

# Available Transfer Capability Evaluation Based on Extended Blind Number

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**Abstract** – In view of the uncertainties in off-line available transfer capability (ATC) evaluation, a method based on blind number was proposed. The study on ATC offers important reference information for secure operation of power system, decision-making of power market participants, system planning and so on. The representation of ATC will be more accurate with full consideration of uncertainties. Blind number is able to comprehensively express various types of uncertainties, and in this paper, the content of blind number is theoretically extended with the provision of probability density information. With the OPF based ATC model, the optimization algorithm based on the expanded blind number theory is detailed and the effectiveness of the approach is justified by the IEEE 30-bus system.

**Index Terms** – Available transfer capability, uncertainty, blind number, probability density, optimal power flow, fuzzy

## I. INTRODUCTION

Available transmission capacity(ATC) between the regional power systems is an important technical index representing the operation state of the system and guiding market resources optimal allocation. The evaluation of ATC may be based on determinate model or probabilistic model, the former is easier to be implemented with fast calculation and suitable for online ATC evaluation. With regard to the offline ATC evaluation forecasting the future scenario of the system, the uncertainties need to be properly considered by probabilistic approach. The stochastic model [1] takes into account the uncertainty of generators and lines failure, which is subject to a two-point distribution and the errors of load forecast, which is subject to normal distribution. To solve the problem of time consuming, Monte Carlo simulation is introduced in [2] to evaluate ATC with huge amount of uncertain variables in large scale system. While Bootstrap approach [3] collects the system data of the latest days and simulates the possible scenario in the next future, then Monte Carlo approach, optimization algorithm and statistic theory are applied together to evaluate the value of ATC. In this paper, a blind number [4,5] based ATC evaluation approach is proposed for dealing uncertainty. Blind number is able to express multiple types of uncertainty

therefore makes the proposed model more capable for uncertainty representation. With regard to the limitation of the blind number, the paper extended its expression formula giving the distribution information in each interval of blind number, thus the extended blind number based ATC evaluation model is much richer in expressing the uncertainty, which will surely increase the accuracy of the model. With the proposed model, probability density distribution of ATC will be obtained and the corresponding result of confidence degree is more reliable with reference to the reality.

The rest of the paper is organized as follows: Section II briefly introduces the concept of blind number and develops the extended blind number. Section III analyzes the uncertainties in ATC evaluation; the model of blind number based ATC evaluation is reported in Section IV and the numerical examples are shown in Section V. Finally, conclusions are drawn in Section VI.

## II. EXTENDED BLIND NUMBER

In reality the information contains several kinds of uncertainties [4], namely fuzziness [6], randomness [7] or grey characteristics [8] etc. The information with multiple types of uncertainties can be defined as blind information, which can be expressed by blind number.

### A. Definition of Blind number [4]

Suppose  $a_i \in g(i)$ ,  $\alpha_i \in [0,1]$ ,  $i = 1, 2, \dots, n$ ,  $f(x)$  is the function defined on the interval function  $g(i)$ , and

$$f(x) = \begin{cases} \alpha_i, & x = a_i (i = 1, 2, \dots, n) \\ 0, & \text{others} \end{cases} \quad (1)$$

where  $n$  is the rank of  $f(x)$ ,  $\alpha_i$  is the probability of  $x = a_i$ , when  $\alpha_i$  is a real number  $f(x)$  is an ‘unascertained number’ and if  $\alpha_i$  is a ‘interval number’,  $f(x)$  is a ‘blind number’, namely, the blind number is composed by the intervals with the corresponding probabilities. Suppose  $\alpha_i = [x_{2i-1}, x_{2i}]$  and  $\sum_{i=1}^n \alpha_i = \alpha \leq 1$ , the distribution of blind number  $f(x)$  is illustrated in Fig.1. While the algebraic operation laws of the blind number are detailed in [4].

