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Optimal VSC Design Based On Nash Strategy for Differential 2-Player Games

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***Abstract*—For optimal vehicle yaw stability control system development, inclusion of driver dynamics seems necessary. In this paper, a novel design approach is proposed for developing optimal solutions to vehicle stability control problems in the presence of the driver-in-the-loop steering models. The design concept is inspired by a Nash strategy for exactly known systems with more than two players. In the presented method, driver, controlling the steering wheel, and vehicle stability control unit, applying braking torques on the wheels, are defined as two dynamic players in a 2-player differential LQ game, and as a result, a novel control algorithm is developed. The results from a numerical simulation of a single lane change maneuver show the effectiveness of this controller over the common LQR control approach.**

***Keywords*—vehicle stability controller, VSC, optimal control, differential game, driver-in-the-loop**

I. Introduction

Vehicle stability control system, VSC, helps the driver maintain a stable vehicle and avoid spin outs during emergency braking and steering maneuvers. These systems have been developed and recently commercialized by several companies. A comprehensive literature review conducted by Ferguson [1] reveals that VSC can effectively reduce single-vehicle crashes in cars and SUVs by 30-50%. And fatal rollover crashes are estimated to be about 70-90% lower with VSC regardless of vehicle type.

It is well-known that direct yaw control is one of the most effective methods of active chassis control which can considerably enhance the vehicle stability and controllability [2]. For vehicle control, braking/traction control is considered as a way of controlling the lateral motion of a vehicle during severe driving maneuvers using active steering, e.g., front and/or four wheel steering [3], or active unilateral braking using ABS [4]. These systems use feedback control to improve vehicle stability and handling performance [5-7]. They utilize the information provided by the sensors onboard the vehicle and command the actuators based on their inference of current situation. The sensor set of the existing stability control systems consists of four wheel speeds, steering angle, yaw rate, lateral acceleration, and in some cases one or two pressure sensors. Therefore, the driver's command and intentions are interpreted by the controller through the use of the sensory information. However, by introducing the driver as part of the control algorithm, it is possible to improve upon the performance of the vehicle stability control system. This is accomplished through forming a common differential LQ game.

For decades, driver modeling has been an interesting issue for either traffic or vehicle control research [8-10]. Driver models are usually based on preview of the road ahead [8] where driver is represented as an optimal preview controller, constructing a path error functional by previewing the road over a known preview distance, and minimizing a weighted integral of squares of differences between the previewed path points and the corresponding estimated lateral position of the vehicle over the preview distance. The main weakness of the theory, as pointed out in MacAdam's article [9], is the need to presume a constant control input over the preview interval in order to estimate the vehicle position. While this draws back the application of preview-based control strategies in actual vehicles, introducing the driver as a single Nash player can improve the performance of the vehicle control system.

In the work presented here, Nash strategy is used to develop an optimal control strategy which takes into account the driver of the vehicle as an inherent part of the controller.

II. Vehicle Dynamics

A. Evaluation Model

In order to accurately evaluate the controller performance, a nonlinear vehicle model with eight degrees of freedom (DOF) is introduced with four degrees representing the rigid body motion of the vehicle, and the other four degrees representing the rotational motion of the four wheels. Nonlinear terms are included in the model to study the effects of various dynamics nonlinearities inherent to a vehicle system. These nonlinearities may come from a nonlinear tire model, a nonlinear suspension spring or damper, or nonlinearities related to the steering system.

Fig. 1 depicts a schematic representation of a four-wheel road vehicle. The vehicle body has four degrees of freedom,

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namely, longitudinal velocity, u, lateral velocity, v, yaw angle, ψ, and roll angle, φ.

