Impacts of Tradable Emission Permits on Oligopoly Electricity Market Production under Complete and Incomplete Information

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Abstract—A method is developed to explore the potential links between an oligopolistic electricity market and a competitive emission permit market in which permits allocated to those highly efficient generation companies (units) with lower emissions could be traded to other companies with higher emissions. The well-developed Cournot non-cooperative game model is employed to describe the behavior of power producers, and to determine the market equilibriums of generation outputs under complete and incomplete information. A numerical example with six power producers is employed to demonstrate the features of the developed model as well as to analyze the impacts of emission permit trading on the oligopoly electric market equilibriums.

Keywords—Electricity market, emissions trading, Cournot game theory, oligopoly, non-cooperative games

I. NOMENCLATURES

\( p_i \): The output of producer \( i \) \( (i = 1, 2, \ldots, n) \);

\( P_L \): The total output of all producers;

\( R \): The market price of electricity;

\( K \): The load-price elasticity coefficient of electricity;

\( e_i \): The initial allocation of permits to producer \( i \);

\( E \): The capped level of total emissions;

\( e_m \): The net demand of emission permit by producer \( i \);

\( \rho \): The price of the tradable emission permit.

II. INTRODUCTION

The potential role of emissions trading to support environmental objectives is receiving increasing policy attention. Considerable research work has been done on emission trading mechanisms, including five key aspects: 1) the fundamental theoretical underpinnings of emissions trading [1]-[3] to establish markets for what are currently environmental externalities; 2) the potential efficiency advantages of such policies [4]-[5] including the respective strengths and weaknesses of emission permit markets compared with direct taxation of emissions [4]; 3) the allocation of pollutant emission rights [6]-[8]; 4) studies of the conduct of emitters within such markets [9]-[11]; and 5) the potential role of global greenhouse gas (GHG) markets [12]-[14].

After some early experience with emissions trading of Sulphuric Oxide (SOx) air pollutants in the United States, greenhouse emissions trading has been established within the international climate framework of the Kyoto Protocol, and with the European Union. This has driven growing consideration of the potential of emissions within developing countries [15]. In China, considerable research work has been undertaken on emissions trading of SOx, with special emphasis on the applicability of such policies, methods for initial allocations, and auction pricing [16]-[20].

Over the last several decades, the electricity industry itself has been transitioning in some countries from a traditional monopolistic industry structure towards a greater role for competitive electricity markets. However, these restructured electricity markets are in many circumstances, best considered as oligopoly rather than perfectly competitive markets. Hence there are considerable risks of market power in such restructured industries. Considerable work has been done on the analysis and mitigation of market power in electricity markets mainly by employing game theory methods [21]-[24]. However, most game theory based market power analysis methods require assumptions of market participants having complete information, and this is not realistic in competitive electricity markets. With this in mind, a game theory based market equilibrium analysis method capable of handling incomplete information is presented in [25].

Based on the research work in [25], this paper presents an extension to current models by incorporating a competitive market for emission permits into an oligopoly electricity market. It is assumed that the power producers behave as price takers in the emission trading market while they act strategically in the electricity market. The focus of this work is on the impacts of tradable emission permits on the equilibrium of an oligopolistic electricity market under complete and incomplete information. One of the well-known classical oligopoly market models, the Cournot model, is employed in this work to study the behavior of power producers under such circumstances.

This paper is organized as follows. In Section III, the developed mathematical models are presented for the situations under complete and incomplete information with emission trading included. In Section IV, a numerical example is utilized to demonstrate the developed models. Finally, Section V concludes the paper.
II. MODEL FORMULATION

A. The market equilibrium under complete information

Before presenting the model developed in this paper, some basic hypotheses are made about the Cournot non-cooperative game model and the assumption of so-called complete information: (1) all producers always pursue profit-maximization; (2) all producers compete on quantity rather than price; (3) the product from all producers is homogeneous; (4) the demand function is linear with the price; (5) each producer knows the market demand function, its own cost function and the cost functions of rivals.

Suppose that the demand function in the studied electricity market takes the following linear form,

$$\max L = P_{L_{\max}} - KR$$ \hspace{1cm} (1)

where, $P_L$ is the total active power load, $K$ is the load-price elasticity of electricity; $R$ is the market price of electricity; $P_{L_{\max}}$ is the maximum possible active power load when $R=0$, and $R_{\max}$ is the upper limit of the electricity price (when $R$ is greater than $R_{\max}$, customers will choose to not consume electricity).

