Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Control of Uncertain Active Suspension System with Anti-lock Braking system Using Fuzzy Neural Controllers

Wei-Yen Wang
Department of Applied Electronics Technology
National Taiwan Normal University
Taipei, Taiwan.
wywang@ntnu.edu.tw

Ming-Chang Chen
Department of Electrical Engineering, National Taiwan University of Science and Technology
maximchen.chen@gmail.com

Yi-Hsing Chien
Department of Electrical Engineering, National Taipei University of Technology
bbxpv8@yahoo.com.tw

Tsu-Tian Lee
Department of Electrical Engineering, National Taipei University of Technology
president@ntut.edu.tw

***Abstract*-This paper proposes anti-lock braking system to integrate with active suspensions system applied in a quarter vehicles model, and can use a road estimate to get the road condition. This estimate is based on the LuGre friction model with a road condition parameter, and can transmit a reference slip ration to slip ratio controller through a mapping function considering the effect of road characteristics. In the controller design, an observer-based direct adaptive fuzzy-neural controller (DAFC) for an ABS is developed. After, this paper will discuss that active suspension system influence on ABS. Active suspension systems are not ideal, unchanging, and certain, as many control systems assume. If parts of the suspension system fail, it becomes an uncertain system. In such cases, we need an approximator to remodel this uncertain system to maintain good control. We propose a new method to on-line identify the uncertain active suspension system and design a T-S fuzzy-neural controller to control it. Finally, integrating algorithm is constructed to coordinate these two subsystems. Simulation results of the ABS with active suspension system, and is shown to provide good effectiveness under varying conditions.**

Keywords—anti-lock braking system, active suspension system, T-S fuzzy-neural, DAFC.

I. Introduction

There are various control techniques for anti lock braking system [1] and active suspension system [2]. These have been applied widely to improve the ride, comfort, and safety of vehicles. Recently, there is some integrating algorithm which combining the previous mentioned subsystem in order to control vehicle dynamic state to reach better efficient. But these control techniques assume that the road is smooth and consistent, and that active suspension system doesn't change. However, vehicles are not always driven on smooth roads-there may be holes, rocks, or the road may be otherwise uneven. As well, the suspension system does not remain static, and parts of the suspension system may even fail. The systems described in [3] can deal with changing road conditions, but not a changing active suspension system. Note that if parts of active suspension system fail, the system becomes uncertain. Thus we need an approximator to remodel this uncertain system to maintain good control.

In this paper, we propose a T-S fuzzy-neural controller to on-line identify the uncertain active suspension system and control it. We will show that the tracking error of a closed-loop system can be made arbitrarily small, no matter the situation. The consequent part of T-S fuzzy-neural model has a linear form. So we first use the mean value theorem to transform the nonlinear active suspension system into a virtual linearized system model. Then, an on-line identification algorithm and robust tracking controller is developed for active suspension system.

In traditional anti-lock braking systems, certain states, such as the wheel acceleration and the brake-line pressure, are difficult to obtain, or are prone to noise and other measurement errors. To resolve this problem, a state observer is required. Thus, the observer-based direct adaptive fuzzy-neural controller (DAFC) [4] is applied to ABS control, and its stability can be guaranteed by the universal approximation theorem. The DAFC also can overcome system uncertainties and disturbances. We propose an antilock-braking system based on a road estimator, using the DAFC to force the wheel slip ratio to follow a reference slip ratio obtained by the road estimator.

This paper is to take advantage of anti-lock braking system combined with active suspension system in the unknown road condition and uncertain system. Simulation results of the ABS with active suspension system, and is shown to provide good effectiveness under varying conditions.

II. System Model and Dynamic

In this section, we design a dynamic equation of an antilock brake system with active suspensions applied to a quarter vehicle model.

The wheel dynamic equation can be derived as follows:

$$\dot{\omega} = \frac{T_b - T_t}{I} = \frac{K_b P_i - F_x R}{I} \tag{1}$$

and the differential equation of vehicle longitudinal dynamics is

$$\dot{v} = \frac{F_x}{m_q} \tag{2}$$

where ω is the angular velocity of the wheel, v is the vehicle velocity, F_x is the longitudinal reactive force, m_q is the mass

978-1-4244-2794-9/09/$25.00 ©2009 IEEE

of the quarter of the vehicle supported by the wheel, R is the tire rolling radius, I is the moment of inertia, and K_b is the gain between the pressure of the ABS. The control objective of ABS is to regulate wheel slip to maximize the coefficient of friction between the wheel and the road for any road condition. The coefficient of friction during braking can be described as a function of the slip ratio λ, which is defined as

$$\lambda = \frac{v - R\omega}{v} \tag{3}$$

A. Second order dynamic system

To design the controller, we define the slip for a braking operation as equation (3). First differentiating equation (3) along with equation (1), we find the derivative of λ to be

$$\dot{\lambda} = \frac{1}{v}\left[\frac{R}{I}\left(F_x R - K_b P_i\right) + (1-\lambda)\dot{v}\right] \tag{4}$$

Defining $\boldsymbol{\lambda} = [\lambda\ \dot{\lambda}]$ and $\mathbf{z} = [z\ \dot{z}\ \ddot{z}]$, then the time derivative function of (4) can be expressed as

$$\begin{aligned}\ddot{\lambda} &= [\frac{R^2\dot{F}_x}{Iv} + (\frac{(1-\lambda)\dot{a}_v - \dot{\lambda}a_v}{v})] - \frac{RK_b\dot{P}_i}{Iv} + d(\lambda, v, a_v, P_i, \mathbf{z}) \\ &\equiv \tilde{f}(\lambda, \dot{\lambda}, v, \mathbf{z}) + b(v)u + d(\lambda, v, a_v, P_i, \mathbf{z})\end{aligned} \tag{5}$$

where

$$\tilde{f} = (\lambda, \dot{\lambda}, v, \mathbf{z}) = \frac{R^2\dot{F}_x}{Iv} + (\frac{(1-\lambda)\dot{a}_v - \dot{\lambda}a_v}{v}),\ b(v) = -\frac{RK_b}{Iv},\ u = \dot{P}_i$$

and $\dot{v} = a_v$ is the acceleration. $d(\lambda, v, a_v, P_i, \mathbf{z})$ represents the nonlinearities obtained by differentiating $\dot{\lambda}$ with respect to v on the premise that other parameters are treated as constants.

