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Adaptive Type-2 Fuzzy Maintenance Advisor for Offshore Power Systems

Zhao-xia Wang, C.S. Chang*, F. Yang and W. W. Tan
Department of Electrical and Computer Engineering,
National University of Singapore,
Republic of Singapore 119260
- eleccs@nus.edu.sg

***Abstract*— Proper maintenance strategies are very desirable for minimizing the operational and maintenance costs of power systems without sacrificing reliability. Condition-based maintenance has largely replaced time-based maintenance because of the former's potential economic benefits. As offshore substations are often remotely located, they experience more adverse environments, higher failures, and therefore need more powerful analytical tools than their onshore counterpart. As reliability information collected during operation of an offshore substation can rarely avoid uncertainties, it is essential to obtain consistent estimates of reliability measures under changing environmental and operating conditions. Some attempts with type-1 fuzzy logic were made with limited success in handling uncertainties occurring in onshore power-system maintenance. An adaptive maintenance advisor using type-2 fuzzy logic is proposed here for handling operational variations and uncertainties for condition-based maintenance of an offshore substation. The maintenance advisor receives maintenance plans for its key components from a system maintenance optimizer, which is optimizing all the maintenance activities in the entire connected grid by considering only major system variables and the overall system performance. During operation, the offshore substation will experience continuing ageing and shifts in control, set-point, weather and load factors, measurement and human-judgment detected from the connected grid and all other equipments; which will certainly contain a lot of uncertainties. The advisor implements the system-optimized maintenance plan within its offshore substation, and estimates the change of load-point reliability due to operational variations and uncertainties of its key components. The maintenance advisor will report any drastic deterioration of load-point reliability within each substation, which may lead to re-optimization of the substation's maintenance activities for meeting its desired reliability during operation. The reliability of an offshore substation connected to a medium-sized onshore grid will be studied here using minimum cut set method. The relative merits between type-2 & type-1 fuzzy logic will also be studied in terms of their versatility, efficiency and ability for reliability modelling of operational variations and uncertainties.**

Keywords* — *Adaptive Maintenance Advisor, Offshore Substation, Load-point Reliability, Type-2 Fuzzy Sets, Hidden Markov Model, Minimum cut set.

I. INTRODUCTION

Offshore power systems are often remotely located and their access for data acquisition, inspection and maintenance may be extremely difficult, especially during adverse weather conditions. Their information collected during operation can rarely avoid uncertainties. Hence more powerful tools are needed to deal with those uncertainties for continuous on-line monitoring or periodic inspections to provide early warning against failure [1-4].

Reliability analysis is an essential part of condition-based maintenance [5]. Because of uncertainties arising from inside and outside the equipment, it is often difficult to obtain exact reliability indices using conventional reliability analysis especially when conditions change. Fuzzy sets theory was proposed by Zadeh [6] to represent and handle imprecise information, and to resemble human reasoning under uncertainties by using approximate information to generate proper decisions. Known as type-1 fuzzy logic, the methodology has been successfully used in many applications [7-9]. Zadeh further proposed the alternative type-2 fuzzy logic [10], which demonstrated greater freedom for designing fuzzy sets and greater success than type-1 fuzzy sets in various fields to handle uncertainties [7, 9-13].

As the reliability information collected during power system operation can rarely avoid uncertainties, it is essential to obtain a consistent estimate of reliability measures under changing environmental and operating conditions [4]. Some attempts were made with type-1 fuzzy logic to handle uncertainties [14-18], and in particular power-system maintenance problems [16-18]. Fuzzy Markov model was employed to describe transition rates [15] and to handle uncertainties related to generating units by incorporating fuzzy mean time to failure and fuzzy mean time to repair [16].

The hidden Markov model that is different from a regular Markov model has been adopted for partial discharge, system monitoring, image classification and fuzzy spatial pattern processing [24-27]. In this paper, type-2 fuzzy logic learning and analysis system is linked to a hidden Markov model and the type-2 fuzzy hidden Markov model analysis is proposed to analyze the reliability indices of the offshore power system.

Our previous works proposed an adaptive maintenance advisor using a spreadsheet for calculating the initial reliability indices at various load points of a substation [4] and proper scheduling preventive maintenance using decision-varying Markov models was proposed for predicting the availability of individual component [20]. Minimum cut set is used for evaluating the overall reliability of interconnected power systems. Based on this, a system maintenance optimizer was developed for optimizing all the maintenance activities within the grid [20].

