A 3D Self-positioning Method for Wireless Sensor Nodes Based on Linear FMCW and TFDA

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Abstract—In wireless sensor networks (WSN), location information acquisition is critical to guarantee their performance. This paper presents a new positioning method in WSN with high precision at reasonable implementation cost for 3D case. Reference nodes with known locations transmit linear frequency modulation continuous wave (FMCW), while other sensor nodes estimate the range difference to them based on the received signals’ frequency difference, called time frequency difference arrival (TFDA). The location information can be obtained by solving a set of hyperbolic equations. Two different positioning methods: Taylor iterative method and Chan’s method are inspected and compared in terms of accuracy, constraints and computational complexity. This proposed technique is cost-effective, scalable and easy to implement. The simulation results show that the new method enjoys high precision.

Index Terms—Location, frequency modulation, 3D positioning, wireless sensor networks.

I. INTRODUCTION

Wireless sensor networks (WSN) have received great attentions recently due to their wide applications in both military and civilian areas, e.g., environmental monitoring and protection, tragedy rescue, wild animal protection, smart building, and interactive virtual world [1]. Positioning is a key enabler for these attractive applications. Sensing data from a node without its location information is less useful and sometimes useless [2]. Moreover, the availability of position information enables one to employ more efficient protocols and routing algorithms [3][4]. Position-based routing has some obvious advantages over node-based and data-based routing: no need to maintain routing tables need to be maintained, good resilience to mobility and so on.

It is well known that a Global Positioning System (GPS) can provide the absolute coordinates. The positioning service is available only when at least four satellites are visible [5]. It cannot be well utilized in some environment, e.g., indoor, urban area with high buildings, tunnels, forest or jungle. Even when nodes are deployed in GPS friendly conditions, there are issues related to power, size and cost that may make the solution undesirable. Sensor nodes can be very small, for example, wearable sensors. Thus, it could be difficult to integrate GPS receivers into the sensor nodes that can afford only limited power, physical size and cost are highly limited. Moreover, political considerations may affect the availability of GPS signals due to external factors. For example, the Selective Availability policy can dramatically decrease the positioning accuracy.

In WSN, there are two classes of GPS free approaches to localization [6]: (1) coarse-grained localization using minimal information, and (2) fine-grained localization using detailed information. For the first class, minimal information could be binary proximity (e.g., can a node hear another?), cardinal direction information (is it in the north, south, west or east of a given node?), or near far information (in a set of nodes, which one is the closest to a given node?). With such information, the course location of a node can be obtained. The example approaches include: binary proximity [7], centroid calculation [8], geometric constraints [9], approximate point in triangle [10], and identifying code construction (ID-CODE) algorithm [11]. They need lower network resources and cost because only minimal information is required. The main drawback is less accurate location information. Next, we discuss the second class of approaches.

The most common fine-grained positioning technologies include: received signal strength (RSS), angle of arrival (AOA), time of arrival (TOA) and time difference of arrival (TDOA) [12][13]. An RSS method measures the received signal’s power that may change if the environment is changing [14]. In WSN with a time-varying channel, the measuring results are not reliable. Therefore, the position information obtained based on RSS is imprecise. An AOA approach relies on antenna array for determining the angle of an arrival signal [15]. Therefore, no such option exists for small-size sensors of only single antenna setting. An TOA method measures the signal arrival time, which is used by the GPS positioning calculation. It requires fine synchronization that is the key component to determine the positioning resolution. TDOA is widely used for cell phone positioning applications, E911. This method also needs the time synchronization among different stations, which is available in cellular networks. However, this requirement is difficult to be satisfied for sensor nodes with limited communication and computational abilities.

In [16][17], we proposed a new positioning method with high precision feature for WSN limited synchronization requirement. A few reference nodes with the knowledge of their own locations broadcast a linear FMCW signals. All other nodes receive transmitted signals from some of them and can estimate the range difference by the received signals’ frequency difference and, therefore, derive their own location.
In this paper, we extend our work to 3D positioning scenario. This paper is organized as follows: in Section II, a system model is introduced. Section III briefly introduce the distance difference estimation based on linear FMCW technique and the positioning method. Section IV investigates the direct and indirect methods for positioning an unknown node and general nodes (GNs) with unknown positions, each of which receives linear FMCW signals from at least four reference nodes autonomously [18].