B. Friction Model

The friction force is shown as follows:

$$\mu = \frac{F_x}{F_z} \tag{6}$$

where F_x is the longitudinal force, and F_z is the normal force. The longitudinal force F_x is effective at road-surface level. That is it allows the driver to apply throttle and then brakes to accelerate and slow the vehicle.

In previous research [5], a road characteristic θ was introduced into the LuGre friction model. The normalized friction force is shown as follows:

$$\dot{z} = v_r - \theta\frac{\delta_0\,|\,v_r\,|}{g(v_r)}z - \kappa R|\omega|z \tag{7}$$

$$F_x = (\delta_0 z + \delta_1\dot{z} + \delta_2 v_r)F_z) \tag{8}$$

$$F_z = m_q g - K_t(z_u - z_r) - C_t(\dot{z}_u - \dot{z}_r) \tag{9}$$

where

$$g(v_r) = F_c + (F_s - F_c)e^{-|\frac{v_r}{v_s}|}, \tag{10}$$

δ_0 is the normalized rubber longitudinal lumped stiffness, δ_1 is the normalized rubber longitudinal lumped damping, δ_2 is the normalized viscous relative damping, F_c is the normalized Coulomb friction, F_s is the normalized static friction, v_s is the Stribeck relative velocity, z is the internal friction state, C_t is the tire damping coefficient, K_t is the tire spring coefficient, z_s and z_u are the displacements of the car body and wheel, z_r is the road disturbance, and v_r is a relative velocity defined as $r\omega - v$, which is equal to $-\lambda v$. The constant $\kappa = L/2 > 0$ captures the effect in the lumped model of the force distribution. L is the tire patch length.

C. Active suspension model

The dynamic equations of motion for this quarter-car active suspension model [1] can be written as follows:

$$\begin{aligned} m_s\ddot{z}_s &= -K_a(z_s - z_u) - C_a(\dot{z}_s - \dot{z}_u) + u^s \\ m_{us}\ddot{z}_u &= K_a(z_s - z_u) + C_a(\dot{z}_s - \dot{z}_u) - u^s - K_t(z_u - z_r) - C_t(\dot{z}_u - \dot{z}_r)\end{aligned} \tag{11}$$

where K_a is the active suspension spring coefficient, C_a is the active suspension damping coefficient, and u^s is the control force.

III. Estimator for Tire/Road Contact Friction

We can derive an one-wheel vehicle model with the LuGre friction model used in previous studies [2], called the LuGre-based ABS,

$$m_q\dot{v} = F_z(\delta_0 z + \delta_1\dot{z}) + F_z\delta_2 v_r \tag{12}$$

$$I\dot{\omega} = -RF_z(\delta_0 z + \delta_1\dot{z}) - \delta_\omega\omega + u_r \tag{13}$$

$$\dot{z} = v_r - \theta\frac{\delta_0\,|\,v_r\,|}{g(v_r)}z \tag{14}$$

$$\dot{\theta} = 0 \tag{15}$$

where u_r is an input variable for the road estimator, and we have neglected the term δ_2 in equation (13). In equations (12)-(15), we assume that only ω is measurable. To set our systems in the same framework as the classical dynamic system, the following change of coordinates is introduced:

$$\begin{aligned}\eta &= Rm_q v + I\omega \\ \chi &= I\omega + RF_z\delta_1 z\end{aligned} \tag{16}$$

Defining the state vector $\mathbf{x}$ and output variable y_r as:

$$\mathbf{x} = \begin{bmatrix}\eta \\ \chi \\ z\end{bmatrix},\ y_r = \omega \tag{17}$$

Substituting equation (16) into (12)-(15), the following equations are obtained:

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{F_z\delta_2}{m_q} & 0 & 0 \\ 0 & -\frac{\delta_0}{\delta_1} & 0 \\ -\frac{1}{Rm_q} & 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix}0 \\ 0 \\ -1\end{bmatrix}\theta\phi(y_r, u_r, \mathbf{x}) + \begin{bmatrix} R^2F_z\delta_2 + I\frac{F_z\delta_2}{m_q} - \delta_\omega \\ I\frac{\delta_1}{\delta_0} - \delta_\omega \\ R + \frac{I}{Rm_q}\end{bmatrix}y_r + \begin{bmatrix}1 \\ 1 \\ 0\end{bmatrix}u_r, \tag{18}$$

$$y_r = [0\quad \frac{1}{I}\quad \frac{-RF_z\delta_1}{I}]\mathbf{x}$$

where

$$\phi(y_r, u_r, \mathbf{x}) = \frac{\delta_0|Ry_r - v|}{g(Ry_r - v)}z \tag{19}$$

and

$$v = \frac{\eta - I\cdot y_r}{Rm_q}. \tag{20}$$

Under the hypotheses from previous methods [6], an estimator structure is proposed for the LuGre-based ABS shown in equation (18) and it can be expressed as:

$$\dot{\hat{\mathbf{x}}} = \begin{bmatrix} -\frac{F_z\delta_2}{m_q} & 0 & 0 \\ 0 & -\frac{\delta_0}{\delta_1} & 0 \\ -\frac{1}{Rm_q} & 0 & 0 \end{bmatrix}\hat{\mathbf{x}} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}\hat{\theta}\phi(y_r, u_r, \mathbf{x}) + \begin{bmatrix} R^2F_z\delta_2 + I\frac{F_z\delta_2}{m_q} - \delta_\omega \\ I\frac{\delta_1}{\delta_0} - \delta_\omega \\ R + \frac{I}{Rm_q} \end{bmatrix} y_r + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u_r$$
$$+ \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}(y_r - \hat{y}_r) + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} 2\theta_{\max}(f_{\max} + f(\|\hat{\mathbf{x}}\|)\operatrm{sgn}(y_r - \hat{y}_r)$$
$$\dot{\hat{\theta}} = \gamma\frac{\delta_0 |\hat{v}_r|}{g(\hat{v}_r)}\hat{z}(\omega - \hat{y}_r)$$
$$\hat{y}_r = [0 \quad \frac{1}{I} \quad \frac{-RF_z\delta_1}{I}]\hat{\mathbf{x}} \tag{21}$$

where

$$f(\|\hat{\mathbf{x}}\|) = \frac{\delta_0}{F_c}((R + \frac{I}{Rm_q})y_{\sup} + \frac{|\hat{\eta}|}{Rm_q})|\hat{z}|, \tag{22}$$

$\hat{\eta}, \hat{\chi}, \hat{z}, \hat{y}_r$, and $\hat{v}_r$ are the estimated values of η, χ, z, y_r, and v_r, respectively, $\theta_{\max}, f_{\max}$, and $y_{\sup}$ are the maximum values of θ, $f(\|\hat{\mathbf{x}}\|)$, and y_r, respectively, and γ is the learning rate. The estimator gain vector of the road estimator is $\mathbf{K} = [k_1\ k_2\ k_3]$.