In this paper a flexible approach is proposed as in Fig. 1 to link our adaptive maintenance advisor for each offshore substation with our system maintenance optimizer for the connected grid.

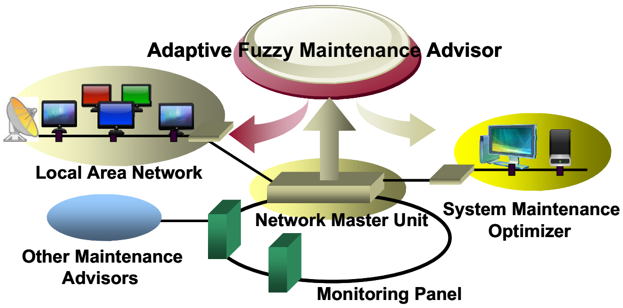


Fig. 1 Intelligent Predictive Maintenance and Condition Monitoring System for Offshore Substation

The maintenance advisor receives the initial or updated maintenance plan from the system maintenance optimizer, which is optimizing all the maintenance activities in the connected grid by considering only major system variables and the overall system performance. During operation, the offshore substation will experience continuing ageing and shifts in control, set-point, weather and load factors, measurement and human-judgment detected from the connected grid and all other equipments; which will certainly contain a lot of uncertainties. The advisor implements the system-optimized maintenance plan within its offshore substation, and estimates the change of load-point reliability due to operational variations and uncertainties of its key components. The maintenance advisor will assess the load-point reliability and report any drastic deterioration within the substation, which may lead to re-optimization of the substation's maintenance activities for meeting its desired reliability during operation. Type-2 fuzzy logic is proposed here for reliability modelling of operational variations and uncertainties for condition-based maintenance of offshore substations.

This paper is organized into 5 sections. Section 2 reviews the basic multi-phase Markov model and proposes the type-2 fuzzy hidden Markov model. It also presents the approach of adaptive maintenance advisor and system maintenance optimizer, and outlines the proposed adaptive fuzzy maintenance advisor. Section 3 describes the studied offshore substation and connected grid, as well as the fuzzy memberships for modeling operational variations and uncertainties of key components. Section 4 presents the relative impacts on the load-point reliability from various operational variations and uncertainties. It also discusses the relative performance of type-2 against type-1 fuzzy logic in their ability for modelling operational variations and uncertainties, their versatility and efficiency. Section 5 concludes the paper.

II. Basic Multi-Phase Stochastic Model and Type-2 Fuzzy Hidden Markov Model

Fig. 2 outlines the type-2 fuzzy hidden Markov model for individual offshore power equipment. The multi-phase reliability model for individual equipment provides a quantitative connection between maintenance and reliability [20]. Discrete process phases are usually used and mostly based on regular Markov models [4, 19-23]. In a regular Markov model, the state is directly visible to the observer, and therefore only the state transition probabilities are the parameters.

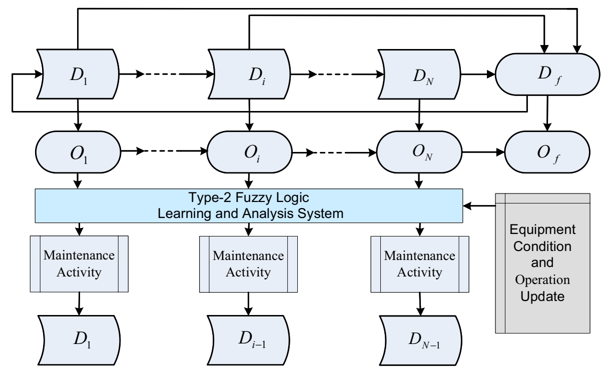


Fig. 2 Type-2 Fuzzy Hidden Markov Model

In the hidden Markov model [24-27], deterioration processes of each equipment are modeled by a discrete N-state Markov process, D_i, $i=1,2,\ldots N$. D_1 denotes the "as good as new" state, $D_2,\ldots,D_n$ are the states with different levels of deteriorations, and D_f is the failed state. The transition rates among different states form the matrix Λ . Following a maintenance activity, the equipment will experience a transition of state to a new one, as shown in Fig. 2.

