Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Fuzzy Logic Control Implementation of Rectilinear Plant with Inverted Pendulum

Harold Rivera
Research Assistant, School of Engineering
University of North Florida
Jacksonville, Florida USA
rivh0002@unf.edu

Daniel Cox
Professor, School of Engineering
University of North Florida
Jacksonville, Florida USA
dcox@unf.edu

***Abstract*—Although many control problems have been solved using linear techniques, the world is inherently non-linear and many systems cannot be adequately treated with traditional linear systems techniques. Controllers based on Fuzzy Logic have been shown to perform well with non-linear systems. They are essentially non-linear controllers whose underlying theory was inspired by the way in which humans make decisions. The Fuzzy Logic Controller (FLC) makes use of expert knowledge and accepts a level of imprecision in the parameters—the fuzzy nature of the variables. When the system is highly non-linear and expert knowledge exists, a controller can be generated using fuzzy logic. The plant used for this analysis is the ECP rectilinear plant with the inverted pendulum feature. This plant offers a benchmark dynamic system used world-wide for the testing of traditional and new control techniques. The system is non-linear and requires fast action, excellent qualities for a benchmark system.**

***Keywords*—fuzzy logic, control, inverted pendulum.**

I. Introduction

Although many control problems have been solved using linear techniques, the world is inherently non-linear and many systems cannot be adequately treated with these traditional techniques. Controllers based on Fuzzy Logic Control (FLC) have been shown to perform well with non-linear systems. They are essentially non-linear controllers whose underlying theory was inspired by the way in which humans make decisions. The FLC makes use of expert knowledge and accepts a level of imprecision in the parameters, the fuzzy nature of the variables. When the system is highly non-linear and expert knowledge exists, a controller can be generated using fuzzy logic. The plant used for this analysis and implementation is the inverted pendulum. It was selected because it offers a benchmark dynamic system used world-wide for the testing of traditional and new control techniques. The system is non-linear and requires fast action, excellent qualities for a benchmark system.

In this paper the approach taken is that of the heuristic controller. This controller generalizes the fuzzy logic concept for the control of dynamic systems. Its main feature is the ability to control systems without a mathematical model. As long as heuristic, expert information is available, a controller can be designed to control the process. The heuristic controller is first simulated then implemented. The simulations are performed using MATLAB and Simulink. The Simulink environment provides an efficient platform to analyze non-linear systems.

II. Background

Fuzzy Logic Control was developed as a platform for decision-making in which uncertainty was embraced. The Fuzzy Logic paradigm is a response to what was described as the effects on the preciseness to which a designer can model a system mathematically as the complexity of the system increases. The idea is that as a system grows in complexity, the mathematical model of the system would grow more uncertain to the point where the model may be inadequate for controlling purposes. The new "fuzzy" ideas were not well received in the beginning, however the use of the concept in the control of dynamic systems is now well established and many references exist [1-6].

FLC is based on the theory of fuzzy sets. Conventional logic is based on two-valued logic. A vast field of philosophy deals with the attack and defense of arguments based on theory developed from the fundamental values "true" and "false". An argument, for example, could be debunked if logical discrepancies are found in it. The discrepancies are found with the use of rules or postulates of the field of logic. This traditional two-valued logic was taken by Boole and others to develop the Boolean Algebra widely used today for computing. This subject of the field of mathematics in conventional logic requires a solid understanding before embarking in fuzzy set approaches. Fuzzy sets allow for multi-valued logic. Instead of an element being either a member or not a member of a set, it is allowed to be a partial member. Fuzzy sets describe linguistic terms.

III. Fuzzy Logic Control Approach

The controllers designed are based on the model-free Mamdani fuzzy logic controller. All the discussions henceforth deal with this type of controller. This controller does not need a model and requires expert knowledge in the form of rules. Without these, the design of the controller is very difficult and demands much trial and error. On the contrary, if the operation of the control of a process can be codified into non-conflicting rules, a robust non-linear controller can be developed as an alternative [3].