Consider a WSN deployed in a predetermined area. A small proportion of devices, called reference nodes (RNs), have a priori information about their coordinates, and other nodes called general nodes (GNs) that do not know their positions in advance, as shown in Fig. 1. RNs may be equipped with GPS receivers or a global reference system, such as accelerometers, compasses and gyros in order to obtain their position. Each RN has a unique identification (ID) and orthogonal to each other. Assume that RNs have perfect synchronization with each other. They broadcast their IDs, location information and a linear FM signals (also called chirp signals) to their neighbors simultaneously. A GN may hear several RNs at the same time, it can recognize the RN based on the received signals, it also receives several chirp signals from these RNs.

A GN then obtains the frequency difference by mixing the received signals. Since the frequency difference is proportional to the range difference, one can obtain the range differences from one GN to two RNs, which can define one hyperbolic equation. In a special case that the range differences from one GN to two different RNs are the same, they define one straight-line equation. With the information of several (M) RNs’ positions and the range differences \( (M - 1) \) to each pair of them, this GN can estimate its own 3D location accurately by solving a set of hyperbolic equations when \( M \geq 4 \).

Fig. 2 shows the block diagram of the time-frequency differential arrival method. A GN can receive linear FMCW signals from M different neighboring RNs. It uses a correlation method to extract the first arrival linear FMCW signal from its nearest RN often with the strongest power. This first arrival signal is used to mix the received signal that may include several linear FMCW signals from RNs. A low-pass filter is then used to remove the high frequency components and let the intermediate frequency (the frequency difference) pass. After windowing, the signal’s power spectrum is calculated by using that fast fourier transform (FFT) algorithm. Therefore, the frequency differences among the first arrival signal and others can be obtained. Due to high-speed digital signal processors and fast algorithms, calculating time frequency difference arrival (TFDAs) is time and cost-efficient. Once a GN knows TFDAs and the locations of these RNs, it can calculate its own position by solving a set of hyperbolic equations. In case one GN can hear three RNs with the same distance, it cannot distinguish these frequency differences. However this GN has the knowledge of these RNs by their IDs, therefore, it can still estimate its own location by calculation the centroid of the triangle determined by these three RNs.

Compare with GPS system, another obvious advantage is that linear FMCW technique is easy to operation, robust and low cost. In this TFDA positioning system, in order to get all information of the difference range, the required hardware is: a mixer and low-pass filter, the required software algorithms are: windowing function, FFT and magnitude square calculating. All these requirements are cost-effective.

In the GPS system, the GPS receiver needs to demodulate the received satellite signals at first, then uses coarse-acquisition (C/A) Gold code to separate them, called C/A decoding. The receiver also obtains the signal receiving time by its local clock. After decoding, the receiver calculates sending time of the signal from each satellite, which takes two steps. 1) the receiver uses the C/A Gold code with the same pseudo-random number sequence as the satellite’s to compute an offset that generates the best correlation. This process is repeated until a correlation peak appears or all 1023 possible cases have been tried. If all 1023 cases have been tried without valid correlation, the frequency oscillator is offset to the next value and the process is repeated; and 2) The receiver begins reading the satellite broadcasting navigation message (including almanac, ephemeris parameters and so on). After it is read and interpreted, the sending time embedded in the message can be acquired. At this time, the receiver can obtain one TOA by computing the difference of the sending and receiving time from one satellite [5]. The hardware and software used in a GPS receiver is much more complex than that of the TFDA technique. It also needs more memory as it must store all satellites’ C/A code.

III. A BRIEF INTRODUCTION FOR TFDA POSITIONING TECHNIQUE

A. Distance difference measurement based on Linear FMCW

Similar to the FMCW radar, each RN transmits a chirp signal for a certain duration \( T \) and starts at a determined time...
A GN node can receive several linear FMCW signals. Two of them can be mixed with each other, and filtered by a low-pass filter to produce a superposition of different frequencies [20].