Unlike the regular Markov model, the state D_i is not directly visible, while the output O_i, which is dependent on the state D_i, is visible. A hidden Markov model can be assumed to be a regular Markov model [19-23] with unobserved states. Each state has a probability distribution over the possible visible output. Therefore the sequence of visible output provides some information associated with the possible invisible states. Similar to a maintenance activity, the equipment will also experience a transition of state to a new one following an operational variation, as represented by:

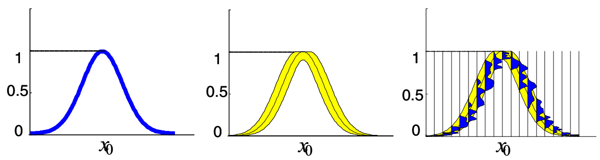
$$\begin{aligned}\Delta\Lambda(t) &= \Lambda(t) - \Lambda_o(t) \\ &= f_{T2}(\Lambda_0(t), W(t), A(t), M(t), L(t), \ldots)\end{aligned} \tag{1}$$

where $W(t)$, $A(t)$, $M(t)$ and $L(t)$ represent the working environment, equipment age, maintenance information and load factor in the time interval "t", respectively.

$W(t)$, $A(t)$, $M(t)$ and $L(t)$ are connected with each other by fuzzy linguistic rules. Once the rules have been established, the fuzzy system can be viewed as a mapping function f_{T2} from inputs to outputs. Fuzzy linguistic rules are derived from both expert knowledge and mathematical strategies [7, 10, and 13].

Type-2, being different from type-1 fuzzy sets, is characterized by fuzzy membership functions in three dimensions. The difference between type-1 and type-2 fuzzy membership functions is shown in Fig. 3 (a)-(c). The uncertainties in the primary membership function of a type-2 fuzzy set, called the footprint of uncertainty [7], and the secondary membership function are shown in Fig. 3(b) and Fig. 3(c), respectively. More details of the type-2 fuzzy are discussed in our previous work [4] and other literature [7, 10-13].

Type-2 fuzzy hidden Markov model diagram is proposed here to analyze the parameters and to handle the uncertainties. The type-2 fuzzy system for dealing with the uncertainties coming from different operations, weather environment & condition updates has been designed as shown in Fig. 6.



(a) (b) (c)

Fig. 3 (a) Membership Function of Type-1 Fuzzy Set, and also Primary Membership Function of Type-2 Fuzzy Set, (b) Footprint of Uncertainty of Type-2 Fuzzy Set, and (c) Secondary Membership Function (blue dots)

Fig. 4 shows the adaptive fuzzy maintenance advisor for an offshore substation using type-2 fuzzy logic learning and analysis system to deal with uncertainties that come from condition and operational updates. Fig. 4 also shows the links between each adaptive maintenance advisor for each offshore substation with the system maintenance optimizer for the entire connected grid. The former implements the day-to-day maintenance of key components in each substation and assess the impacts on the load point arising from operational variations and uncertainties of these components.

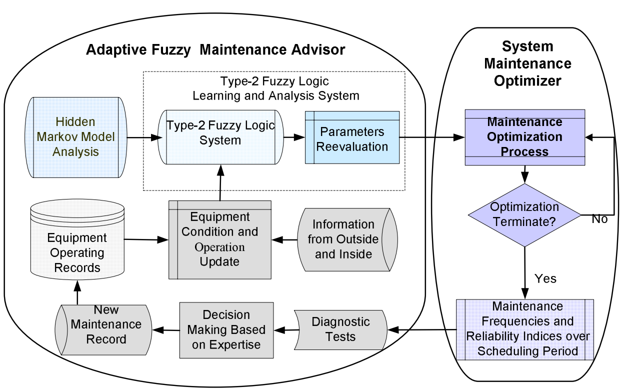


Fig. 4 Layout Of Adaptive Fuzzy Maintenance Advisor

III. STUDY PARAMETERS AND TYPE-2 FUZZY MODEL FOR MAINTENANCE ADVISOR

A. *Study Configuration and Parameters*

Fig. 5 shows Bus 07 of IEEE-RTS [28], which is assumed to be an offshore substation for case studies in this paper. The load-point reliability is affected by the reliabilities of the transformer Ts and five circuit breakers CB1-CB5 in the substation. The adaptive maintenance advisor obtains the initial maintenance plan from the system maintenance optimizer. The study period is set at 30 years.