IV. Observer-Based Direct Adaptive Fuzzy-Neural Control

In this paper, the control objective is to design a DAFC such that the slip ratio, $y = \lambda$, follows a reference slip ratio, $y_m = \lambda_d$. First, we convert the tracking problem to a regulation problem, so equation (4) is rewritten as

$$\dot{\boldsymbol{\lambda}} = \mathbf{A}\boldsymbol{\lambda} + \mathbf{B}(f(\boldsymbol{\lambda}) + bu + d)$$
$$y = \mathbf{C}^T\boldsymbol{\lambda} \tag{23}$$

where $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $\boldsymbol{\lambda} = [\lambda \quad \dot{\lambda}]^T = [\lambda_1 \quad \lambda_2]^T \in R^2$ is a vector of state. Define the output tracking error $e = y_m - y = \lambda_d - \lambda$, the reference vector $\mathbf{y}_m = [y_m \quad \dot{y}_m]^T = [\lambda_d \quad \dot{\lambda}_d]^T$ and the tracking error vector $\mathbf{e} = [e \quad \dot{e}]^T = [e_1 \quad e_2]^T$.

Based on the certainty equivalence approach, an optimal control law is

$$u^* = \frac{1}{b}[-f(\lambda) + \ddot{y}_m + \mathbf{K}_c^T\hat{\mathbf{e}}] \tag{24}$$

where $\hat{\mathbf{e}}$ denotes the estimate of $\mathbf{e}$, and $\mathbf{K}_c = [k_2^c \quad k_1^c]^T$ is the feedback gain vector, chosen such that the characteristic polynomial of $\mathbf{A} - \mathbf{B}\mathbf{K}_c^T$ is Hurwitz because **(A, B)** is controllable. Since only the system output (the slip ratio), $y = \lambda$, is assumed to be measurable, and $f(\lambda)$ is assumed to be unknown, the optimal control law (24) cannot be implemented.

Thus, we suppose a control input u is

$$u = u_f + u_{sv} \tag{25}$$

where u_f is designed to approximate the optimal control law (24), and the control term u_{sv} is employed to compensate for the external disturbances and modeling error.

From (23), (24) and (25), we have

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} - \mathbf{B}\mathbf{K}_c^T\hat{\mathbf{e}} + \mathbf{B}[bu^* - bu_f - bu_{sv} - d]$$
$$e_1 = \mathbf{C}^T\mathbf{e} \tag{26}$$

Thus, the tracking problem has been converted into a regulation problem of designing an observer for estimating the vector $\mathbf{e}$ in (26) in order to regulate e_1 to zero.

The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input linguistic vector $\mathbf{e} = [e_1 \quad e_2] \in \mathbf{R}^2$ to an output linguistic variable $u_f \in \mathbf{R}$.

The ith fuzzy IF-THEN rule is written as

$$\mathrm{R}^i: \text{If } e_1 \text{ is } A_1^i \text{ and } e_2 \text{ is } A_2^i \text{ then } u_f \text{ is } B^i. \tag{27}$$

where A_1^i, A_2^i and B^i are fuzzy sets. By using the production inference, center-average and singleton fuzzifier methods, the output of the fuzzy-neural network can be expressed as

$$u_f = \frac{\sum_{i=1}^{h} p^i \left[\prod_{j=1}^{2} \mu_{A_j^i}(e_j)\right]}{\sum_{i=1}^{h}[\prod_{j=1}^{2} \mu_{A_j^i}(e_j)]} = \boldsymbol{\theta}_c^T\varphi(\mathbf{e}) \tag{28}$$

where $\mu_{A_j^i}(e_j)$ is the membership function value of the fuzzy variable, h is the total number of IF-THE rules, p^i is the point at which $\mu_{B^i}(p^i) = 1$, $\boldsymbol{\theta}_c = [p^1\ p^2 ...\ p^h]^T$ is an adjustable parameter vector, and $\varphi = [\varphi^1\ \varphi^2 \cdots \varphi^h]^T$ is a fuzzy basis vector, where φ^i is defined as

$$\varphi^i(\mathbf{e}) = \frac{\prod_{j=1}^{2} \mu_{A_j^i}(e_j)}{\sum_{i=1}^{h}[\prod_{j=1}^{2} \mu_{A_j^i}(e_j)]} \tag{29}$$

First, we replace u_f in (25) by the output of the fuzzy-neural network, $\boldsymbol{\theta}_c^T\varphi(\hat{\mathbf{e}})$, in (28), i.e.,

$$u_f(\hat{\mathbf{e}} | \boldsymbol{\theta}_c^T) = \boldsymbol{\theta}_c^T\varphi(\hat{\mathbf{e}}) \tag{30}$$

where $\hat{\mathbf{e}}$ denotes the estimate of $\mathbf{e}$.

Next, consider the following observer that estimates the state vector $\mathbf{e}$ in (26):

$$\dot{\hat{\mathbf{e}}} = \mathbf{A}\hat{\mathbf{e}} - \mathbf{B}\mathbf{K}_c^T\hat{\mathbf{e}} + \mathbf{B}(b\overline{v} - bu_{sv}) + \mathbf{K}_o(e_1 - \hat{e}_1)$$
$$\hat{e}_1 = \mathbf{C}^T\hat{\mathbf{e}} \tag{31}$$

where $\mathbf{K}_o = [k_o^1 \quad k_o^2]^T$ is the observer gain vector, chosen such that the characteristic polynomial of $\mathbf{A} - \mathbf{K}_o\mathbf{C}^T$ is strictly Hurwitz because **(C, A)** is observable. The control term $\overline{v}$ is employed to compensate for the external disturbance d and the modeling error. We define the observation errors as $\tilde{\mathbf{e}} = \mathbf{e} - \hat{\mathbf{e}}$ and $\tilde{e}_1 = e_1 - \hat{e}_1$. Subtracting (31) from (26), we have

$$\dot{\tilde{\mathbf{e}}} = (\mathbf{A} - \mathbf{K}_o\mathbf{C}^T)\tilde{\mathbf{e}} + \mathbf{B}[bu^* - bu_f(\hat{\mathbf{e}} | \boldsymbol{\theta}_c) - b\overline{v} - d]$$
$$\tilde{e}_1 = \mathbf{C}^T\tilde{\mathbf{e}} \tag{32}$$