Fig. 3 shows three transmitted signals from four different RNs. The travel time differences are available for accurate positioning.

The travel time differences are related by the following formula:

\[ R_{i1} = \frac{f_{d1} T}{B}, \quad \text{with } i = 2, 3 \text{ and } 4 \]  

(1)

The time difference of arrival \( \tau_{1} \), and the distance difference, \( R_{1} \), are related by the following formula:

\[ R_{i1} = c \tau_{i1}, \quad \text{with } i = 2, 3 \text{ and } 4 \]

where \( c \) is the speed of light. The received The range difference \( R_{1} \) can be obtained by:

\[ R_{i1} = c \tau_{i1} = c \cdot \frac{f_{d1} T}{B} \]  

(2)

and the range difference resolution is

\[ \Delta R = \frac{c}{2B} \]

\( \Delta R \) is small if the bandwidth is wide enough. Compared to other ranging methods, it is easy to achieve high range resolution with a linear FMCW technique [21].

Moreover, all received signals can be used to obtain TFDA by applying the FFT concurrently. Then the multiple range differences are available for accurate positioning.

**B. Positioning based on TFDA for 2D case**

Consider a GN with unknown position coordinate \((x, y)\) in a WSN. Assume it can receive signals from \( M (M \geq 3) \) RNs.

The distance from this node to the \( i \)th reference node is

\[ R_i = \sqrt{(X_i-x)^2 + (Y_i-y)^2} \quad i = 1, 2, \ldots, M \]  

(3)

where \( X_i \) and \( Y_i \) are the known coordinates of \( RN_i \).

The range difference between this node to RN \( i \) and RN 1 is

\[ R_{i1} = R_i - R_1 \]

\[ = \sqrt{(X_i-x)^2 + (Y_i-y)^2} - \sqrt{(X_1-x)^2 + (Y_1-y)^2} \]

where \( R_1 \) is the range between RN 1 and this GN. This defines the set of nonlinear hyperbolic equations whose solution gives the 2-D coordinates of the unknown node [17].

**IV. 3D LOCATION DETERMINATION**

Consider a GN with unknown position coordinate \((x, y, z)\) in a WSN. Assume it can receive signals from \( M (M \geq 4) \) RNs. The distance from this node to the \( i \)th reference node is

\[ R_i = \sqrt{(X_i-x)^2 + (Y_i-y)^2 + (Z_i-z)^2} \quad i = 1, 2, \ldots, M \]  

(5)

where \( X_i, Y_i \) and \( Z_i \) are the known coordinates of \( RN_i \). The range difference between this node to RN \( i \) and RN 1 is

\[ R_{i1} = R_i - R_1 \]

\[ = \sqrt{(X_i-x)^2 + (Y_i-y)^2 + (Z_i-z)^2} - \sqrt{(X_1-x)^2 + (Y_1-y)^2 + (Z_1-z)^2} \]  

(6)

where \( R_1 \) is the range between RN 1 and this GN. This defines the set of nonlinear hyperbolic equations whose solution gives the 3-D coordinates of the unknown node.

**A. Taylor Iterative Method**

When four or more RNs can be heard by a RN. An iterative Taylor-series method is used for solving the hyperbolic equations. With an initial coordinate values \((x_0, y_0, z_0)\) and the estimate at the each iteration by determining the local linear least-square (LS) solution as the deviations [22][23].

To serve Eq. (6) by taking partial derivative of \( x \) and \( y \) on the both sides, we can have

\[ \Delta R_{i1} = \left( \frac{X_i-x}{R_i} \right) \Delta x + \left( \frac{Y_i-y}{R_i} \right) \Delta y \]

\[ + \left( \frac{Z_i-z}{R_i} \right) \Delta z \]  

(7)