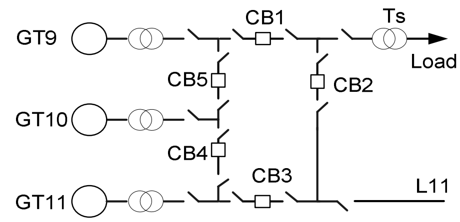


Fig. 5 Configuration of Bus 07 in IEEE-RTS

The failure data of circuit breaker are tabulated in Table I, which are obtained from published research work [29].

TABLE I. CIRCUIT BREAKER FAILURE DATA

	F/Yr Active Failure rate	F/Yr Passive Failure rate	F/Yr Total failure rate	Hr Repair time	Hr Switching time
Circuit breaker	0.0066	0.0005	0.0071	108	1

B. *Transformer Fuzzy Memberships*

Fig. 6 shows the type-2 membership functions used for the transformer. The input uncertainties arising from current weather condition, age, load factor, and last maintenance records will affect the transformer reliability, which are expressed here in probability of failure per year, failure rate per year and mean time between failures (MTBF). The number of antecedent variables in the universe of the discourse can be increased and decreased by considering current situations. The number of membership functions can also be increased and the different footprints of uncertainties of the type-2 fuzzy logic can be used according to different uncertainties.

Type-2 fuzzy logic represents the impact of the above operational variations using the footprint of uncertainty of the membership functions, which makes type-2 fuzzy sets uniquely different from type-1 fuzzy sets, as shown in Fig. 6. Three Gaussian membership functions and five Gaussian membership functions are used as primary membership functions in antecedent variables and in consequent variables in Fig. 6 (a1)-(a4) & (b1)-(b3), respectively. Three member ship functions are used as secondary membership functions in both antecedent variables and consequent variables in Fig. 6 (c2)-(c4) & (d1)-(d3). The shape looks similar as shown in Fig. 6 (c2)-(c4) & (d1)-(d3), but the universe of the discourses in the secondary membership functions are different. Fig. 6 (c1) & (c3) show the shape of the secondary membership function of the equipment age and load factor, respectively. These two figures are different from other secondary membership functions as there is no new information or no additional uncertainty about the equipment age and load factor in this paper.

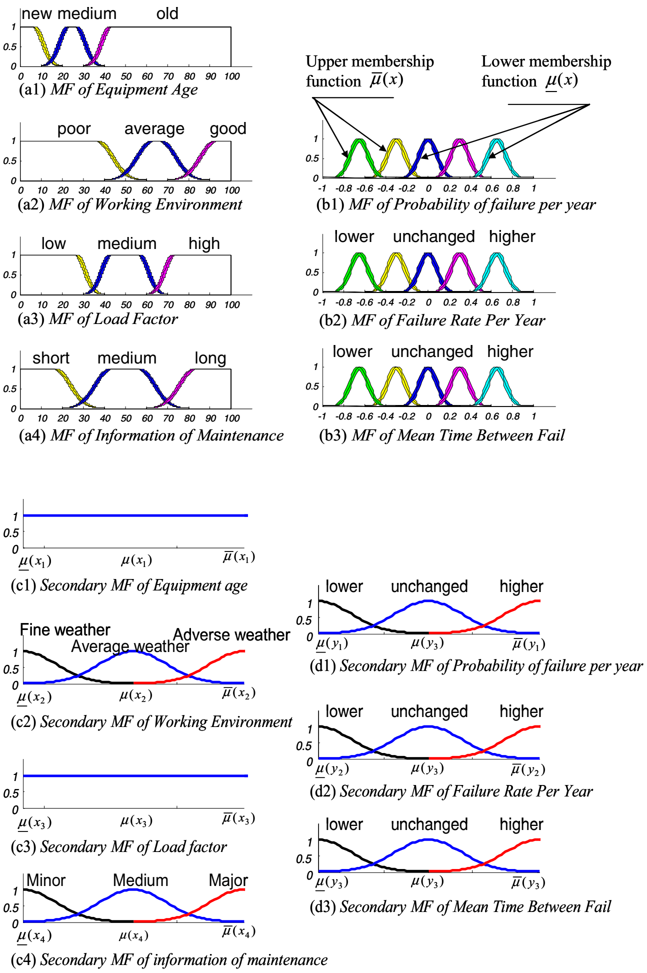


Fig. 6 Membership Functions (MF) Used in Type-2 Fuzzy Logic

The uncertainties of the input for working environment are quantized into three linguistic variables, namely, fine weather, average weather and adverse weather, as shown in Fig. 6 (c2), which are represented by secondary membership functions. The classes of the maintenance (minor, medium, major) as three linguistic variables are represented by three secondary membership functions shown in Fig. 6 (c4). All the secondary membership functions of the output are given in Fig. 6 (d1)-(d3).