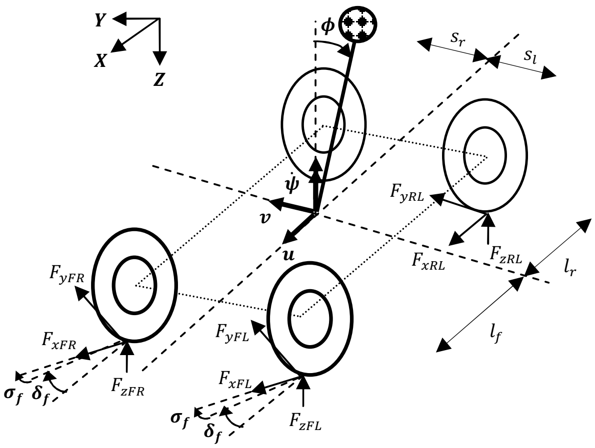


Figure 1. Vehicle Schematics

The final equations of motion are as follows;

$$\begin{aligned} & m\dot{u}-mv\dot{\psi}-m_{\phi}h\phi\ddot{\psi}-2m_{\phi}h\dot{\psi}\dot{\phi}= \\ & \quad F_{xF}\cos\delta_f-F_{yF}\sin\delta_f+F_{xR}, \end{aligned} \tag{1}$$

$$\begin{aligned} & m\dot{v}+mu\dot{\psi}+m_{\phi}h\ddot{\phi}-m_{\phi}h\phi\dot{\psi}^2= \\ & \quad F_{xF}\sin\delta_f+F_{yF}\cos\delta_f+F_{yR}, \end{aligned} \tag{2}$$

$$\begin{aligned} & I_z\ddot{\psi}+(I_{z,\phi}\theta_R-I_{xz,\phi})\ddot{\phi}-m_{\phi}h\dot{u}\phi+m_{\phi}v\phi\dot{\psi}= \\ & \quad l_f\left(F_{xFR}+F_{xFL}\right)\sin\delta_f+l_f\left(F_{yFR}+F_{yFL}\right)\cos\delta_f \\ & \quad -l_r\left(F_{yRR}+F_{yRL}\right) \\ & \quad +s_l\left(F_{xRL}+F_{xFL}\cos\delta_f-F_{yFL}\sin\delta_f\right) \\ & \quad -s_r\left(F_{xRR}+F_{xFR}\cos\delta_f-F_{yFR}\sin\delta_f\right), \end{aligned} \tag{3}$$

$$\begin{aligned} & (I_{x,\phi}+m_{\phi}h^2)\ddot{\phi}+m_{\phi}h\dot{v}+m_{\phi}hu\dot{\psi}+C_{\phi}\dot{\phi} \\ & \quad +(I_z\theta_R-I_{xz})\dot{\psi}-(m_{\phi}h^2+I_{y,\phi}-I_{z,\phi})\phi\dot{\psi}^2 \\ & \quad +(K_{\phi}-m_{\phi}gh)\phi=0. \end{aligned} \tag{4}$$

where f denotes the front axle while r stands for the rear axle of the vehicle, and $F_{xF}, F_{xR}, F_{yF}, F_{yR}$ denote the sum of the front (F) or rear (R) axle components of longitudinal and lateral forces.

The simple equation of motion that governs wheel dynamics is given by:

$$I_p\dot{\omega}=-F_xR_e+T_{Driving}-T_{Braking} \tag{5}$$

where I_p is the wheel y-axis moment of inertial, F_x is the longitudinal tire force applied on each wheel, ω is the wheel rotational velocity, R_e is the effective radius of the wheel, and $T_{Driving}, T_{Braking}$ represent the applied driving and braking torque on each wheel, respectively.

Longitudinal slip of the tire, κ, is defined as the difference between the tire tangential speed and the speed of the axle relative to the road, which is represented by the following equation:

$$\begin{aligned} & \kappa_i=\left(R_e\omega_i-u\right)/\max\{R_e\omega_i,u\}.\mathrm{sign}(\omega_i) \\ & \quad , i=1,..,4\,(each\ wheel) \end{aligned} \tag{6}$$

Slip angle, α, is defined as the angular difference between the treads in the contact patch and the direction the wheel is turned, and is given by:

$$\alpha=\begin{cases} -\dfrac{v+l_f\dot{\psi}}{u}+\sigma_f+\delta_f & : front\ wheels \\ -\dfrac{v-l_r\dot{\psi}}{u}+\sigma_r & : rear\ wheels \end{cases} \tag{7}$$

Tire forces are calculated by Pacejka's "Magic Formula" tire model which has shown to suitably match the experimental data [11].

$$\begin{aligned} & F=D\sin\left(C\arctan(B(1-E)(x+S_h)\right. \\ & \quad \left.+E\arctan(B(x+S_h)))\right)+S_v \end{aligned} \tag{8}$$

where BCD indicates the slope near origin, and will be used for linear force estimation.

