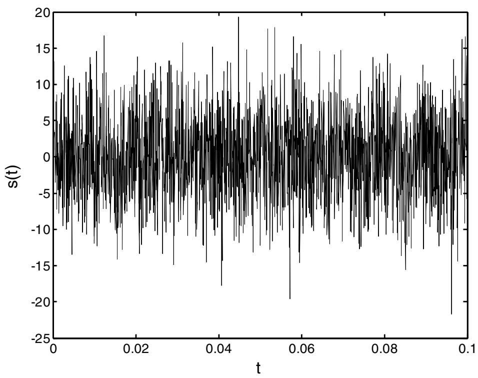
The requirement for small parameters is too strict in practical use. Actually, it is more useful to generate the SR under the large parameters case. There is large damping in the non-linear bistable system described by the Langevin equation. Under the large parameters case, especially when $f >> 1$, it is difficult to generate the SR phenomenon through the Langevin equation. However, under the large parameters case, the numerical simulation shows that it is easy to generate the SR by the W system, via adjusting the parameters. From the simulation we find that while the frequency of the signal increase, the c should be increased to obtain a fast system response if we fix the b unchanged. If c is increased, the potential well will be deeper, and thus, the gain of the amplifier, K, should be increased. In the following, we will give the simulation results.

In Fig. 7 we show the simulation result of the W system for $f = 100HZ$. Under this case the amplitude of the signal and the intensity of the noise are very high. The amplitude of the signal is $A_0 = 1$, and the intensity of the noise is D = 16. From Fig. 7 we see that we cannot directly distinguish the original signal in the time or the frequency domains without the help of the W system. While we adjust the parameters to b = 1, a = 0.05 + b, c = 60 and K = 2000, the SR occurs and then the weak signal is effectively detected.

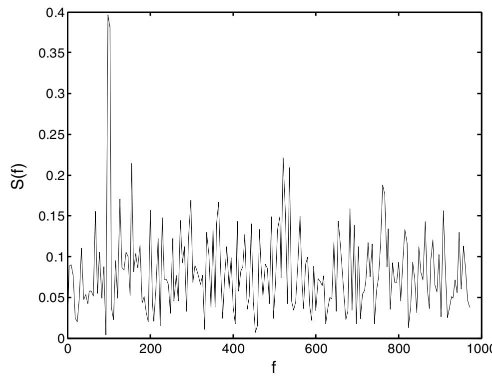
In Fig. 8 we show the SR result under the case that $f = 1000HZ$, A_0 = 0.3, D = 6, b = 1, a = 0.01 + b, c = 200 and K = 20000. It can be seen that the signal is totally overwhelmed by the strong noise. However, after inputting the signal into the W system, we can clearly detect the original signal at the output side.

IV. CONCLUSIONS

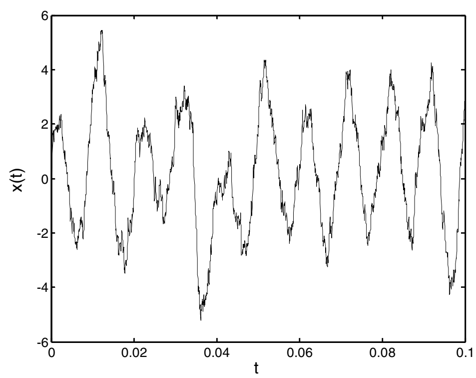
In the above we put forward a piecewise-linear bistable system. We gave its potential function and dynamic equation. Compared with the non-linear bistable system, its parameters are less correlated, so it is more convenient to adjust. The numerical simulations illustrate that through this system the SR can be generated under the small and the large parameters cases. Through the numerical simulation we proved that this system is very helpful for detecting the weak signal under the



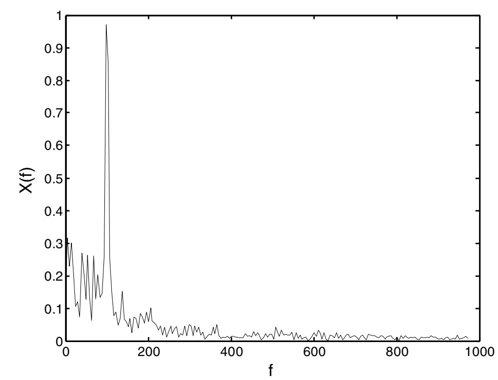
(a) The input signal



(b) The spectrum of the input signal



(c) The output signal



(d) The spectrum of the output signal

Figure 7. The SR of the W system under the large parameters case, while f=100,D=16, A_0 =1， f_s =20000

strong noise background not only for the small parameters case, but also for the large parameters case, which is very common in the engineering fields.

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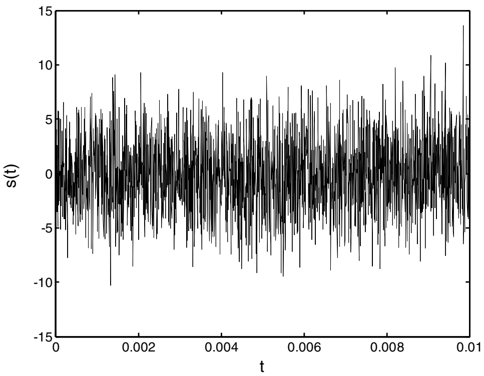
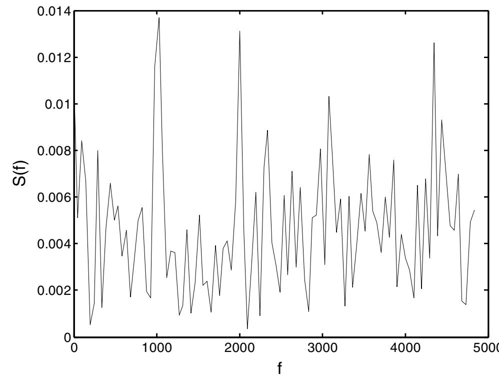
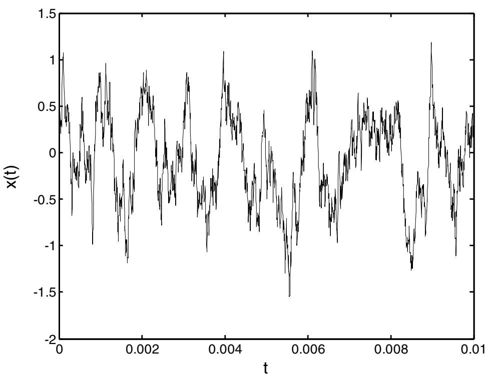


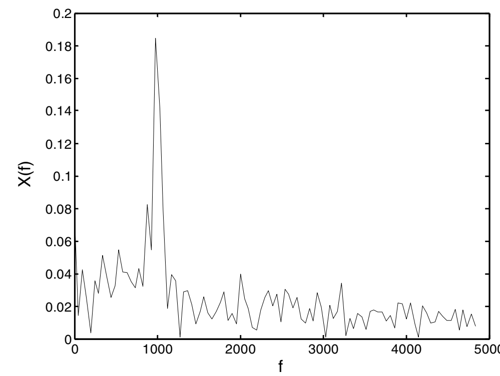
(a) The input signal.



(b) The spectrum of the input signal.



(c) The output signal.



(d) The spectrum of the output signal.

Figure 8. The SR of the W system, while f=1000, D=6, A_0 =0.3， f_s =200000.

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