As well, the output error dynamics of (32) can be given as

$$\tilde{e}_1 = H(s)[bu^* - bu_f(\hat{\mathbf{e}} \mid \boldsymbol{\theta}_c) - b\bar{v} - d] \quad (33)$$

where s is the Laplace variable, and $H(s) = \mathbf{C}^T(s\mathbf{I} - (\mathbf{A} - \mathbf{K_o C}^T))^{-1}\mathbf{B}$ is the transfer function of (33). Equation (32) can be rewritten as

$$\dot{\tilde{\mathbf{e}}} = (\mathbf{A} - \mathbf{K}_o\mathbf{C}^T)\tilde{\mathbf{e}} + \mathbf{B}[bu_f(\hat{\mathbf{e}} \mid \boldsymbol{\theta}_c^*) - bu_f(\hat{\mathbf{e}} \mid \boldsymbol{\theta}_c) - b\bar{v} + \tau_m - d] \quad (34)$$
$$\tilde{e}_1 = \mathbf{C}^T\tilde{\mathbf{e}}$$

where $\tau_m = bu^* - bu_f(\hat{\mathbf{e}} \mid \boldsymbol{\theta}_c^*)$ is an approximation error. According to (30), (34) can be rewritten as

$$\dot{\tilde{\mathbf{e}}} = (\mathbf{A} - \mathbf{K}_o\mathbf{C}^T)\tilde{\mathbf{e}} + \mathbf{B}[b\tilde{\boldsymbol{\theta}}_c^T\varphi(\hat{\mathbf{e}}) - b\bar{v} + \tau_m - d] \quad (35)$$
$$\tilde{e}_1 = \mathbf{C}^T\tilde{\mathbf{e}}$$

where $\tilde{\boldsymbol{\theta}}_c^* = \boldsymbol{\theta}_c^* - \boldsymbol{\theta}_c$. Since only the output $\tilde{e}_1$ in (35) is assumed to be measurable, we use the SPR-Lyapunov design approach to analyze the stability of (35). Equation (35) can be rewritten as

$$\tilde{e}_1 = H(s)[b\tilde{\boldsymbol{\theta}}_c^T\varphi(\hat{\mathbf{e}}) - b\bar{v} + \tau_m - d] \quad (36)$$

where $H(s) = \mathbf{C}^T(s\mathbf{I} - (\mathbf{A} - \mathbf{K}_o\mathbf{C}^T))^{-1}\mathbf{B}$ is a known stable transfer function. In order to employ the SPR-Lyapunov design approach, (36) can be written as

$$\tilde{e}_1 = H(s)L(s)[\tilde{\boldsymbol{\theta}}_c^T\varphi(\hat{\mathbf{e}}) - v_f + \tau_f] \quad (37)$$

where $\tau_f = L^{-1}(s)\tau_T$, $\tau_T = \tau_m - d - L(s)\tilde{\boldsymbol{\theta}}_c^T\varphi(\hat{\mathbf{e}}) + b\tilde{\boldsymbol{\theta}}_c^T\varphi(\hat{\mathbf{e}})$, and $v_f = L^{-1}(s)[b\bar{v}]$. $L(s)$ is chosen so that $L^{-1}(s)$ is a proper stable transfer function and $H(s)L(s)$ is a proper SPR transfer function. Suppose that $L(s) = s + b_1$, such that $H(s)L(s)$ is a proper SPR transfer function. Then the state-space realization of (37) can be written as

$$\dot{\tilde{\mathbf{e}}} = \mathbf{A}_c\tilde{\mathbf{e}} + \mathbf{B}_c[\tilde{\boldsymbol{\theta}}_c^T\varphi(\hat{\mathbf{e}}) - v_f + \tau_f] \quad (38)$$
$$\tilde{e}_1 = \mathbf{C}_c^T\tilde{\mathbf{e}}$$

where $\mathbf{A}_c = (\mathbf{A} - \mathbf{K}_o\mathbf{C}^T) \in \Re^{2\times 2}, \mathbf{B}_c^T = [0 \quad b_1] \in \Re^2$ and $\mathbf{C}_c^T = [1\ 0] \in \Re^2$.

Theorem 1: From the following adaptive law

$$\dot{\boldsymbol{\theta}}_c = \begin{cases} \kappa\tilde{e}_1\varphi(\hat{\mathbf{e}}), & if \parallel \boldsymbol{\theta}_c \parallel < m_{\boldsymbol{\theta}_c}\ or (\parallel \boldsymbol{\theta}_c \parallel < m_{\boldsymbol{\theta}_c} \text{ and } \tilde{e}_1\boldsymbol{\theta}_c^T\varphi(\hat{\mathbf{e}}) \ge 0) \\ \mathbf{P_r}(\kappa\tilde{e}_1\varphi(\hat{\mathbf{e}})), & if \parallel \boldsymbol{\theta}_c \parallel < m_{\boldsymbol{\theta}_c} \text{ and } \tilde{e}_1\boldsymbol{\theta}_c^T\varphi(\hat{\mathbf{e}}) < 0 \end{cases} \quad (39)$$

with the projection operator

$$\mathbf{P_r}(\kappa\tilde{e}_1\varphi(\hat{\mathbf{e}})) = \kappa\tilde{e}_1\varphi(\hat{\mathbf{e}}) - \kappa\frac{\tilde{e}_1\boldsymbol{\theta}_c^T\varphi(\hat{\mathbf{e}})}{\|\boldsymbol{\theta}_c\|^2}\boldsymbol{\theta}_c \quad (40)$$

where κ is a positive number called the learning rate which determines the rate of learning, and $m_{\boldsymbol{\theta}}$ is a boundary of the adjusted parameter $\boldsymbol{\theta}_c$. The supervised control law is

$$\bar{v} = \begin{cases} \rho \ \ if \ \ \tilde{e}_1 \ge 0 \text{ and } |\tilde{e}_1| > \alpha \\ -\rho \ \ if \ \ \tilde{e}_1 < 0 \text{ and } |\tilde{e}_1| > \alpha, \text{ where } \alpha \text{ is a positive constant} \\ \rho\tilde{e}_1/\alpha \ \ if \ |\tilde{e}_1| \le \alpha \end{cases} \quad (41)$$

where ρ means supervised control. If ρ is chosen arbitrarily large, the control law can achieve stabilization. Then, with the supervised control law $\bar{v}$, $\tilde{e}_1(t)$ converges as $t \to \infty$.♦