B. Control Model

The commonly used single track bicycle model for steering is considered here. This model will capture the needed dynamic information for yaw as well as lateral degrees of freedom.

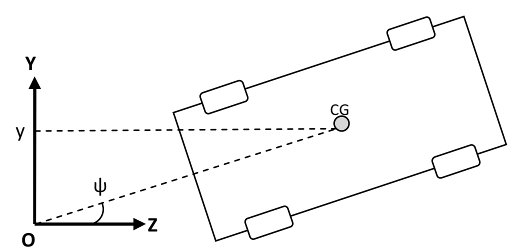


Figure 2. Conventional vehicle bicycle model

Assume that vehicle motion is represented by its global lateral position and velocity, and the yaw angle and yaw rate at the vehicle center of mass as shown in Fig. 2. The state variable vector becomes

$$x=\left(y\ \ \dot{y}\ \ \psi\ \ \dot{\psi}\right)^T, \tag{9}$$

where y is the global lateral position of the vehicle CG, $\dot{y}$ is the global lateral velocity in Y direction with respect to a fixed ground coordinate O, and $\psi, \dot{\psi}$ are yaw angle and yaw rate, respectively.

For the sake of simplicity, the mathematical model is linearized around the operating conditions, $x^* = \mathbf{0}_{4\times1}$, $u^* = \mathbf{0}_{2\times1}$; thus, the equations of motion for a constant forward speed is given by:

$$\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}_\delta \delta_{SW} + \mathbf{B}_M M, \\ x(t_0) = x_0, \end{cases} \tag{10}$$

with

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & u & 0 \\ 0 & -\dfrac{C_{\alpha f} + C_{\alpha r}}{mu} & 0 & -u - \dfrac{l_f C_{\alpha f} - l_r C_{\alpha r}}{mu} \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{l_f C_{\alpha f} - l_r C_{\alpha r}}{I_z u} & 0 & -\dfrac{l_f{}^2 C_{\alpha f} + l_r{}^2 C_{\alpha r}}{I_z u} \end{bmatrix},$$

$$\mathbf{B}_\delta = \begin{bmatrix} 0 & \dfrac{C_{\alpha f}}{i_s m} & 0 & \dfrac{l_f C_{\alpha f}}{i_s I_z} \end{bmatrix}^T,$$

$$\mathbf{B}_M = \begin{bmatrix} 0 & 0 & 0 & \dfrac{1}{I_z} \end{bmatrix}^T,$$

where M is the compensated yaw moment, and steering wheel angle, δ_{SW}, is related to the front wheel steering angle, δ_f, by a constant gear ratio, i_s, as follows;

$$\delta_{SW} / \delta_f = i_s. \tag{11}$$

III. Optimal Vehicle Stability Controller

A. Nash Strategy for Certain Systems

Consider a two-player linear quadratic differential nonzero-sum game on a finite time horizon. The dynamics are described by:

$$\begin{cases} \dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}_1 u_1 + \mathbf{B}_2 u_2, \\ x(t_0) = x_0, \end{cases} \tag{12}$$

and for each player a quadratic cost function is given as:

$$J_i = x(t_f)^T \mathbf{K}_{if} x(t_f) + \int_{t_0}^{t_f} \left(x^T \mathbf{Q}_i x + u_1^T \mathbf{R}_{i1} u_1 + u_2^T \mathbf{R}_{i2} u_2 \right) d\tau, \tag{13}$$

where all weighting matrices are constant and symmetric with $\mathbf{Q}_i \geq 0, \mathbf{R}_{ij} = \mathbf{D}_{ij}^T \mathbf{D}_{ij} \geq 0$ and $\mathbf{R}_{ii} = \mathbf{D}_i^T \mathbf{D}_i > 0$, and $i = 1, 2$ is the player number.

Nash equilibrium concept, as in [12,13], is defined as the pair (γ_1^*, γ_2^*) which corresponds to a Nash equilibrium if the following relations are satisfied for each admissible strategy (γ_1, γ_2):

$$\begin{cases} J_1(\gamma_1, \gamma_2^*) \geq J_1(\gamma_1^*, \gamma_2^*), \\ J_2(\gamma_1^*, \gamma_2) \geq J_2(\gamma_1^*, \gamma_2^*). \end{cases} \tag{14}$$

The Nash equilibrium is defined such that it has the property that there is no incentive for any unilateral deviation by any one of the players. In the other words, at Nash equilibrium with (γ_1^*, γ_2^*), the player who chooses to change his/her strategy cannot improve his/her payoff.