Substituting (2) into (1) yields,

$$R = R_{\max} - KP_L$$ \hspace{1cm} (3)

Generation costs are represented as quadratic functions

$$C_i(p_i) = a_i + b_i p_i + c_i p_i^2 \hspace{1cm} i = 1, 2, ..., n$$ \hspace{1cm} (4)

where $a_i$, $b_i$ and $c_i$ are production cost coefficients.

The payoff function of each power producer could be written as

$$U_i(P) = p_i R - C_{1i}(p_i) - C_{2i}(p_i) - \rho e_{mi}$$ \hspace{1cm} (5)

where $C_{2i}(x_i) = d_i x_i^2$ represents the cost function for emission reduction by the $i$th power producer, $e_{mi}$ is the net demand of emission permit by this producer; $\rho$ is the price of the tradable emission permit.

The objective function is then

$$G = \max_i U_i(P)$$ \hspace{1cm} (6)

Subject to

$$e_m + e_i = k_i p_i - \alpha_i p_i$$ \hspace{1cm} (7)

$$\sum_{i=1}^{n} e_{mi} = 0$$ \hspace{1cm} (8)

In order to apply the Cournot non-cooperative game model, the above optimization problem must be transformed into an unconstrained one.

Rearranging equation (7) we have

$$e_{mi} = k_i p_i - \alpha_i p_i - e_i$$ \hspace{1cm} (9)

Substituting (9) into (5) and (8) yields,

$$U_i'(P) = p_i R - C_{1i}(p_i) - C_{2i}(p_i) - \rho (k_i p_i - \alpha_i p_i - e_i)$$ \hspace{1cm} (10)

and

$$\sum_{i=1}^{n} (k_i p_i - \alpha_i p_i - e_i) = 0$$ \hspace{1cm} (11)

By introducing Lagrange multiplier $\lambda$ and adding (10) and (11) together, we have

$$U_i^*(P) = p_i R - C_{1i}(p_i) - C_{2i}(p_i) - \rho (k_i p_i - \alpha_i p_i - e_i) + \lambda \sum_{i=1}^{n} (k_i p_i - \alpha_i p_i - e_i)$$ \hspace{1cm} (12)

The Cournot equilibrium occurs at the solution of the equations:

$$\frac{\partial U_i^*(P)}{\partial p_i} = R_{\max} - K \sum_{j=1}^{n} p_j - b_i - 2(K + c_i) p_i$$

$$= 2d_i \alpha_i^2 p_i + (\lambda - \theta)(k_i - \alpha_i) = 0$$

$$i = 1, 2, ..., n.$$ \hspace{1cm} (13)

where

$$\frac{\partial U_i^*(P)}{\partial \alpha_i} = -2d_i \alpha_i^2 p_i + (\theta - \lambda) p_i = 0,$$

and

$$\sum_{i=1}^{n} (k_i p_i - \alpha_i p_i - e_i) = 0.$$}

B. The market equilibrium under incomplete information

In [25], the expected electricity market equilibriums under three different representations of incomplete information were investigated. Due to space constraints, only the third representation in [25] is employed here; that is, each power producer has an estimated distribution of the cost function for
The actual production cost function and the cost function associated with emission-reduction of the \( i \)th producer respectively are

\[
C_{i}(p_{i}^{(i)}) = a_{i} + b_{i}p_{i}^{(i)} + c_{i}(p_{i}^{(i)})^2 \quad i = 1, 2, \ldots, n. \quad (14)
\]

\[
C_{2i}(p_{i}^{(i)}) = d_{i}(p_{i}^{(i)})^2 \quad i = 1, 2, \ldots, n. \quad (15)
\]

The cost function of the \( j \)th producer as estimated by the \( i \)th producer is

\[
C_{ij}^{(i)}(p_{j}^{(i)}) = a_{ij}^{(i)} + b_{ij}^{(i)}p_{j}^{(i)} + c_{ij}^{(i)}(p_{j}^{(i)})^2 + \varepsilon_{ij}^{(i)} \quad 
\varepsilon_{ij}^{(i)} \in \mathcal{N}[0, \sigma^2(C_{ij}^{(i)})] \nonumber \
i, j = 1, 2, \ldots, n; i \neq j \quad (16)
\]

where \( \mathcal{N}[0, \sigma^2(C_{ij}^{(i)})] \) represents a normal distribution with mean 0 and variance \( \sigma^2(C_{ij}^{(i)}) \).