### B. The extended blind number

In the formula of the traditional blind number as referred in II-A, the detailed distribution inside the interval is not given,

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which is usually taken as even distribution during algebraic operation [4,5]. In fact, the actual distribution may be various. Therefore the expression of the blind number was extended so as to allow the description of the detailed distribution inside the interval. The extended blind number is formulated as follows:

$$f_x(x) = \begin{cases} g_i(x) & x \in [x_{2i-1}, x_{2i}] (i=1,2,3,\dots,n) \\ 0 & \text{others} \end{cases} \quad (2)$$

Where,  $f_x(x)$  is the probability density function of the variable  $x$ ,  $g_i(x)$  is the probability density of  $x$  in the interval  $[x_{2i-1}, x_{2i}]$ , therefore the confidence degree of  $x \in [x_{2i-1}, x_{2i}]$  is

$$\int_{x_{2i-1}}^{x_{2i}} f_x(x) dx = \int_{x_{2i-1}}^{x_{2i}} g_i(x) dx.$$

Of course the sum of confidence degrees with respect to all the intervals should be less than 1, namely  $\sum_{i=1}^n \int_{x_{2i-1}}^{x_{2i}} g_i(x) dx \leq 1$ .

### III. UNCERTAINTIES IN ATC EVALUATION

The uncertainty of ATC between the regional power system may caused by various factors, which should be fully considered for the offline ATC evaluation. In this paper, load uncertainty will mainly be considered.

#### A. Load uncertainty

Usually the uncertainty of the load can be described with a stochastic variable with normal distribution. But the load may vary with the mutation of the related factors, which can be weather, or other social economic activities. Moreover, the forecast result may be various with different forecast technologies. Reasonable and full description of the load is very important to the accuracy of the ATC evaluation. Blind number is applied in this paper for the representation of the load uncertainty.

#### B. Blind number representation of the load

The necessities of the blind number representation of the load can be analyzed in the following two respects:

- 1) Mutation of the driven factors of the load: although the load uncertainty is usually subject to normal distribution, the actual load may contain multiple uncertainties. For instance, whether high energy-consuming industry will be built will tremendously impact the future load of the region, the corresponding uncertainty cannot be expressed by a single normal distribution, while the extended blind number is able to describe this uncertainty conveniently.
- 2) Result expression of different forecast technologies: different forecast technologies get different forecast value with different using of the input data. The approaches of Regression analysis, exponential smoothing, time series try to discover the tendency of the load evolution, while artificial neural network, support vector machine are the approaches to identify the mapping between the load and its driven factors. The comprehensive approach [9] gets a final result by

assigning the weights to the various forecast values obtained with different forecast approaches,

$$3) \quad L^F = \alpha_1 L_1^F + \alpha_2 L_2^F + \dots + \alpha_n L_n^F \quad (3)$$

$$\alpha_1 + \alpha_2 + \dots + \alpha_n = 1$$

Where,  $L_i^F$  is the forecasted load with approach  $i$ ,  $\alpha_i$  is its assigned weight,  $L^F$  is the final result. A determinate forecast value is obtained with this approach, but the expression of blind number fully keeps the uncertain information,

$$L^F(x) = \begin{cases} \alpha_1 & x = L_1^F \\ \dots\dots & \\ \alpha_n & x = L_n^F \end{cases} \quad (4)$$

Where,  $x$  is the load variable. With the extended blind number, the expression may be more precise with the distribution information,

$$f(p) = \begin{cases} g_i(p) & p \in [p_{2i-1}, p_{2i}] (i=1,2,3,\dots,n) \\ 0 & \text{others} \end{cases} \quad (5)$$

Where,  $f(p)$  is the load distribution function,  $g_i(p)$  is the probability density in the load interval  $[p_{2i-1}, p_{2i}]$ , hence the confidence level of the load  $p \in [p_{2i-1}, p_{2i}]$  is  $\int_{p_{2i-1}}^{p_{2i}} g_i(p) dx$ .

### IV. ATC EVALUATION BASED ON EXTEND BLIND NUMBER

The evaluation of ATC is a typical optimization problem and it is directly related to the operation state of the uncertain and time varying system. The uncertain factors changing the state of the system will influence the actual ATC value. In this paper, the uncertainty of the load is represented by the extended blind number, hence the detailed ATC distribution is obtained.

#### A. ATC evaluation model based on OPF

ATC evaluation can be based on OPF (Optimal Power Flow), the objective functions of which may be various but are equivalent, e.g. maximize the total active power output of the power generation region, maximize the total load of the load region and maximize the active power flow of the interconnecting lines. In this paper, we take the maximization of the total active power output of the power generation region as the objective function:

$$f = \text{Max} \sum_{i \in A} (P_{Gi}) \quad (6)$$

S.T.