<u>Theorem 1</u>: Let the strategies (γ_1^*, γ_2^*) be such that there exist solutions $(\mathbf{P}_1, \mathbf{P}_2)$ to the differential equations

$$\dot{\mathbf{P}}_i = -\frac{\partial \mathbf{H}_i}{\partial \mathbf{x}}\left(x^*, \gamma_1^*, \gamma_2^*, \mathbf{P}_i\right) - \frac{\partial \mathbf{H}_i}{\partial u_i}\left(x^*, \gamma_1^*, \gamma_2^*, \mathbf{P}_i\right) . \frac{\partial \gamma_j^*}{\partial x}, \tag{15}$$

where $i = 1, 2$,

$$\mathbf{H}_i\left(x, u_1, u_2, \mathbf{P}_i\right) = x^T \mathbf{Q}_i x + u_1^T \mathbf{R}_{i1} u_1 + u_2^T \mathbf{R}_{i2} u_2 + \mathbf{P}_i^T \left(\mathbf{A}x + \mathbf{B}_1 u_1 + \mathbf{B}_2 u_2\right), \tag{16}$$

with terminal conditions, for $i = 1, 2$,

$$\mathbf{P}_i(t_f) = \mathbf{K}_{if} x^*(t_f), \tag{17}$$

such that for $i = 1, 2$,

$$\frac{\partial \mathbf{H}_i}{\partial u_i}\left(x^*, \gamma_1^*, \gamma_2^*, \mathbf{P}_i\right) = 0, \tag{18}$$

and x^* satisfies

$$\begin{cases} \dot{x}^*(t) = \mathbf{A}x^*(t) + \mathbf{B_1}\gamma_1^* + \mathbf{B_2}\gamma_2^*, \\ x^*(t_0) = x_0. \end{cases} \tag{19}$$

Then (γ_1^*, γ_2^*) is a Nash equilibrium with respect to the memoryless perfect state information structure, and the following equalities hold [14]:

$$u_i^* = \gamma_i^* = -\mathbf{R}_{ii}^{-1} \mathbf{B}_i^T \mathbf{P}_i(t). \tag{20}$$

Restricting the admissible strategies to the class of linear feedback strategies:

$$\Gamma_i^{fd} = \left\{ \gamma_i \in \Gamma_i \middle| \gamma_i(x, t) = \mathbf{F}_i(t) x \right\}, \tag{21}$$

there exists a generically unique linear feedback Nash equilibrium [14,15].

<u>Theorem 2</u>: Suppose $(\mathbf{K}_1, \mathbf{K}_2)$ satisfy the coupled Riccati equations given by

$$\dot{\mathbf{K}}_1 = \Im_1(\mathbf{K}_1, \mathbf{K}_2) = -\mathbf{A}^T \mathbf{K}_1 - \mathbf{K}_1 \mathbf{A} - \mathbf{Q}_1 + \mathbf{K}_1 \mathbf{S}_1 \mathbf{K}_1 + \mathbf{K}_1 \mathbf{S}_2 \mathbf{K}_2 + \mathbf{K}_2 \mathbf{S}_2 \mathbf{K}_1 - \mathbf{K}_2 \mathbf{S}_{12} \mathbf{K}_2,$$

$$\dot{\mathbf{K}}_2 = \Im_2(\mathbf{K}_1, \mathbf{K}_2) = -\mathbf{A}^T\mathbf{K}_2 - \mathbf{K}_2\mathbf{A} - \mathbf{Q}_2$$
$$+\mathbf{K}_2\mathbf{S}_2\mathbf{K}_2 + \mathbf{K}_2\mathbf{S}_1\mathbf{K}_1 + \mathbf{K}_1\mathbf{S}_1\mathbf{K}_2 - \mathbf{K}_1\mathbf{S}_{21}\mathbf{K}_1,$$
$$\mathbf{K}_1(t_f) = \mathbf{K}_{1f},$$
$$\mathbf{K}_2(t_f) = \mathbf{K}_{2f}, \quad (22)$$

where

$$\mathbf{S}_i = \mathbf{B}_i\mathbf{R}_{ii}^{-1}\mathbf{B}_i^T,$$
$$\mathbf{S}_{ij} = \mathbf{B}_j\mathbf{R}_{jj}^{-1}\mathbf{R}_{ij}\mathbf{R}_{jj}^{-1}\mathbf{B}_j^T.$$