As seen from Fig. 6, type-2 fuzzy logic learning and analysis system has greater flexibility than type-1 fuzzy logic in incorporating the influence of many factors by adding secondary membership function or changing the footprint of uncertainty of their membership functions. In contrast, type-1 fuzzy system used only the upper membership functions of the type-2 fuzzy system as in Fig. 6.

The universes of discourse for the secondary membership function, whose values are from the lower membership function $\underline{\mu}(x)$ to upper membership function $\overline{\mu}(x)$, are set as discrete universes too. All the secondary membership functions of the antecedent variables and consequent variables are set to be interval type-2 fuzzy sets. Three discrete values are used in a vertical slice, which represent the three universes of discourse, e.g. the fine weather, average weather and adverse weather for the secondary membership functions of Fig. 6 (c2).

Table II shows the factors considered in type-2 fuzzy rules about the condition updates and operation uncertainties considered in this paper. Equipment age, working environment, load factor and information of the maintenance are all the antecedent variables used in the simulations. The primary membership functions of them are shown in Fig. 6 (a1)-(a4).

It is assumed that the uncertainties related to the MTBF, the probability of failure per year and failure rate per year can be incorporated through the evaluation of the type-2 fuzzy experts system, which is well acquainted with the changes of those reliability indices [4]. Basically, aging the unit will decrease the MTBF; adverse working environment causes the decrease of the MTBF; the increase in the load also decreases the MTBF.

The inputs are combined through "if then" rules given by type2-fuzzy experts using fuzzy inference system to get output of the changes of the MTBF. For example, if the input of age corresponds to "old", working environment to "poor", load factor to "higher" and the time of previous maintenance to "long", the output is the "higher" of the MTBF.

In fact, the additional uncertainties of the working environment from "fine weather" to "adverse weather" is continuous discourse variables, which is shown in the corresponding footprint of uncertainties in the primary membership functions (Fig. 6 (a2)). The uncertainties and the primary membership functions are connected with the rules given by the type-2 fuzzy experts. For example, if the higher order uncertainties (weather information) are "fine", then the primary membership functions (working environment) will be moved to "good" by the given parameters from type-2 fuzzy experts.

TABLE II. OPERATION UPDATES AND UNCERTAINTIES IN T-2 FUZZY RULES

Type-2 Fuzzy Rule	Operational variations and uncertainties	
	Primary MF representing operational variations	Secondary MF representing uncertainties
Equipment Age	Ageing update(a1)	No uncertainty at present (c1)
Working Environment	Working environment update (a2)	Weather variation (c2)
Load Factor	Load factor update (a3)	No uncertainty at present (c3)
Information of Maintenance	Time of previous maintenance (a4)	Extent of previous maintenance (c4)

The type-2 fuzzy system in this paper reduces the computational complexity in terms of the number of rules compared to the type-1 fuzzy system. Normally, the number of rules increases exponentially as the number of input increases in the type-1 fuzzy system. If another input of the type-1 fuzzy system is added to represent the additional uncertainties, the number of rules is multiplied by the number of membership functions in the added input. For example, if a type-1 fuzzy system has N_1 inputs and one output, and there are N_2 membership functions associated with each input, the number of rules is ${N_2}^{N_1}$. When one more input which is quantized into N membership functions is added to represent one uncertainty, the number of rules of the system will be ${N_2}^{N_1} \times N$. However, in our type-2 fuzzy system, each uncertainty is superimposed on one additional input rather than be treated as another input. If these uncertainties are represented by N secondary membership functions, the number of rules in the type-2 fuzzy system is ${N_2}^{N_1} + N$

IV. RESULT AND DISCUSSIONS

One of the three reliability indices, MTBF (Figs.7-11), is used to show the relative impacts of operational variations and uncertainties of the transformer, circuit-breaker reliability and the choice between type-1 & type-2 fuzzy logic. Other reliability indices may also be used for other case studies.