The cost function associated with emission-reduction of the \( j \)th producer as estimated by the \( i \)th producer is

\[
C_{ij}^{(i)}(p_{j}^{(i)}) = d_{ij}^{(i)}(\alpha_{j}^{(i)}p_{j}^{(i)})^2 + \varepsilon_{ij}^{(i)} \quad 
\varepsilon_{ij}^{(i)} \in \mathcal{N}[0, \sigma^2(C_{ij}^{(i)})] \nonumber \
i, j = 1, 2, \ldots, n; i \neq j \quad (17)
\]

where \( \mathcal{N}[0, \sigma^2(C_{ij}^{(i)})] \) represents a normal distribution with mean 0 and variance \( \sigma^2(C_{ij}^{(i)}) \).

Suppose that the standard deviations take the following forms

\[
\sigma(C_{ij}^{(i)}) = gp_{j}^{(i)}
\]

and

\[
\sigma(C_{ij}^{(i)}) = h\alpha_{j}^{(i)}p_{j}^{(i)}
\]

where \( g \) and \( h \) are specified positive constants.

The \( i \)th producer’s estimate of the total generation output is

\[
p_{L}^{(i)} = \sum_{j=1}^{n} p_{j}^{(i)} \quad (19)
\]

The payoff function of the \( i \)th producer is

\[
U_{i}(P) = p_{i}^{(i)}(R - C_{1i}(p_{i}^{(i)}) - C_{2i}(p_{i}^{(i)}) - \rho e_{ni}) \quad i = 1, 2, \ldots, n. \quad (20)
\]

The payoff function of the \( j \)th producer as estimated by the \( i \)th producer is

\[
U_{ij}^{(i)}(P) = p_{i}^{(i)}(R - C_{1i}(p_{i}^{(i)}) - C_{2i}(p_{i}^{(i)}) - \rho e_{ni}) \quad i, j = 1, 2, \ldots, n; i \neq j \quad (21)
\]

The mean and standard deviation of \( U_{ij}^{(i)}(P) \), respectively, are

\[
\bar{U}_{ij}(P) = p_{i}^{(i)}(R - a_{ij}^{(i)} - b_{ij}^{(i)}p_{j}^{(i)} - c_{ij}^{(i)}(p_{j}^{(i)})^2 - d_{ij}^{(i)}(\alpha_{j}^{(i)}p_{j}^{(i)})^2 - \rho e_{ni}) \quad i, j = 1, 2, \ldots, n; i \neq j
\]

\[
\sigma(U_{ij}^{(i)}(P)) = (g + h\alpha_{j}^{(i)})p_{j}^{(i)} \quad i, j = 1, 2, \ldots, n; i \neq j \quad (22)
\]

Each power producer’s objective is to maximize the mean and to minimize the standard deviation. In engineering terms, this can be expressed as a weighted sum maximization problem.

Maximize \( \Psi_{ij}^{(i)}(P) = \bar{U}_{ij}(P) - \alpha \sigma(U_{ij}^{(i)}(P)) \quad i, j = 1, 2, \ldots, n; i \neq j \quad (24) \]

where \( \alpha \) is a specified constant between 3 and 4 [26].

The objective functions:

Maximize \( \Psi_{ij}^{(i)}(P) \quad i, j = 1, 2, \ldots, n; i \neq j \)

and

Maximize \( U_{i}(P) \quad i = 1, 2, \ldots, n. \)

Subject to

\[
e_{ni} + \sum_{j=1, j \neq i}^{n} e_{nj} = 0 \quad i, j = 1, 2, \ldots, n; i \neq j
\]

\[
\sum_{j=1}^{n} e_{ni} = 0 \quad i = 1, 2, \ldots, n. \quad (25)
\]

Similarly, the above constrained optimization problem could be transformed into an unconstrained one.