Then the pair of strategies $\left(\gamma_1^*, \gamma_2^*\right) = \left(-\mathbf{R}_{11}^{-1}\mathbf{B}_1^T\mathbf{K}_1(t)x, -\mathbf{R}_{22}^{-1}\mathbf{B}_2^T\mathbf{K}_2(t)x\right)$ are linear feedback Nash equilibriums. The functions of Theorem 1 are given by $\mathbf{P}_i = \mathbf{K}_i(t)x(t)$.

When we allow for more general, e.g., nonlinear feedback, strategies, there may exist many more (feedback) Nash equilibria for the memoryless perfect state information structure [16].

If there exist positive definite time-invariable matrices $\left(\mathbf{K}_{1f}, \mathbf{K}_{2f}\right)$, satisfying the Coupled Algebraic Riccati-type Equations (CARE):

$$\Im_1(\mathbf{K}_{1f}, \mathbf{K}_{2f}) = \Im_2(\mathbf{K}_{1f}, \mathbf{K}_{2f}) = \mathbf{0}_n, \quad (23)$$

the controls

$$u_i^*(t) = \mathbf{F}_i x(t) = -\mathbf{R}_{ii}^{-1}\mathbf{B}_i^T\mathbf{K}_{if}x(t), \quad (24)$$

constitute a set of Nash equilibrium strategies within the class of admissible control (21) functions if the game has a closed-loop information structure.

B. Control Derivation

Considering cooperative control efforts $u_1 = \delta_{SW}$ and $u_2 = M_B$, system of Eq. (12) forms a two-player linear quadratic differential game. Therefore, using Theorem 1 and Theorem 2 the optimal set of control efforts are:

$$\delta_{SW} = -\mathbf{R}_{\delta\delta}^{-1}\mathbf{B}_\delta^T\mathbf{K}_\delta\left(x - x_d\right) = \mathbf{G}_\delta\left(x - x_d\right), \quad (25)$$
$$M_B = -\mathbf{R}_{MM}^{-1}\mathbf{B}_M^T\mathbf{K}_M\left(x - x_d\right) = \mathbf{G}_M\left(x - x_d\right). \quad (26)$$

where $\mathbf{K}_\delta, \mathbf{K}_B$ are the solutions of the following Riccati-like equations:

$$-\mathbf{A}^T\mathbf{K}_\delta - \mathbf{K}_\delta\mathbf{A} - \mathbf{Q}_\delta + \mathbf{K}_\delta\mathbf{S}_\delta\mathbf{K}_\delta$$
$$+\mathbf{K}_\delta\mathbf{S}_M\mathbf{K}_M + \mathbf{K}_M\mathbf{S}_M\mathbf{K}_\delta - \mathbf{K}_M\mathbf{S}_{\delta M}\mathbf{K}_M = \mathbf{0}, \quad (27)$$
$$-\mathbf{A}^T\mathbf{K}_M - \mathbf{K}_M\mathbf{A} - \mathbf{Q}_M + \mathbf{K}_M\mathbf{S}_M\mathbf{K}_M$$
$$+\mathbf{K}_M\mathbf{S}_\delta\mathbf{K}_\delta + \mathbf{K}_\delta\mathbf{S}_\delta\mathbf{K}_M - \mathbf{K}_\delta\mathbf{S}_{M\delta}\mathbf{K}_\delta = \mathbf{0}. \quad (28)$$

IV. Simulation and Results

Based on Wong's findings [2], handling stability is guaranteed provided that the controller can keep the vehicle yaw rate close to the desired value that can be dynamically calculated based on the driver's steering input and vehicle forward speed:

$$\dot{\psi}_d = \frac{u/(l_f + l_r)}{1 + u^2/u_{char}^2}\delta_f, \quad (29)$$

where u_{char} is the characteristic speed of the vehicle.