- (1) Power flow equation constraints:

$$P_{Gi} - P_{Di} - V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \quad (7)$$

$$Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0$$

(2) Active and reactive power constraints of the generators:

$$P_{Gi}^{base} \leq P_{Gi} \leq P_{Gi}^{max} \quad i \in A \quad (8)$$

$$Q_{Gi}^{base} \leq Q_{Gi} \leq Q_{Gi}^{max} \quad i \in A \quad (9)$$

(3) Constraint of active power demand:

$$P_{Di}^{base} \leq P_{Di} \leq P_{Di}^{max} \quad i \in B \quad (10)$$

(4) Voltage limit :

$$V_i^{min} \leq V_i \leq V_i^{max} \quad i \in A \cup B \quad (11)$$

(5) Line flow limit :

$$|P_{ij}| \leq P_{ij}^{max} \quad i \in A, j \in B \quad (12)$$

Where,  $P_{Gi}$ ,  $Q_{Gi}$  are the active power output and reactive power output of the generator at bus  $i$ ;  $P_{Di}$ ,  $Q_{Di}$  are the active and reactive power demand at bus  $i$ ;  $G_{ij}$ ,  $B_{ij}$  are the respectively the real part and reactive part of the element of the admittance matrix;  $\theta_{ij}$  is the angle difference between bus  $i$  and  $j$ ;  $V_i$  is the voltage of the bus  $i$ ,  $P_{ij}$  is the power flow in line between bus  $i$  and  $j$ ;  $A$  and  $B$  are respectively the zones of power export and import; the superscripts “base” “max” and “min” mean respectively the base case, the upper limit and the lower limit.

### B. Computation algorithm for ATC evaluation based on blind number

When the extended blind number represented load is introduced in the ATC evaluation, the final result of ATC is in the form of extended blind number as well. The algebraic operation of the blind number drastically increases the computation burden. It is reported in [4, 5] that with the traditional algorithm the computation is exponentially increased with the expansion of the blind number dimension, which is apparently not suitable for large scale system application. Moreover, theoretically speaking, the traditional algorithm of blind number operation cannot be applied to the operation of the extended blind number. Therefore a new approach was proposed as below:

1) Suppose an extended blind number  $x$  as shown in Fig.1, with its distribution along the interval(s).  $x$  is divided into a number of small intervals computing their respective confidences degrees with the integration of the probability density functions.

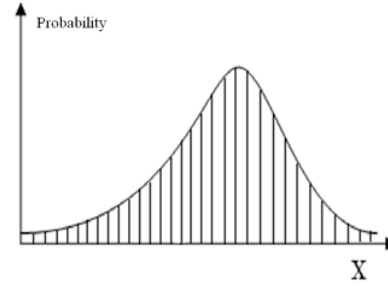


Fig.1 Distribution of the extended blind number  $x$

- 2) Central value of the small interval is taken to represent the value of the interval and is introduced to the evaluation of the ATC obtaining the ATC value with reference to each small interval and its corresponding confidence degree. All combinations of the intervals in the operation between the blind numbers should be considered.
- 3) Sort all the ATC intervals and merge the overlapped intervals taking into account the corresponding confidence degrees to get the final blind number expressed ATC, which can be easily illustrated by its distribution.

## V. NUMERICAL EXAMPLES

In this Section, a simple example was used to compare the difference between the operation of extended and traditional blind number accounting for the more detailed and precise information expression of the extended blind number. Then the extended blind number based ATC evaluation approach was illustrated by its application to the IEEE 30-bus system.

### A. Comparison between the operation of the extended and traditional blind number

With comparison to the traditional blind number, the extended blind number gives detailed probability density distribution. To illustrate the difference, the following example was considered:

$$Q = x + y \quad (13)$$

In which,  $x$  and  $y$  may be with the traditional blind number expression:

$$f(x) = \begin{cases} 0.4 & x \in [0,2] \\ 0.6 & x \in [2,6] \end{cases} \quad (14)$$

$$f(y) = \begin{cases} 0.3 & y \in [3,5] \\ 0.7 & y \in [5,7] \end{cases}$$

or with the extended blind number expression:

$$g(x) = \begin{cases} \frac{0.4}{0.3 \times \sqrt{2\pi}} e^{-\frac{(x-1)^2}{2 \times 0.3^2}} & x \in [0,2] \\ 0.15 & x \in [2,6] \end{cases} \quad (15)$$

$$g(y) = \begin{cases} \frac{0.3}{0.3 \times \sqrt{2\pi}} e^{-\frac{(y-4)^2}{2 \times 0.3^2}} & y \in [3,5] \\ 0.35 & y \in [5,7] \end{cases}$$