V. Active suspension system using on-line adaptive T-S fuzzy neural controller

We define the state variables $x_1^s = z_s$, $x_2^s = \dot{z}_s$, $x_3^s = z_u$, $x_4^s = \dot{z}_u$, and separate u^s into u_1^s and u_2^s. The quarter-car active suspension system can be modeled as:

$$\begin{aligned} \dot{x}_1^s &= x_2^s \\ \dot{x}_2^s &= \frac{-K_a(x_1^s - x_3^s) - C_a(x_2^s - x_4^s) + (u_1^s + u_2^s)}{m_s} \\ \dot{x}_3^s &= x_4^s \\ \dot{x}_4^s &= \frac{K_a(x_1^s - x_3^s) + C_a(x_2^s - x_4^s) - (u_1^s + u_2^s) - K_t(x_3^s - z_r^s) - C_t(x_4^s - \dot{z}_r^s)}{m_{us}} \end{aligned} \quad (42)$$

Let the state vector be $\mathbf{x}^s = [x_1^s, x_2^s, x_3^s, x_4^s]^T$ so that the output vector of generalized coordinates becomes $\mathbf{y}^s = [y_1^s, y_2^s]^T = [x_2^s, x_4^s]^T$.

A. T-S Fuzzy Neural Model

The mean value theorem [7] says that there are points x_i^{s*} $(i = 1,2,3,4)$ and u_j^{s*} $(j = 1,2)$ on the linear segments joining x_i^s to $\bar{x}_i^s$ $(i = 1,2,3,4)$ and u_j^s to $\bar{u}_j^s$ $(j = 1,2)$. Thus, system (42) can be exactly formed as follows:

$$\dot{\mathbf{x}}^s = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ 0 & 0 & 0 & 1 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{bmatrix}\mathbf{x}_\xi^s + \begin{bmatrix} 0 & 0 \\ \beta_{21} & \beta_{22} \\ 0 & 0 \\ \beta_{41} & \beta_{42} \end{bmatrix}\mathbf{u}_\xi^s + \begin{bmatrix} 0 \\ d_{d1}^s \\ 0 \\ d_{d2}^s \end{bmatrix} + \begin{bmatrix} \bar{x}_2^s \\ F_1^s(\bar{\mathbf{x}}^s, \bar{\mathbf{u}}^s) \\ \bar{x}_4^s \\ F_2^s(\bar{\mathbf{x}}^s, \bar{\mathbf{u}}^s) \end{bmatrix} \quad (43)$$

Then, we can reduce (43) as follows:

$$\begin{bmatrix} \dot{x}_2^s \\ \dot{x}_4^s \end{bmatrix} = \begin{bmatrix} \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{bmatrix}\mathbf{x}_\xi^s + \begin{bmatrix} \beta_{21} & \beta_{22} \\ \beta_{41} & \beta_{42} \end{bmatrix}\mathbf{u}_\xi^s + \begin{bmatrix} d_{d1}^s + F_1^s(\bar{\mathbf{x}}^s, \bar{\mathbf{u}}^s) \\ d_{d2}^s + F_2^s(\bar{\mathbf{x}}^s, \bar{\mathbf{u}}^s) \end{bmatrix} = \mathbf{A}^s\mathbf{x}_\xi^s + \mathbf{B}^s\mathbf{u}_\xi^s + \mathbf{d}_d^s \quad (44)$$

where $\bar{\mathbf{x}}^s = [\bar{x}_1^s, \bar{x}_2^s, \bar{x}_3^s, \bar{x}_4^s]^T = t_1\mathbf{x}^s$ and $\bar{\mathbf{u}}^s = [\bar{u}_1^s, \bar{u}_2^s]^T = t_2\mathbf{u}^s$ ($0 < t_1, t_2 < 1$), are vectors of critical points, $\mathbf{x}_\xi^s = [x_{\xi 1}^s, x_{\xi 2}^s]^T = \mathbf{x}^s - \bar{\mathbf{x}}^s$, $\mathbf{u}_\xi^s = [u_{\xi 1}^s, u_{\xi 2}^s]^T = \mathbf{u}^s - \bar{\mathbf{u}}^s$, $\alpha_{ij} = \partial F_k^s / \partial x_j^s \mid_{(\mathbf{x}^{s*}, \mathbf{u}^{s*})}$ $(i = 2,4,\ j = 1,2,3,4,\ k = 1,2)$, $\beta_{ij} = \partial F_k^s / \partial u_j^s \mid_{(\mathbf{x}^{s*}, \mathbf{u}^{s*})}$ $(i = 2,4,\ j = 1,2,\ k = 1,2)$ and $\mathbf{d}_d^s = [d_1^s + F_1^s(\bar{\mathbf{x}}^s, \bar{\mathbf{u}}^s),\ d_2^s + F_2^s(\bar{\mathbf{x}}^s, \bar{\mathbf{u}}^s)]^T = [d_{d1}^s, d_{d2}^s]^T$.

The T-S fuzzy model defined in [8] is

$$\begin{aligned} &R^{(i)}: \text{If } z_1^s \text{ is } F_1^{si} \text{ and} \ldots z_6^s \text{ is } F_6^{si} \\ &\text{Then } \bar{y}_l^s = p_{l1}^{si}z_1^s + p_{l2}^{si}z_2^s + \ldots + p_{l6}^{si}z_6^s \end{aligned} \quad (45)$$

where $\mathbf{z}^s = [z_1^s, z_2^s, \cdots z_6^s]^T \in \mathrm{R}^6$ is a vector of state, $\bar{y}_l^s$ is the output of the T-S fuzzy model, F_j^{si} $(j = 1,2,\cdots,6)$ are fuzzy sets, and p_{lk}^{si} $(i = 1,2,\cdots,h, l = 1,2, k = 1,2,\cdots,6)$ are adjustable parameters. The weighting p_{lk}^s of the fuzzy-neural network is

$$p_{lk}^s = \frac{\sum_{i=1}^{h} p_{lk}^{si}(\prod_{j=1}^{6}\mu_{F_j^{si}}(z_j^s))}{\sum_{i=1}^{h}(\prod_{j=1}^{6}\mu_{F_j^{si}}(z_j^s))} \quad (46)$$

where $\mu_{F_j^{si}}(z_j^s)$ is the value of the membership function. For the tuning of the weighting factors p_{lk}^s, we define

$$w^{si}=\frac{\prod_{j=1}^{6}\mu_{F_j^{si}}(z_j^s)}{\sum_{i=1}^{h}(\prod_{j=1}^{6}\mu_{F_j^{si}}(z_j^s))},\ i=1,2,\ldots,h. \tag{47}$$