Computer simulations are carried out to verify the effectiveness of the control algorithms. Therefore, the presented controller is evaluated using the nonlinear model that was introduced in section 2.1, with the control objective to guarantee handling performance in a single lane change maneuver using the following desired states:

$$x_d = \begin{pmatrix} 5 & 0 & 0 & \dot{\psi}_d \end{pmatrix}^T. \quad (30)$$

Initial vehicle conditions are set to:

$$x(t_0) = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}^T. \quad (31)$$

The optimal strategies defined in equations (25) and (26) are computed for a sedan at a nominal speed $u = 80\,km/hr$ and the following values of other involved parameters:

$m = 1418$ kg,
$I_x = 550$ kg.m^2,
$I_y = 2000$ kg.m^2,
$I_z = 1819$ kg.m^2,
$I_{xz} = 70$ kg.m^2,
$l_f = 1.012$ m,
$l_r = 1.568$ m,
$i_s = 19.5$,
$C_{\kappa f} = 35000$ N,
$C_{\kappa r} = 35000$ N,
$\mathrm{I}_p = 1\ kg.m^2$,
$R_e = 0.35\ m$,
$h = 0.54\ m$,
$C_\phi = 2767\ N.m.s/rad$,
$K_\phi = 65947\ N.m/rad$,
$\varepsilon_f = 0.14$,
$\varepsilon_r = 0.1$,
$c_f = 2.45e-7\ rad/N$,
$c_r = 6.11e-7\ rad/N$,
$\theta_R = 0.0155\ rad$.

To evaluate the performance of the proposed driver-in-the-loop control method based on Nash theory, a common linear quadratic optimization strategy (LQR) is also developed and evaluated in such a way that driver's and controller's gains are optimized independently for the system using the parameters provided below:

$$\mathbf{Q}_{\delta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix},$$

$$\mathbf{Q}_{M} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{32}$$

$$\mathbf{R}_{\delta\delta} = 10,\ \mathbf{R}_{\delta M} = 0,$$

$$\mathbf{R}_{MM} = 1e-5,\ \mathbf{R}_{M\delta} = 1e3.$$

The final state feedback gains for both strategies are calculated and listed as follows;

- Nash Strategy:

$$\begin{cases} \mathbf{G}_{\delta} = [-0.1003 \quad -0.0168 \quad -1.0634 \quad -0.0667], \\ \mathbf{G}_{M} = 1e3 \times [0 \quad -0.0388 \quad -3.9525 \quad -0.3469]. \end{cases} \tag{33}$$

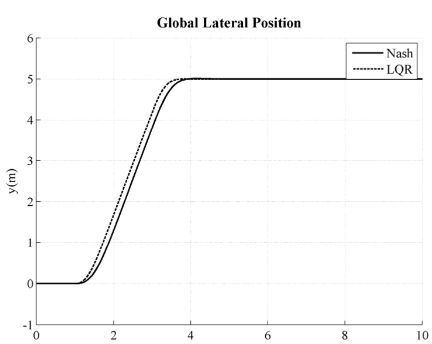
- LQR:

$$\begin{cases} \mathbf{G}_{\delta} = [-0.3162 \quad -0.0413 \quad -2.3383 \quad -0.1334], \\ \mathbf{G}_{M} = 1e3 \times [0 \quad -0.0072 \quad -1.0012 \quad -0.1105]. \end{cases} \tag{34}$$

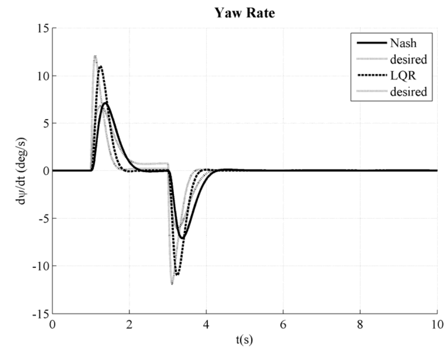
It must be mentioned that the gain $\mathbf{G}_M$ for global lateral position is always zero since the controller has no information regarding the desired lateral position.

Fig. 3 shows the simulation results of the vehicle performance with Nash and LQR optimization strategies for a single lane change maneuver.