\( R_i \) can be calculated by (5) when \( x = x_0, y = y_0 \) and \( z = z_0 \), i.e.,

\[ R_i = \sqrt{(X_i-x_0)^2 + (Y_i-y_0)^2 + (Z_i-z_0)^2} \]
Then we can obtain a set of linearized hyperbolic equations
\[
\begin{bmatrix}
\Delta R_{2,1} \\
\Delta R_{3,1} \\
\vdots \\
\Delta R_{M,1}
\end{bmatrix} = \mathbf{h}
\]
\[
\mathbf{h} = \begin{bmatrix}
X_{1,-x} - X_{1,x} & Y_{1,-y} - Y_{1,y} & Z_{1,-z} - Z_{1,z} \\
X_{2,-x} - X_{2,x} & Y_{2,-y} - Y_{2,y} & Z_{2,-z} - Z_{2,z} \\
\vdots \\
X_{M,-x} - X_{M,x} & Y_{M,-y} - Y_{M,y} & Z_{M,-z} - Z_{M,z}
\end{bmatrix}
\] (8)

Hence, the LS solution for this equation is when \( M \geq 4 \),
\[
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}^T = (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \mathbf{h}
\] (9)

If the covariance matrix of the estimated TFDAs is known, the LS solution to (8) is,
\[
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}^T = (\mathbf{G}^T \mathbf{C}^{-1} \mathbf{G})^{-1} \mathbf{G}^T \mathbf{h}
\] (10)

where \( \mathbf{C} \) is the covariance matrix of the estimated TFDAs. In the next iteration, \( x_0, y_0 \) and \( z_0 \) are set to be \( x = x_0 + \Delta x, y = y_0 + \Delta y \) and \( z = z_0 + \Delta z \). The whole process is repeated until the deviation is sufficiently small.

The Taylor method is robust and can provide accurate location results if \( M \geq 5 \). It can also use the redundant measurements to improve the positioning solution. The main drawback is that it depends on the initial estimation and the computational complexity is high.

### B. Analytical Methods

For 2D case, there are two analytical methods can be used to solve the non linear hyperbolic equation set: Fang’s method and Chan’s method, and Chan’s method can be extended to 3D case [24].

For a four-RN case, assume RN4 to be at the origin, i.e., \( (X_4, Y_4, Z_4) = (0, 0, 0) \), while the other two at \( (X_1, Y_1, Z_1) \), \( (X_2, Y_2, Z_2) \) and \( (X_3, Y_3, Z_3) \). Following Schau’s method [25], we can obtain,
\[
\begin{bmatrix}
R_{2,1}^2 - K_1 + K_1 & R_{3,1}^2 - K_3 + K_1 & R_{4,1}^2 - K_4 + K_1 \\
R_{2,1}^2 - K_1 + K_1 & R_{3,1}^2 - K_3 + K_1 & R_{4,1}^2 - K_4 + K_1 \\
R_{2,1}^2 - K_1 + K_1 & R_{3,1}^2 - K_3 + K_1 & R_{4,1}^2 - K_4 + K_1 \\
\end{bmatrix} - 2(x^2 + y^2 + z^2)^{1/2}
\begin{bmatrix}
R_{2,1} \\
R_{3,1} \\
R_{4,1}
\end{bmatrix} = 2 \begin{bmatrix}
X_{2,1} \\
X_{3,1} \\
X_{4,1}
\end{bmatrix} \begin{bmatrix}
X_{2,1} \\
X_{3,1} \\
X_{4,1}
\end{bmatrix} \begin{bmatrix}
Z_{2,1} \\
Z_{3,1} \\
Z_{4,1}
\end{bmatrix}
\] (11)

where \( K_1 = X_1^2 + Y_1^2 + Z_1^2, \ K_2 = X_2^2 + Y_2^2 + Z_2^2, \ K_3 = X_3^2 + Y_3^2 + Z_3^2 \) and \( K_4 = X_4^2 + Y_4^2 + Z_4^2 \).