978-1-4244-2794-9/09/$25.00

The overall structure of a Mamdani controller consists of three general modules: the fuzzification module, the fuzzy inference module, and the defuzzification module. The first module is associated with converting the numerical values (not assuming stochastic values) from the sensors into linguistic variables. The numerical values are essentially converted into "words" with a degree of "truth". At this juncture it is necessary to qualify and to explain the reason of the use of the word "truth". In the context of fuzzy control, fuzzy sets must pass tests in order to be used. The sets used to demonstrate the fuzzification process are all normal and convex. These two conditions are necessary. Here, the reference to fuzzy sets is actually a reference to fuzzy numbers, which are normal and convex fuzzy sets. The normal condition demands the highest value to be one and the convex condition, without much mathematical detail, requires only one "peak" and that no "dips" exist in the numbers. The reason to use "truth" (many other words could be used) is that membership functions essentially provide a definition to a word. When fuzzifying, the numerical values are graded against the "definition" established by the expert in the designation of the membership function. This is a very common and often controversial argument as many tend to equal membership functions and probability density functions. Although the principles are very similar, they are not the same. The probability density functions are based on probability theory whereas the membership functions are based on possibility theory. A good explanation is that probability density functions describe a data set while membership functions describe the definition of a word [5]. Thus, a membership value could be seen as the degree of "truthfulness" or accuracy of that value against the definition set by the expert without any numerical, analytical statistical analysis, and with no data sets in hand.

The next module in the Mamdani fuzzy controller is the fuzzy inference module. This module is the most significant in terms of the performance of the final controller. A mistake here can doom the controller to fail its task. The creation of a heuristic rule set is as much as science as it is an art. The process can be conducted by interviewing operators and obtaining information about what physical actions they take, when they take them, and why. These human-like rules are one of the benefits of using fuzzy controllers. The rule set may resemble the instructions a trainee may be given before controlling a process. Some more involved processes have insurmountable modeling problems but seem to be controllable by human operators. This is an area where time and effort could be saved using fuzzy controllers. If the operator can be interrogated and a comprehensive set of rules can be obtained, a fuzzy controller is feasible. In this case, comprehensive means that the rule can "cover" for all the expected states the process is to go through while in operation.

The field of fuzzy logic is vast and much literature exists on multi-valued logic. A thorough explanation is outside the scope of this paper. However, an important problem in fuzzy logic control is how to "blend" or combine signals. A geometric or analytical approach could be taken to prove the results provided by fuzzy logic literature. For example, binary signals can be blended or analyzed using AND and OR Boolean functions. These functions provide with the basis of two-valued logic. If an action is to be taken if two conditions are true (if it is "hot" and it is "humid") then the Boolean function AND can be used to decide. A simple program or hardware piece can be built for the easy four case scenario (the truth table has only four entries for the two-input case). The same goes for the case in which an action is to be taken if at least one condition is met (if it is either "hot" or "humid"). In these cases, the OR Boolean function can be used.

Several formulations have been devised for the "blending" or analysis of the fuzzy sets. The most common operations used in Fuzzy Logic are the MIN and MAX operations. The MIN operation implements the AND function and the MAX function implements the OR function. In this work and in many applications the MIN-MAX scheme is used for fuzzy implication. However, it is important to note that many other variants of the MAX and MIX norms have been successfully used. The general name used to encapsulate the ways in which fuzzy sets can be operated (fuzzy generalization of the AND operation) is a triangular-norm. Many t-norms, as they are known in fuzzy literature, are available [1,5]. The many generalizations of the OR operation in fuzzy logic are termed triangular conorms or t-conorms. Commonly used t-conorms are the algebraic sum conorm and the Lukasiewicz conorm. Most development software for Fuzzy logic supports these. Other t-norms used less sparingly include the Hamacher, the Frank, the Schweizer-Sklar, the Aczel-Alsina, and the Dombi norms [1,5].