As shown in Fig. 3a, both methods lead the vehicle through a single lane change maneuver of 5m. Fig. 3b indicates that while in both cases the vehicle yaw rate is close to its desired value for better handling, the maximum value of the yaw rate is lower with Nash controller compared to the common LQR controller.



(a)



(b)

Figure 3. Time history of vehicle response to a lane change maneuver: (a) global lateral position, (b) yaw rate

Fig. 4 shows the longitudinal and lateral velocities of the vehicle body for the Nash controller simulation.

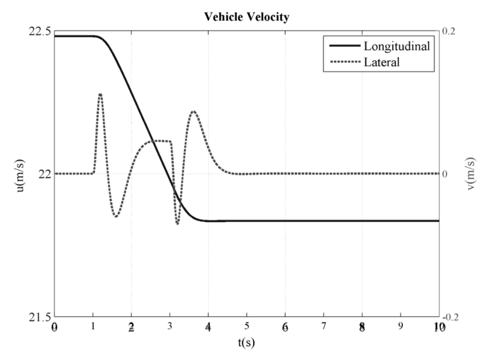
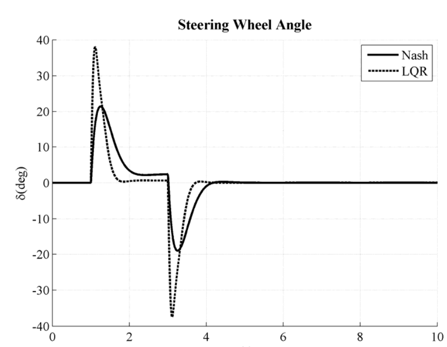


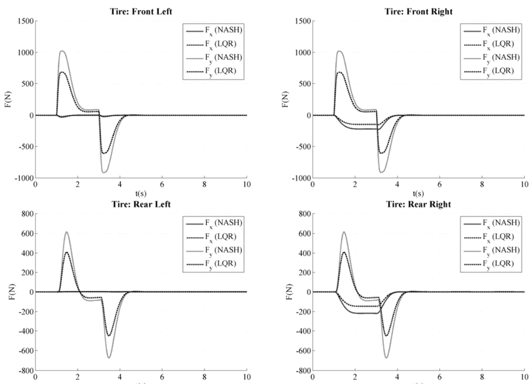
Figure 4. Time history of vehicle longitudinal and lateral velocities

During the lane change the vehicle velocity has a negligible drop in longitudinal direction due to the activation of vehicle braking system. The change in vehicle velocity in lateral direction is small and indicates a stable maneuver in lateral motion.

Fig. 5 reports the driver's steering angle and the tire forces in which ABS braking system is activated to maneuver the vehicle for a single lane change using Nash strategy as well as LQR approach.



(a)



(b)

Figure 5. Time history of control inputs for certain driver model: (a) steering wheel angle, and (b) compensated yaw moment

As shown in Fig. 5a, with Nash optimization theory the driver steering wheel input which represents driver effort is less, and therefore, the controller is more involved than with LQR controller.

Fig. 5b that plots the force components of four wheels in Newton shows that the ABS systems exerted braking torques with maximum braking force of 212 N on the front-right and rear-right wheels, while leaving the other wheels to rotate freely.

V. Conclusion

Based on Game Theory, a novel cooperative optimal control strategy is introduced. Using definition of a linear quadratic differential game, driver's steering input and the controller's braking torques are defined as two dynamic players of the game "vehicle stability", and their corresponding control efforts are optimized through Nash optimal strategy.

In a certain game system, driver's input and the controller optimal solution are plotted and compared with a common LQR's. As shown in the results presented, the proposed 'Game Theory'-based controller not only stabilizes the system, but also involves the controller more in stabilizing the vehicle compared to common LQR controllers hence reducing the driver's workload.

An advantage of the proposed controller not to be overlooked is the stability aspect. The good correspondence between vehicle yaw rate and the desired yaw rate in the case with the proposed controller is indicative of improved vehicle handling stability. Although not demonstrated in this work, more extensive vehicle testing is likely to show that a closed-loop yaw control system based on Nash strategy not only improves vehicle performance, but also increases driver confidence in an emergency maneuver, because the controller takes more responsibility when Nash strategy is applied.

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