Eq. (11) can be written as,
\[
\mathbf{b} - 2 \mathbf{Rd} = 2 \mathbf{Ax}
\] (12)

where
\[
\mathbf{b} = \begin{bmatrix}
R_{2,1}^2 - K_2 + K_1 \\
R_{3,1}^2 - K_3 + K_1 \\
R_{4,1}^2 - K_4 + K_1
\end{bmatrix} - 2(x^2 + y^2 + z^2)^{1/2}
\begin{bmatrix}
R_{2,1} \\
R_{3,1} \\
R_{4,1}
\end{bmatrix}
\]

\[
\mathbf{d} = \begin{bmatrix}
R_{2,1} \\
R_{3,1} \\
R_{4,1}
\end{bmatrix}, \ \mathbf{x} = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}, \ \mathbf{R} = (x^2 + y^2 + z^2)^{1/2}
\]

of this GN and \( \mathbf{A} = \begin{bmatrix}
X_{2,1} & Y_{2,1} & Z_{2,1} \\
X_{3,1} & Y_{3,1} & Z_{3,1} \\
X_{4,1} & Y_{4,1} & Z_{4,1}
\end{bmatrix} \).

In (12) the unknown \((x, y, z)\) appears in both side of the equation, the GN position \( \mathbf{x} \) on the right-hand side, and GN radius \( \mathbf{R} \) on the left-hand side. If we assume that \( \mathbf{R} \) is known, Eq. (12) has the solution,
\[
\mathbf{x} = \frac{1}{2} \mathbf{A}^{-1} (\mathbf{b} - 2 \mathbf{Rd})
\] (13)

recall
\[
\mathbf{R} = (\mathbf{x}^T \mathbf{x})^{1/2}
\] (14)

Insert (12) into (14) with \( i = 1 \), a quadratic equation in terms of \( \mathbf{R} \) is obtained [25]. Then we have
\[
\mathbf{R} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\] (15)

where
\[
a = [4 - 4d^T (A^{-1})^T A^{-1} \mathbf{d}] \\
b = [2d^T (A^{-1})^T A^{-1} b + 2b^T (A^{-1})^T A^{-1} \mathbf{d}] \\
c = [2b^T (A^{-1})^T A^{-1} b]
\] (16)

one root will be selected based on prior information. This is one disadvantage of Chan’s method, the ambiguity problem. Then the position of a RN can be obtained by using Eq. (13).

When there are 5 RNs and more available and the measurements are not noiseless, the system is over determined and Fang’s method does not work any more. We can use Taylor series iterative method or Chan’s method to solve the equation set and find the solutions.

Let \( \mathbf{z}_a = [\mathbf{x}^T, \mathbf{R}] \) be the unknown vector, where \( \mathbf{x} = [x, y, z]^T \).
From (8), the error vector with TFDA noise is
\[
\mathbf{y} = \mathbf{b} - \mathbf{G_a} \mathbf{z}_a
\] (17)

where
\[
\mathbf{b} = \begin{bmatrix}
R_{2,1}^2 - K_2 + K_1 \\
R_{3,1}^2 - K_3 + K_1 \\
R_{4,1}^2 - K_4 + K_1
\end{bmatrix}
\]

\[
\mathbf{G_a} = \begin{bmatrix}
X_{2,1} & Y_{2,1} & Z_{2,1} \\
X_{3,1} & Y_{3,1} & Z_{3,1} \\
X_{4,1} & Y_{4,1} & Z_{4,1}
\end{bmatrix}
\]

the approximate answer of \( \mathbf{z}_a \) is
\[
\mathbf{z}_a \approx (\mathbf{G_a}^T \mathbf{G_a})^{-1} \mathbf{G_a}^T \mathbf{y}
\] (18)

where \( \mathbf{C} \) is the covariance matrix of \( \mathbf{d} \).

Chan’s method provides a closed-form solution and is approximate to the maximum likelihood (ML) estimator when TFDA estimation errors are small. Furthermore, it can take the advantage of the redundant measurements. It is computational less intensive compared to Taylor iterative method. However, it suffers from the ambiguity problems.
TABLE I
COMPARISON OF THE COMPUTATIONAL COMPLEXITY OF TAYLOR, 
CHAN’S AND FANG’S METHODS, M IS THE NUMBER OF RNs, AND L IS 
THE NUMBER OF ITERATION FOR TAYLOR METHOD