For the purpose of approximating the system in (44), the *i*th fuzzy implication can be described as

$$\begin{aligned}R^{(i)}:\quad & \text{If } x_{\xi 1}^s \text{ is } F_1^{si} \text{ and} \ldots x_{\xi 4}^s \text{ is } F_4^{si}\\ & \text{and } u_{\xi 1}^s \text{ is } F_5^{si} \text{ and} \cdots u_{\xi 2}^s \text{ is } F_6^{si}\\ & \text{Then } [\dot{x}_2^s,\dot{x}_4^s]^T=\hat{\mathbf{A}}^{si}\mathbf{x}_\xi^s+\hat{\mathbf{B}}^{si}\mathbf{u}_\xi^s\end{aligned} \tag{48}$$

where

$$\hat{\mathbf{A}}^{si}=\begin{bmatrix}p_{11}^{si} & p_{12}^{si} & p_{13}^{si} & p_{14}^{si}\\ p_{21}^{si} & p_{22}^{si} & p_{23}^{si} & p_{24}^{si}\end{bmatrix} \tag{49}$$

and

$$\hat{\mathbf{B}}^{si}=\begin{bmatrix}p_{15}^{si} & p_{16}^{si}\\ p_{25}^{si} & p_{16}^{si}\end{bmatrix} \tag{50}$$

After applying (46), (47) and some commonly used defuzzification strategies, the system in (44) becomes

$$\begin{aligned}[\dot{x}_2^s,\dot{x}_4^s]^T &= \sum_{i=1}^{h} w^{si}\{\hat{\mathbf{A}}^{si}\mathbf{x}_\xi^s+\hat{\mathbf{B}}^{si}\mathbf{u}_\xi^s\}+\mathbf{d}_d^s+\mathbf{d}_f^s\\ &= \begin{bmatrix}p_{11}^{si} & p_{12}^{si} & p_{13}^{si} & p_{14}^{si}\\ p_{21}^{si} & p_{22}^{si} & p_{23}^{si} & p_{24}^{si}\end{bmatrix}\mathbf{x}_\xi^s+\begin{bmatrix}p_{15}^{si} & p_{16}^{si}\\ p_{25}^{si} & p_{26}^{si}\end{bmatrix}\mathbf{u}_\xi^s\mathbf{d}_d^s+\mathbf{d}_f^s\end{aligned} \tag{51}$$

where $\mathbf{d}_f^s=(\mathbf{A}^s-\sum_{i=1}^{h}w^{si}\hat{\mathbf{A}}^{si})x_\xi^s+(\mathbf{B}^s-\sum_{i=1}^{h}w^{si}\hat{\mathbf{B}}^{si})\mathbf{u}_\xi^s$, and p_{ij}^s ($i=1,2,\ j=1,2,\cdots,6$) is used to approximate α_{ij} ($i=1,2,\ j=1,2,3,4$) and β_{ij} ($i=1,2,\ j=1,2$) of the system in (44).

B. Controller Design for On-line Modeling and Robust Tracking

We define the optimal adjustable matrices as

$$\hat{\mathbf{A}}_g^{s*i}=\begin{bmatrix}p_{g11}^{s*i} & p_{g12}^{s*i} & p_{g13}^{s*i} & p_{g14}^{s*i}\\ p_{g21}^{s*i} & p_{g22}^{s*i} & p_{g23}^{s*i} & p_{g24}^{s*i}\end{bmatrix},\ \hat{\mathbf{B}}_g^{s*i}=\begin{bmatrix}p_{g15}^{s*i} & p_{g16}^{s*i}\\ p_{g25}^{s*i} & p_{g26}^{s*i}\end{bmatrix} \tag{52}$$

Define the reference signal vector as $\mathbf{r}^s=[r_1^s,r_2^s]^T$. Thus the error vector is $\mathbf{e}^s=\mathbf{y}^s-\mathbf{r}^s=[e_1^s,e_2^s]^T$. Let $\boldsymbol{\omega}^s=[\omega_1^s,\omega_2^s]^T$ and $\omega_j^s=\dot{r}_j^s-\lambda_j^s e_j^s$ ($j=1,2$). In order to compensate for the total effect of external disturbances $\mathbf{d}_d^s=[d_{d1}^s,d_{d2}^s]^T$, and modeling error $\mathbf{d}_f^s=[d_{f1}^s,d_{f2}^s]^T$, redefine the T-S fuzzy-neural approximation including total error

$$\widetilde{\mathbf{d}}_d^s=\begin{bmatrix}\widetilde{d}_{d1}^s\\ \widetilde{d}_{d2}^s\end{bmatrix}=\begin{bmatrix}d_{d1}^s+d_{f1}^s\\ d_{d2}^s+d_{f2}^s\end{bmatrix} \tag{53}$$

as follows:

$$[\dot{x}_2^s,\dot{x}_4^s]^T=\sum w^{si}\{\hat{\mathbf{A}}^{si}\mathbf{x}_\xi^s+\hat{\mathbf{B}}^{si}\mathbf{u}_\xi^s\}+\widetilde{\mathbf{d}}_d^s \tag{54}$$