Equation (14) describes the uncertainty with the intervals and the corresponding confidence degrees; while (15) presents intervals and the detailed probability density distribution. Equations (14) and (15) are with equal confidence degrees in the same interval. Therefore, (14) is a more universal expression containing the case of (15) but is roughly estimated as even distribution in actual operation of traditional blind number. The detailed description of the probability density distribution of the extended blind number accounts for its more accurate uncertainty representation.

Fig. 2 shows the probability density distributions of the two cases,  $Q_1$  and  $Q_2$  are respectively the results with the former one contains only the intervals and corresponding confidence degrees and the latter is with the detailed probability density distribution of the intervals. Obviously,  $Q_1$  and  $Q_2$  are different, theoretically speaking the latter is a particular case of the former one and is more accurate for the consideration of the known uncertainty with respect to the rough estimation of even distribution. The accuracy can be demonstrated by the difference of confidence degree, e.g.  $p(Q_1 > 7) = 0.68$ , while  $p(Q_2 > 7) = 0.6966$ ;  $p(Q_1 > 9) = 0.20$ , while  $p(Q_2 > 9) = 0.3615$ ;  $p(Q_1 > 11) = 0.14$ ; while  $p(Q_2 > 11) = 0.1060$ .

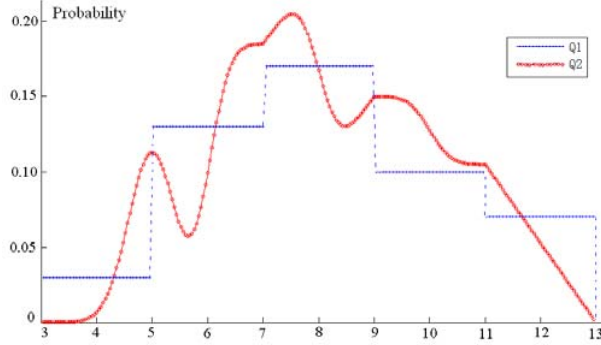


Fig.2 probability density distributions with traditional and extended blind number

### B. Analysis on the extended blind number application to ATC evaluation

The extended blind number was applied to ATC evaluation with the IEEE 30-bus system as a test system as shown in Fig. 3. The system is composed of three zones. There are 6 generation-buses and 20 load-buses and 41 lines. ATC value will be computed between regions 1 and 3 under difference load condition.

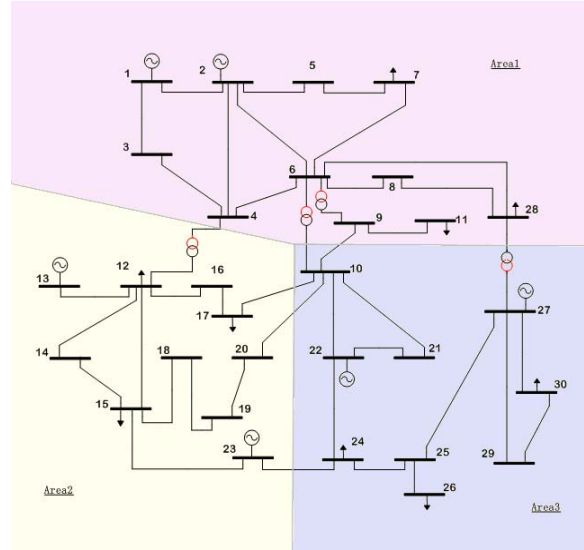


Fig. 3 IEEE 30-bus system

With reference to the OPF based ATC evaluation model introduced in IV.A, the objective function for the ATC evaluation is to maximize the generation in zone 1,

$$\text{Max} \Sigma (P_{G1} + P_{G2}) \quad (16)$$

Where, the constraints are as explained in IV.A. The load  $P$  is assumed to be composed of a constant number  $P_0$  and a stochastic number  $\Delta P$ , namely  $P = P_0 + \Delta P$ .  $P_0$  is the load expectation and  $\Delta P$  may be subject to the normal distribution with zero average value. Of course the actual distribution can be various and the mutation of the load can be considered as well. The extended blind number will be used for the load expression, besides the uncertainty of the load itself, it is able to fully keep all the load information obtained by several forecast technologies as explained in III.B.