<table>
<thead>
<tr>
<th>Method</th>
<th>$M = 3$</th>
<th>$M = 4$</th>
<th>$M &gt; 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of $\times$</td>
<td>number of $+$</td>
<td>Number of $\times$</td>
</tr>
<tr>
<td>Fang</td>
<td>34</td>
<td>16</td>
<td>11M – 2</td>
</tr>
<tr>
<td>Chan</td>
<td>27</td>
<td>30</td>
<td>(22M – 1)L</td>
</tr>
<tr>
<td>Taylor</td>
<td>65L</td>
<td>66L</td>
<td>(14M + 23)L</td>
</tr>
</tbody>
</table>

C. Computational complexity analysis

In this section, we analyze the computational complexity for each algorithm. In order to determine the time and other resources to execute these three algorithms, we investigate the number of additions and multiplications needed for each method.

Table I shows the number of additions and multiplications. When only three RNs are available, all three methods can be applied. We can find that Fang’s method is the most time-efficient one and Taylor method need the most computation. When there are redundant measurements available ($M \geq 4$), Chan’s and Taylor methods are investigated here. It is shown that the former is more efficient than the latter.

V. SIMULATION RESULTS

In this section, the performance analysis of the proposed algorithm is conducted. In the simulation, we deploy 100 nodes (uniform distribution) in $100 \times 100m \times 100m^3$ area. 12 RNs are placed at coordinates (0,0,0), (0,0,50), (50,0,0), (50,50,0), (0,100,0), (0,0,100), (0,100,0), (100,0,0), (100,100,0) and (100,100,100). The deployment is shown in Fig. 4. Each GN sets its original position value for iteration at (50,50,50), and then measures the TFDAs and calculates its own location. Assume that the sweep bandwidth is 1.5GHz, and the distance difference measurement error is of Gaussian distribution with mean 0 and variance 0.1m. In order to compare the performance of a different number of RNs and different methods, the first simulation setup deals with three available RNs (RN1-RN3), Taylor, Fang’s and Chan’s methods are used to obtain GN’s position. Then we add RN4 to RNs set, use Taylor and Chan’s methods for positioning. Furthermore, we use total 6 RNs and repeat the previous simulation.

Fig. 5 shows cumulative distribution function (CDF) vs. location error for different methods when 3 RN are used. Taylor method uses five iterative

Fig. 6 shows cumulative distribution function (CDF) vs. location error for all GN nodes for 8 RN case when different positing methods are used. Based on the statistics, we can find that the 50% GNs have an error distance of around 0.1m while 75% have an error distance of around 0.2m for Chan’s method, while 98% GNs have an error distance of around 0.1m while 99% have an error distance of around 0.2m for Taylor method(Taylor with 5 iterations). Fig. 6 shows CDF vs. location errors for different methods when we use 12 RNs. It is obvious that more RNs can achieve higher resolution. For example, if Taylor method is used, 98% GNs have an error distance of around 0.1m when we use 8 while 99% have an error distance around 0.1m when 12 RN are used. Using Chan’s method, 79% GNs have an error around 0.1m when 12 RNs are used. In the same RN number scenario (RN=8 for
example), Taylor method’s positioning accuracy is higher than Chan’s method.

Table II compares the positioning error between the TFDA method and GPS. It is observed that TFDA method can achieve 0.2m positioning accuracy, that is much higher than that of GPS.

VI. CONCLUSIONS

In this paper, we extended a new method to estimate the location of a sensor node given several reference nodes based on time frequency difference arrival using linear frequency modulation continuous wave technique. Compared with traditional GPS algorithms, the proposed method can provide significant improvements on positioning accuracy and avoid the fine time synchronization requirements except for a small portion of reference nodes. It is also easy to operate and maintain. The location information of each node can be determined by solving a set of hyperbolic equations. Iterative and analytical positioning methods are discussed and compared. The noisy measurement situation is also investigated. The simulation results prove that this new framework is time-efficient and performs precise positioning.

REFERENCES


<table>
<thead>
<tr>
<th>Method</th>
<th>GPS (C/A)</th>
<th>GPS (Y)</th>
<th>TFDA</th>
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</thead>
<tbody>
<tr>
<td>Positioning error (m)</td>
<td>15</td>
<td>1</td>
<td>0.2</td>
</tr>
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