A. Advantage of type-2 fuzzy logic in reducing computational complexity

There are 4 inputs (Equipment Age, Working Environment Load Factor and Information of maintenance), each of which has 3 membership functions (Fig.6) in this type-2 fuzzy system. Two additional uncertainties are respectively superimposed on two inputs, which are represented by 3 fuzzy sets. Altogether, this type-2 fuzzy system produces $87 (= 3^4 + 3 + 3)$ rules. The same uncertainties can be represented by type-1 fuzzy system with $729 (= 3^{(4+2)})$ rules. Having such dimension reduction, type-2 fuzzy is superior in computational complexity to type-1 fuzzy logic in dealing with additional uncertainties.

B. Impact of Transformer Load Factors on Load-point Reliability

According to Table II, the changing transformer load factor is modeled here as an operational variation rather than an uncertainly; only the primary membership functions are hence used to represented such changes, and both type-1 and type-2 fuzzy logic have produced the same results as shown in Fig. 7. Higher load factors are seen leading to lower MTBF than the lower ones.

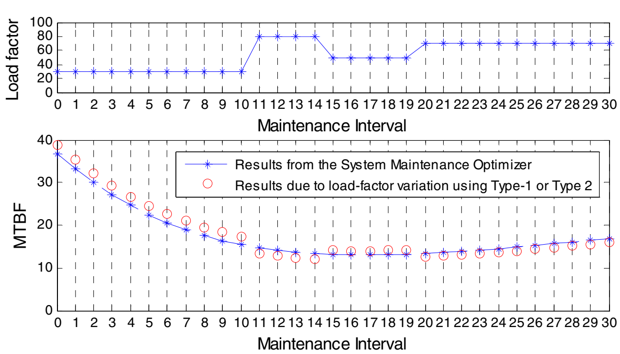


Fig. 7 Updated MTBF Due to Load-factor Variation

C. Impact of Transformer Working Environment on Load-point Reliability

According to Table II, the weather uncertainty (fine, average and adverse) are superimposed to the working-environment variations (poor, average and good), as shown in Fig.6 (c2). As shown in Fig. 8, type-2 fuzzy logic addresses both the operational variation and the uncertainty. In contrast, type-1 fuzzy logic addresses only the former. This is consistent with other reported research with type-1 fuzzy [7, 11] showing insensitivity for uncertainties. Type-2 fuzzy logic indicates correctly that bad weather worsens the MTBF, as can be seen in the later part of the MTBF-plot.

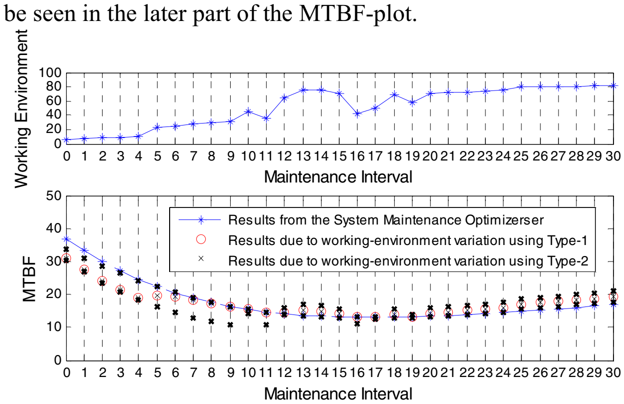


Fig. 8 Updated MTBF Due to Working-environment Variation

D. Impact of Transformer Previous Maintenance on Load-point Reliability

With the three extents of maintenance (minor, medium and major) modeled as "uncertainties", which are presented by the secondary membership functions as shown in Fig.6 (c4). As a result, MTBF as obtained from type-1 fuzzy logic are insensitive to the extent of maintenance. As expected, the type-2 fuzzy system correctly relates the highest MTBF with major maintenance and the lowest MTBF with minor maintenance as shown in Fig. 9.