It was supposed that the stochastic part  $\Delta P$  of all the load in IEEE 30-bus system was subject to  $N(\mu_1, \sigma_1)$  with 40% and subject to  $N(\mu_2, \sigma_2)$  with 60%, while  $P_0$  was the base value as it was in the original IEEE 30-bus system. The expanded blind number expression of  $\Delta P$  may be formulated as below:

$$g(\Delta P) = \begin{cases} \frac{0.6}{\sqrt{2\pi}\sigma_2} e^{-\frac{(\Delta P - \mu)^2}{2\sigma_2^2}} & \Delta P \in [-3\sigma_2 + \mu_2, 3\sigma_2 + \mu_2] \\ \frac{0.4}{\sqrt{2\pi}\sigma_1} e^{-\frac{(\Delta P - \mu)^2}{2\sigma_1^2}} & \Delta P \in [-3\sigma_1 + \mu_1, 3\sigma_1 + \mu_1] \\ 0 & \text{others} \end{cases} \quad (17)$$

In which, the distribution outside the interval of  $[-3\sigma + \mu, 3\sigma + \mu]$  was neglected, since it has little influence on the computation result. If the uncertainty is expressed by the traditional blind number, then

$$f(\Delta P) = \begin{cases} 0.6 & \Delta P \in [-3\sigma_2 + \mu_2, 3\sigma_2 + \mu_2] \\ 0.4 & \Delta P \in [-3\sigma_1 + \mu_1, 3\sigma_1 + \mu_1] \\ 0 & \text{others} \end{cases} \quad (18)$$

Where  $\sigma_1 = \sigma_2 = 0.002$ ,  $\mu_1 = 0.003$  and  $\mu_2 = -0.003$ , the distribution of  $\Delta P$  is shown in Fig. 4.

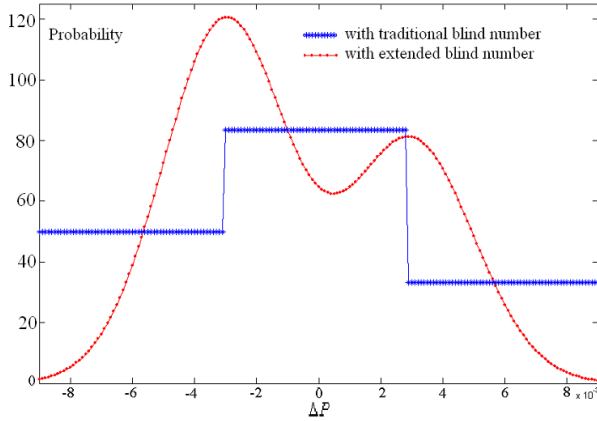


Fig. 4: The distribution of  $\Delta P$

By following the calculation algorithm of IV.B, the ATC distribution with the traditional blind number ( $A_1$ ) and the extended blind number ( $A_2$ ) were shown in Fig.5. The curve  $A_1$  presents the detailed distribution of ATC according to the extended blind number representation. The curve  $A_2$  follows its original input load form of traditional blind number reflecting only the confidence degree of the interval and even distribution is assumed during the operation.

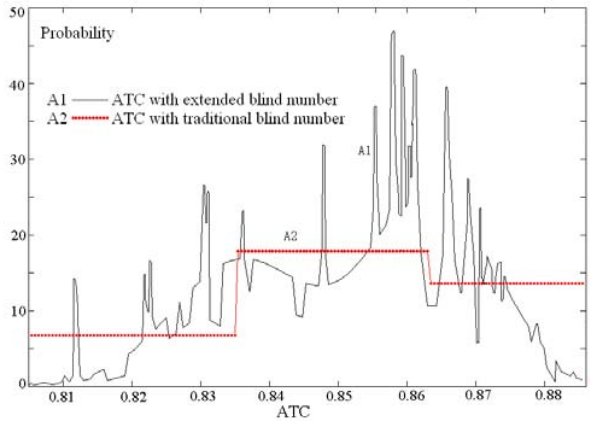


Fig.5 Distribution of ATC with extended blind number ( $A_1$ ) and traditional blind number ( $A_2$ )