Define a coefficient matrix

$$\mathbf{\Lambda}=\begin{bmatrix}-\lambda_1 & 0\\ 0 & -\lambda_2\end{bmatrix} \tag{55}$$

where the coefficients, λ_1, λ_2, are selected such that the matrix $\mathbf{\Lambda}$ is a Hurwitz matrix. From an error dynamic equation $\dot{\mathbf{e}}^s=\mathbf{\Lambda}\mathbf{e}^s$, we could define $\boldsymbol{\omega}^s=[\omega_1^s\ \omega_2^s]^T=\dot{\mathbf{r}}^s+\mathbf{\Lambda}\mathbf{e}^s=\dot{\mathbf{x}}^s$. Since there are external disturbances $\mathbf{d}_d^s$ in the system (44) and considering the design of the controller, we redefine $\boldsymbol{\omega}^s=\dot{\mathbf{r}}^s+\mathbf{\Lambda}\mathbf{e}^s=\dot{\mathbf{x}}^s+\mathbf{u}_s^s-\mathbf{d}_d^s$, where $\mathbf{u}_s^s$ is an error compensator designed to compensate for $\mathbf{d}_d^s$. From (44), based on the certainty equivalence approach, a control input can be chosen as

$$\mathbf{u}_\xi^s=\mathbf{B}^{s^{-1}}(\dot{\mathbf{x}}^s-\mathbf{A}^s\mathbf{x}_\xi^s-\mathbf{d}_d^s)\,. \tag{56}$$

Since $\mathbf{d}_d^s$ is unknown, we redesign $\mathbf{u}_\xi^s$ as follows:

$$\mathbf{u}_\xi^s=\mathbf{B}^{s^{-1}}(-\mathbf{A}^s\mathbf{x}_\xi^s+\boldsymbol{\omega}^s-\mathbf{u}_s^s)\,. \tag{57}$$

Then, the error dynamics can be derived as

$$\dot{\mathbf{e}}^s=\dot{\mathbf{y}}^s-\dot{\mathbf{r}}^s=\dot{\mathbf{x}}^s-\dot{\mathbf{r}}^s=\mathbf{\Lambda}\mathbf{e}^s-\mathbf{u}_s^s+\mathbf{d}_d^s$$

Because the right side of (44) is unknown, we replace $\mathbf{A}^s$ and $\mathbf{B}^s$ by $\sum_{i=1}^{h}w^{si}\hat{\mathbf{A}}^{si}$ and $\sum_{i=1}^{h}w^{si}\hat{\mathbf{B}}^{si}$ in (51), respectively. From (57) and $\mathbf{u}_\xi^s=\mathbf{u}^s-\overline{\mathbf{u}}^s$, a fuzzy-neural control input can be derived as

$$\mathbf{u}^s=(\sum_{i=1}^{h}w^{si}\hat{\mathbf{B}}^{si})^{-1}(-\sum_{i=1}^{h}w^{si}\hat{\mathbf{A}}^{si}\mathbf{x}_\xi^s+\dot{\mathbf{r}}^s+\mathbf{\Lambda}\mathbf{e}^s-\mathbf{u}_s^s)+\overline{\mathbf{u}}^s\,. \tag{58}$$

Substituting (58) for (51), the error dynamic equation of the VLS model becomes

$$\begin{aligned}\dot{\mathbf{e}}^s &= \dot{\mathbf{x}}^s-\dot{\mathbf{r}}^s.\\ &= \Lambda\mathbf{e}^s+\sum_{i=1}^{h}w^{si}\widetilde{\mathbf{A}}^{si}\mathbf{x}_\xi^s+\sum_{i=1}^{h}w^{si}\widetilde{\mathbf{B}}^{si}\mathbf{u}_\xi^s+\widetilde{\mathbf{d}}^s-\mathbf{u}_s^s\end{aligned} \tag{59}$$

where $\widetilde{\mathbf{A}}^{si}=\hat{\mathbf{A}}_g^{s*i}-\hat{\mathbf{A}}^{si},\widetilde{\mathbf{B}}^{si}=\hat{\mathbf{B}}_g^{s*i}-\hat{\mathbf{B}}^{s}$, and $\widetilde{\mathbf{d}}^s=\mathbf{d}_d^s+(\mathbf{A}^s-\sum_{i=1}^{h}w^{si}\hat{\mathbf{A}}_g^{s*i})\mathbf{x}_\xi^s+(\mathbf{B}^s-\sum_{i=1}^{h}w^{si}\hat{\mathbf{B}}_g^{s*i})\mathbf{u}_\xi^s=[\widetilde{d}_1^s\ \widetilde{d}_2^s]^T$.

We define $\mathbf{u}_s^s$ (the error compensator) and $\mathbf{e}_\Delta^s$ as

$$\mathbf{u}_s^s=\begin{bmatrix}sign(e_{\Delta 1}^s) & 0\\ 0 & sign(e_{\Delta 2}^s)\end{bmatrix}\mathbf{k}=[u_{s1}^s,u_{s2}^s]^T \tag{60}$$

and

$$\mathbf{e}_\Delta^s=\mathbf{e}^{sT}\mathbf{\Gamma}=[e_{\Delta 1}^s\ e_{\Delta 2}^s] \tag{61}$$

where $\mathbf{\Gamma}>0$ is a Lyapunov matrix and $\mathbf{k}=[k_1,k_2]^T$. Let $\overline{\mathbf{d}}^s=\widetilde{\mathbf{d}}^s+\sum_{i=1}^{h}w^{si}\widetilde{\mathbf{B}}^{si}u_\xi^s=[\overline{d}_1^s\ \overline{d}_2^s]^T$, $k_i\geq\max(\widetilde{d}_i^{su},\overline{d}_i^{su})$, $\widetilde{d}_i^{su}$ be the upper bound of $|\widetilde{d}_i^s|$, and $\overline{d}_i^{su}$ be the upper bound of $|\overline{d}_i^s|$.

Theorem 2: Consider the general MIMO unknown nonaffine nonlinear system (42), which is approximated as (51) and satisfies assumptions 1. If the controller is designed as (58) with update laws

$$\dot{\hat{\mathbf{A}}}^{si}=\eta_1 w^{si}\mathbf{e}^s\mathbf{x}_\xi^{sT},\ i=1,2,\cdots,h \tag{62}$$

$$\dot{\hat{\mathbf{B}}}^{si}=\eta_2 w^{si}\mathbf{e}^s\mathbf{u}_\xi^{sT}\ \text{ if } |(\sum_{i=1}^{h}w^{si}\hat{\mathbf{B}}^{si})|\neq 0,\ \mathrm{i}=1,2,\cdots,h \tag{63}$$

where η_1 and η_2 are positive constants, then the closed-loop system is robust stable and $\lim_{t\to\infty}\|\mathbf{e}^s(t)\|=0$.