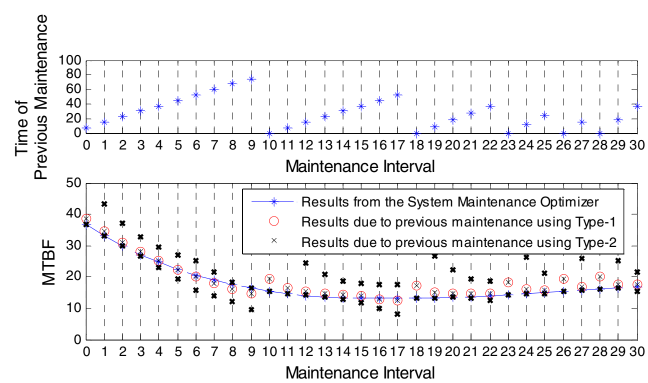


Fig. 9 Updated MTBF Due to Previous Maintenance

E. Impact of Circuit-breaker Reliability on Load-point Reliability

The configuration and protection of a power system directly affect the reliability of the power supply to the load points. Minimum cut sets method is used in this study to analyze the impact of configuration of this system (Fig. 5), taking into consideration the open and short-circuit failure modes, as well as protecting practices of circuit breakers. Minimum cut set method [20] has been used to obtain an estimate of reliability for complex system which cannot be simplified into simple configurations (series or parallel). By definition, a minimum cut set is one irreducible set of components whose failures will definitely cause the system failure. From the reliability point of view, all the minimum cut sets can be viewed as connected in series, and all the events within one minimum cut set can be viewed as connected in parallel. Under the assumption that the generating units are always available and line 11 (L11) is a transmission line supplying power to other areas, the minimum cut sets associated with the load point are shown in Fig. 10. It can be seen in Fig. 10 that besides the transformer, failure or combination of failures of circuit breaker also causes the unavailability of load point.

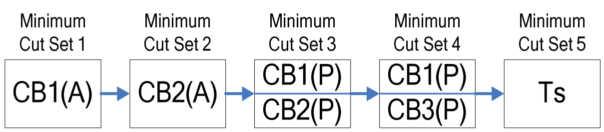


Fig. 10 Minimum Cut Sets of the Load Point.

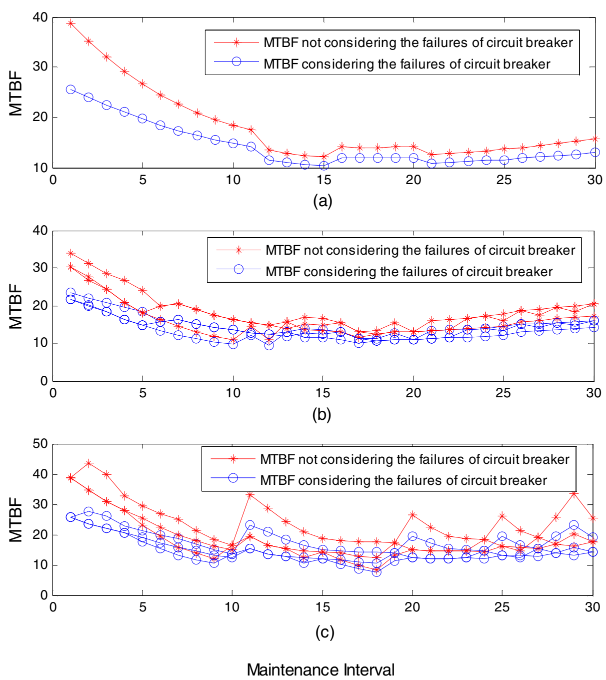


Fig. 11 Load-point MTBF Due to Circuit-breaker Failures & Three Sets of Transformer Operational Variations & Uncertainties of (a) Load Factor, (b) Environment, and (c) Time from Previous Maintenance

Fig. 11 shows the consistent reductions of the load-point MTBF due to the inclusion of circuit-breaker failures, which exist in all the three cases of transformer operational variations/ uncertainties. The substation configuration and circuit-breaker failures are thus seen to have a high impact in the reliability assessment of offshore substation.

V. CONCLUSIONS

This paper proposes an approach for implementing a system-optimized maintenance plan on each offshore substation, and for estimating the change of load-point reliability due to operational variations and uncertainties of its key components. The maintenance advisor will report any drastic deterioration of load-point reliability within the substation, which may lead to re-optimization of the substation's maintenance activities for meeting its desired reliability during operation.

Type-2 fuzzy logic is demonstrated to be superior to type-1 fuzzy logic for modeling operational variations and uncertainties of substation transformers arising from load factor, ageing, working environment and previous maintenance. Circuit-breaker failures and the substation configuration are shown to have a significant impact on the load-point reliability. The other contribution of this paper is the proposed type-2 fuzzy hidden Markov model for the offshore substation for modeling the non-linear performance characteristics and relationships between offshore equipments.

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