To clearly explain the difference, the confidence degree was defined as the probability when ATC was above a certain number. For example, if the ATC value was 0.85, the corresponding confidence degree was  $P(ATC \geq 0.85)$ . Because the ATC result obtained with the extended blind number made full use of the distribution information of the interval and reflected more accurately the actual situation,

more reliable reference was provided for the system operation and planning. It can be explained by the different ATC curves of  $A_1$  and  $A_2$  with the horizontal axis of confidence degree as shown Fig. 6. The curve of confidence degree with respect to the traditional blind number is a polyline, which quite fitted with the even distribution assumption during the operation of the blind number. On the other hand, the confidence degree curve with respect to the extended blind number fluctuated around the polyline of the traditional blind number. Therefore, as pointed out in V.A, the extended blind number was actually the particular case of the traditional blind number and presents more detailed and accurate uncertain information. The polyline in Fig.6 can be taken as the fitting curve of the fluctuate line. Fig. 7 showed the error curve of the fluctuate line with respect to the polyline, which depended on the difference between the actual distribution and the assumed even distribution. In Fig.6, it may be found that there are several intersections of the two curves, which were shown in Fig.7 as the intersections of the error curve, namely only when the confidence degree equals to 0.392, 0.701, 0.807 and 1, the ATC values obtained with the load representation of traditional blind number and extended blind number were equal. In other words, at the most confidence degree values, there were difference ATC values with different load uncertainty representation. The ATC value obtained with the traditional blind number was a rough solution based on the assumption of even distribution in the intervals, while the ATC value obtained with extended blind number was an accurate solution using the detailed actual distribution information in the intervals hence it was more reliable and reasonable than the former.

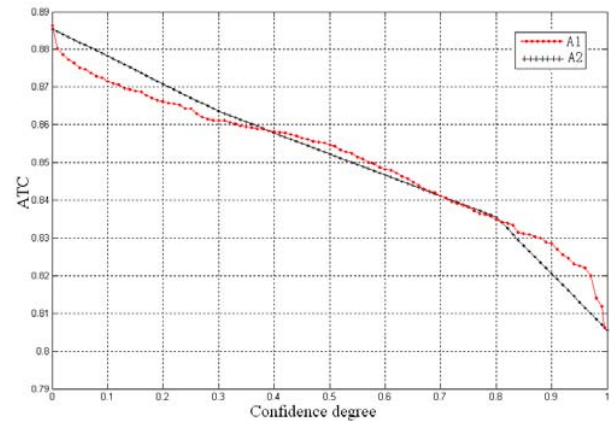


Fig.6 ATC curve with the horizontal axis of confidence degree

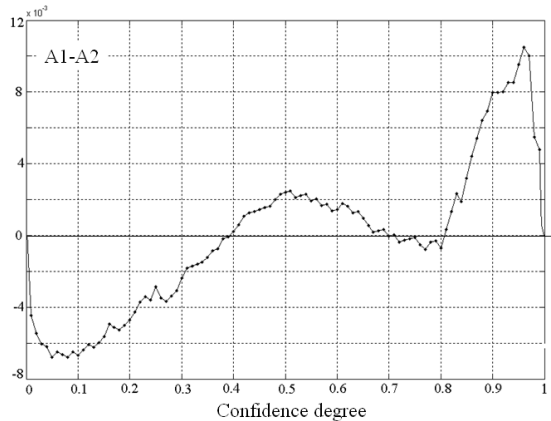


Fig.7 Error of A1 and A2 with the horizontal axis of confidence degree

## VI. CONCLUSIONS

An ATC evaluation approach based on extended blind number was proposed and practical computation algorithm had been applied. With the theoretical method and the numerical examples, the following conclusions had been made:

- 1) It was necessary to use blind number to express the load uncertainty for offline ATC evaluation, because of the mutation of load driven factors and the full consideration of the load forecasts with various forecast technologies.
- 2) The extended blind number was actually the particular case of the traditional blind number with accurate description of the distribution information. A traditional blind number covered many extended blind numbers, while the latter provided more detailed description of the uncertainty.
- 3) With the horizontal axis of confidence degree, the ATC curve obtained with the extended blind number fluctuated around the ATC curve obtained with the traditional blind number. The latter was the rough expression of the former, which was the accurate solution with distribution information fully reflecting the actual situation. More accurate ATC value would be a more reliable reference for system operation and planning and market decision making.

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