VI. Simulation Results

From equation (42), we can get a dynamic model of the active suspension system, where $F_{gi}^{s}(\mathbf{x}^{s},\mathbf{u}^{s})$, $i=1,2$ are uncertain nonlinear functions, u_1^s and u_2^s are the control inputs, and both d_{d1}^s and d_{d2}^s are external disturbances. Five fuzzy sets over the interval [-10000, 10000] are defined for $\mathbf{x}_\xi^s=[x_{\xi1}^s,x_{\xi2}^s,x_{\xi3}^s,x_{\xi4}^s]^T$ with the term sets (PB, PS, Z, NS, NB) and three fuzzy sets over the interval [-28000, 28000] for $u_\xi^s=[u_{\xi1}^s,u_{\xi2}^s]^T$. The design parameters are selected as $\eta=0.01$, $\lambda_1=2$, $\lambda_2=2$ and $\mathbf{Q}=[2\,0;0\,2]$. The initial states of the system are assumed to be $\mathbf{x}^s=[0,0,0,0]^T$.

In the example, a car is driving along a road with a consistent road surface. Figure 1 shows the displacement (z_s) of the car body. We control the displacement of the vehicle body to track the optimum balance point which is defined to be z_s=0. At 1.4 seconds, the tire spring K_t and the active suspension spring K_a "break" (the spring coefficients are multiplied by 10). Figure 2 shows the displacement (z_u) of the car wheel. Figure 3 shows that the road estimator is effective by t= 0.4 seconds, and that observer tracks the actual road condition. Figure 4 shows the curves of the vehicle speed (v) as it was reduced from 33.3 *m/sec* to 0 *m/sec* in 2.8 seconds and the corresponding velocity of the wheel (ω).

VII. CONCLUDING

In this paper, a novel on-line adaptive T-S fuzzy-neural modeling approach to the design of a robust tracking controller for uncertain active suspension systems has been proposed. Besides, we introduce a road estimator based on the LuGre friction model into an ABS system called the LuGre-based ABS. This solves the traditional problem of slip ratio control when the road condition is unknown. And using the observer-based direct adaptive fuzzy-neural controller (DAFC) is applied to ABS control, and its stability is guaranteed by the universal approximation theorem.

In addition, an algorithm for integrating these two subsystems has been proposed, and two individual controllers has been designed and coordinated with this integrated algorithm. Using the proposed controller, the simulation results show that the tracking performance is good throughout even when the springs break. From the simulation results, we can see that the proposed controller can track the reference signal quickly and control the states of the active suspension system well.

REFERENCES

[1] X. He and J. W. Hodgson, " Modeling and simulation for hybrid electric vehicles-part I: Simulation," *IEEE Trans. on Intelligent Transportation System,* vol. 3, no. 4, pp. 244-251, Dec. 2002.

[2] W. E. Ting and J. S. Lin, "Nonlinear Control Design of Anti-lock Braking Systems Combined with Active Suspensions," *Asian Control Conference,* vol. 1, pp. 611-616, July 2004.

[3] Wei-Yen Wang; I-Hsum Li; Ming-Chang Chen; Shun-Feng Su; Shi-Boun Hsu, "Dynamic Slip-Ratio Estimation and Control of Antilock Braking Systems Using an Observer-Based Direct Adaptive Fuzzy–Neural Controller," ***IEEE** TRANSACTIONS ON INDUSTRIAL ELECTRONICS*, Vol. 56, No. 5, MAY 2009.

[4] Wei-Yen Wang, I-Hsum Li, Ming-Chang Chen, Shun-Feng Su and Yih-Guang Leu, "A New Time-Efficient Structure for Observer-Based Adaptive Fuzzy-Neural Controllers for Nonaffine Nonlinear Systems," *International Journal of Innovative Computing, Information and Control,* Vol.6, No.4, April 2010.

[5] Y. G. Leu, W. Y. Wang, and T. T. Lee, "Observer-based direct adaptive fuzzy-neural control for nonaffine nonlinear systems," *IEEE Trans. on Neural Networks*, vol. 16, no. 4, pp. 853-861, July. 2005.

[6] C. Canudas-De-Wit and R. Horowitz, "Observers for tire/road contact friction using only wheel angular velocity information," *Proceedings of the 35th Decision & control Conference,* Phoenix, Arizona USA, pp. 3932-3937, December 1999.

[7] S. I. Grossman, and W. R. Derrick, *Advanced Engineering Mathematics*, Happer & Row, 1998.

[8] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its application to modeling and control," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 15, pp. 116-132, Jan. 1985.

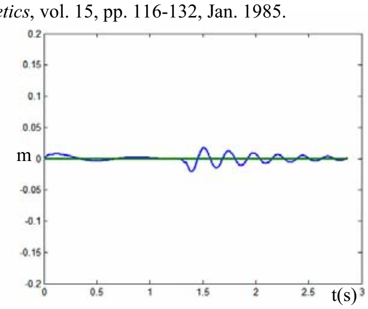

Fig. 1. z_s the displacement of vehicle body using the proposed control

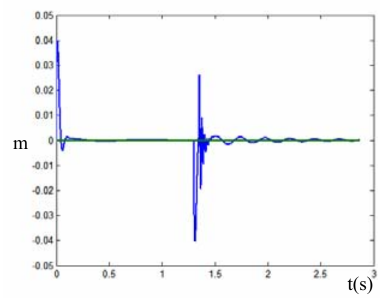

Fig. 2. z_u the displacement of vehicle body using the proposed control

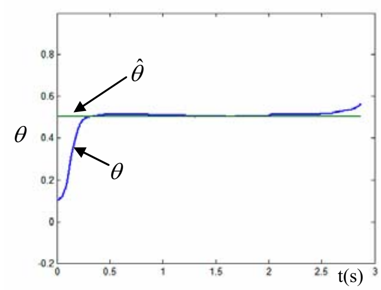

Figure 3 Estimated parameter $\hat{\theta}$ and curve of θ

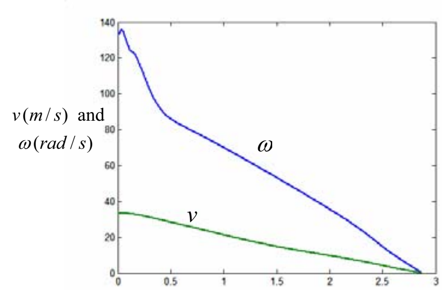

Fig. 4. Vehicle velocity(v) and rotation rate (ω).