\[
U_{ij}^{(P)} = p_{i}^{(i)}(R - C_{1i}(p_{i}^{(i)}) - C_{2i}(p_{i}^{(i)}) - \rho(k_{i}p_{i}^{(i)} - \alpha_{i}p_{j}^{(i)} - e_{i}) + \sum_{j=1}^{n} ((k_{j} - \alpha_{j})p_{j}^{(i)} - e_{j})] + \sum_{j=1, j \neq i}^{n} ((k_{j} - \alpha_{j})p_{j}^{(i)} - e_{j})}
\]

\[
\Psi_{ij}^{(P)} = \bar{U}_{ij}(P) - \alpha \sigma(U_{ij}^{(i)}(P)) + \sum_{j=1}^{n} ((k_{j} - \alpha_{j})p_{j}^{(i)} - e_{j})
\]

3467
The Cournot equilibrium occurs at the solution of the equations

\[ \frac{\partial U_i^i(P)}{\partial p_i^i} = \frac{\partial U_j^j(P)}{\partial \alpha_j^j} = 0, \]

\[ \frac{\partial \Psi_j^j(P)}{\partial p_j^j} = \frac{\partial \Psi_i^i(P)}{\partial \alpha_i^i} = 0 \]

\[ [(k_i - \alpha_i) p_i^i - e_i] + \sum_{j(i),j\neq i}^n [(k_j - \alpha_j) p_j^j - e_j] = 0 \quad i, j = 1, 2, \ldots, n \]

and

\[ \sum_{j=1}^n [(k_i - \alpha_i) p_i^i - \alpha_j p_j^j - e_j] = 0 \quad i = 1, 2, \ldots, n. \]

C. The solution method

To obtain the Cournot non-cooperative game equilibrium, a set of nonlinear equations needs to be solved for both the situation under complete and incomplete information.

Numerical algorithms in MATLAB are employed here for solving the nonlinear equation sets. Specifically, the function ‘fsolve’ in MATLAB is utilized for this purpose.

IV. NUMERICAL EXAMPLES

The methods described in the previous sections are illustrated in the following two subsections, and an example with six power producers is employed for demonstrating the developed models.

A. The market equilibrium under complete information

The cost coefficients together with technological parameters of the six power producers are listed in Table I. We specify \( R_{\text{max}} = 20 \) and \( K = 0.04 \). Each power producer is assumed to have the same emission factor, that is \( k_i = 0.28 \text{t}/\text{kwh} \) (\( i = 1, 2, \ldots, 6 \)). Further, we respectively assume that the total allowable emission and competitive prices of emission trading is 50 t and 2.2 \( \$7/\text{t} \). Moreover, the model also involves an initial allocation of emission permits; in this case it is assumed that the quota is allocated evenly among generation units \([9]\).

For the situation without emission trading, the expected market equilibriums are listed in Table II. Test results for the situation with complete information and emission trading are shown in Table III. From Table III, it is clear that each power producer’s optimal output has decreased compared to that in the situation without the emission trading as shown in Table II, and accordingly the overall generation output level is decreased.

### Table I

<table>
<thead>
<tr>
<th>Producer no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( b )</td>
<td>2.0</td>
<td>1.75</td>
<td>1.0</td>
<td>3.25</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>( c )</td>
<td>0.00375</td>
<td>0.0175</td>
<td>0.0625</td>
<td>0.00834</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>( d )</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.2</td>
<td>0.3</td>
<td>0.45</td>
</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th>Producer no.</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimal output</td>
<td>Market share (%)</td>
</tr>
<tr>
<td>1</td>
<td>96.9</td>
<td>411.1</td>
</tr>
<tr>
<td>2</td>
<td>64.7</td>
<td>240.9</td>
</tr>
<tr>
<td>3</td>
<td>34.0</td>
<td>118.3</td>
</tr>
<tr>
<td>4</td>
<td>59.2</td>
<td>169.3</td>
</tr>
<tr>
<td>5</td>
<td>40.1</td>
<td>104.3</td>
</tr>
<tr>
<td>6</td>
<td>40.1</td>
<td>104.3</td>
</tr>
</tbody>
</table>

Electricity price 6.6 6.74
Expected total generation output 335 332
Total payoff 1148 1181

*TOO=Expected optimal output
TABLE III
TEST RESULTS WITH EMISSION TRADING UNDER COMPLETE INFORMATION

<table>
<thead>
<tr>
<th>Producer no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal output</td>
<td>90.72</td>
<td>60.79</td>
<td>32.17</td>
<td>53.97</td>
<td>36.77</td>
<td>36.77</td>
</tr>
<tr>
<td>payoff</td>
<td>527.47</td>
<td>321.94</td>
<td>158.48</td>
<td>209.05</td>
<td>140.64</td>
<td>151.30</td>
</tr>
<tr>
<td>Market share (%)</td>
<td>29.15</td>
<td>19.53</td>
<td>10.34</td>
<td>17.34</td>
<td>11.82</td>
<td>11.82</td>
</tr>
<tr>
<td>Net demand</td>
<td>13.37</td>
<td>4.25</td>
<td>-4.87</td>
<td>-4.32</td>
<td>-5.44</td>
<td>-2.97</td>
</tr>
<tr>
<td>Reduction efficiency</td>
<td>0.07</td>
<td>0.10</td>
<td>0.18</td>
<td>0.11</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>Electricity price</td>
<td>7.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected total Generation output</td>
<td>311</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total payoff</td>
<td>1509</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The marginal cost of reducing emission 2.2 2.2 2.2 2.2 2.2 2.2

TABLE IV
TEST RESULTS WITH EMISSION TRADING UNDER INCOMPLETE INFORMATION

<table>
<thead>
<tr>
<th>Producer no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal output</td>
<td>87.10</td>
<td>64.94</td>
<td>34.47</td>
<td>44.57</td>
<td>36.42</td>
<td>40.53</td>
</tr>
<tr>
<td>payoff</td>
<td>518.70</td>
<td>347.80</td>
<td>177.88</td>
<td>145.00</td>
<td>168.97</td>
<td></td>
</tr>
<tr>
<td>Market share (%)</td>
<td>28.28</td>
<td>21.08</td>
<td>11.19</td>
<td>14.47</td>
<td>11.82</td>
<td>13.16</td>
</tr>
<tr>
<td>Net demand</td>
<td>12.44</td>
<td>5.51</td>
<td>-4.10</td>
<td>-6.69</td>
<td>-5.36</td>
<td>-1.80</td>
</tr>
<tr>
<td>Reduction efficiency</td>
<td>0.04</td>
<td>0.07</td>
<td>0.16</td>
<td>0.24</td>
<td>0.20</td>
<td>0.12</td>
</tr>
<tr>
<td>Electricity price</td>
<td>7.68</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected total generation output</td>
<td>308</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total payoff</td>
<td>1529</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The marginal cost of reducing emission 2.2 2.2 2.2 2.2 2.2 2.2

B. The market equilibrium under incomplete information

Since each power producer has to estimate the cost function of its rivals under incomplete information, we suppose that the cost function coefficients and technological parameters of producer $j$ as estimated by producer $i$ are as follows,

$$a^{(i)}_j = (1 - \gamma_1) a_j, \quad b^{(i)}_j = (1 - \gamma_2) b_j, \quad c^{(i)}_j = (1 - \gamma_3) c_j, \quad d^{(i)}_j = (1 - \gamma_4) d_j, \quad \alpha = 4, \quad \gamma = 0.1$$

where $\gamma_1$, $\gamma_2$, $\gamma_3$ and $\gamma_4$ are uniformly distributed random numbers between [0,0.2]. The simulation results are given in Table IV.

Similarly, the expected overall generation output level is less than that in the case with complete information, and this results in an increase in the electricity price. On the other hand, the profit of each producer is also increased. The generation outputs of producers 5 and 6 are no longer the same due to the existence of the stochastic parameter $\gamma$.

V. CONCLUSION

In this work, two models are developed for analyzing the impacts of tradable emission permits on oligopoly electricity market production respectively under complete and incomplete information. The Cournot non-cooperative game is employed to determine the electric market equilibrium. A numerical example with six power producers is used to demonstrate the feasibility of these models.

Simulation results indicate that the inclusion of tradable emission permits within the Cournot electricity market equilibrium model leads to a decrease of overall production levels. Furthermore, the impact of tradable emission permits on the generation output level of a generating unit is highly dependent on the accuracy of its estimations about the production and emission abatement cost functions of all rivals.

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