Using the t-norms discussed above, it is possible to arrange control rules into a rule base. The principle is that each rule is evaluated using the t-norms to find the "AND" fuzzy generalization of the terms implicated in the firing rule. Once this is achieved, there should be n results for n rules. These results are then operated using t-conorms to arrive at the final values for the output membership terms. This is the main objective of the fuzzy inference module. Using different t-norms and conorms directly affects the performance of the system. The need for a defuzzification module is justified by the fact that the output of the fuzzy inference module is a list of output fuzzy terms with membership values ascribed to them. This list of actions corresponds to the multiple fuzzy decisions made by the separate "fuzzy deciders" or fuzzy rules. In unison, they provide with the fuzzy control effort based on the complete rule base and given inputs. However, they cannot be implemented directly because they are in the linguistic domain. For example, it is not possible to direct a DC motor to implement 0.8 "fast" and therefore, a conversion is necessary from the linguistic domain back into the numerical domain from which the system retrieved the sensory information and to which the control action is directed to make an effect on the transient response of the system. This conversion is called defuzzification.

The last module of the basic FLC is the defuzzification module. This module is tasked with converting the fuzzy, linguistic control actions output by the fuzzy inference module to the numerical or crisp representation used to control the real-world application. In the fuzzification step, a single numerical number (a number per input) is fuzzified or "runned" by however many membership functions describing that input in order to obtain an expression of membership of this single number with respect to the terms described by the membership functions. In other words, one numeric input was converted into n fuzzy linguistic terms for the case where the input space is divided by n membership functions. In the defuzzification process, the linguistic terms describing the output must now be converted into a single value, assuming one output.

IV. ECP 210 Inverted Pendulum Control

The dynamic system used for this report is the inverted pendulum. It is one of the classic and interesting control problems widely available to students [2,6]. The specific system used is the ECP 210 rectilinear plant with the inverted pendulum feature [7]. A limitation of the system is the restriction on the range of motion of any given cart of the plant. The plant is also designed to support a variety of rectilinear, or translational, dynamic systems and control experiments that do not require large translational ranges of motion.

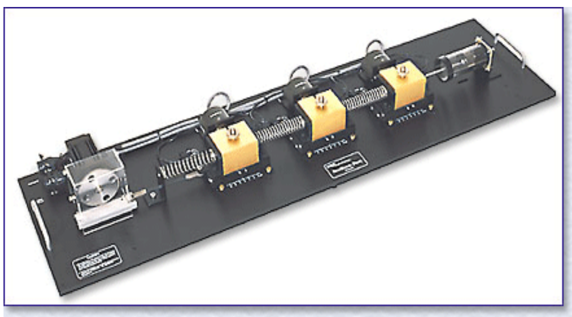

Figure 1. ECP 210 rectilinear plant

Although the FLC approach taken is model free, at the onset the determination of an appropriate model for the inverted pendulum is useful for understanding and insight into the plant. Many of the models encountered in the literature make simplifying assumptions. The model of the inverted pendulum on a cart is the classic configuration. In a traditional model [6] as shown in equation 1 and equation 2, the dynamic equations of motion are found using Lagrange's equations for the two generalized coordinates θ and x, representing the angle of the pendulum and the displacement of the cart, respectively.

$$\ddot{x} = -mal\ddot{\theta}\cos\theta + mal\dot{\theta}^2\sin\theta + au \quad (1)$$

$$\ddot{\theta} = \frac{mgl\sin\theta - m^2l^2a\dot{\theta}^2\sin(2\theta)/2 + mal\cos\theta u}{I - m^2l^2a\cos^2\theta + ml^2} \quad (2)$$

Before the controllers are implemented with the plant hardware, they have first been simulated using MATLAB. This design tool provides a complete fuzzy logic support and is widely used for control courses. For all the controllers presented herein, the same test was applied. The test consisted of:

- A regulation test with initial angle 15 degrees as initial condition.
- The same test with four impulses as a disturbance.
- The same test with a sinusoidal disturbance
- The same test with both disturbances summed in.

The plot of Figure 2 below shows a plot of the impulse disturbance presented at the output of the controllers. This is analogous to a 50 ms tap on the pendulum as it is being controlled. The magnitude of the impulses range from 5 to about 9 degrees.

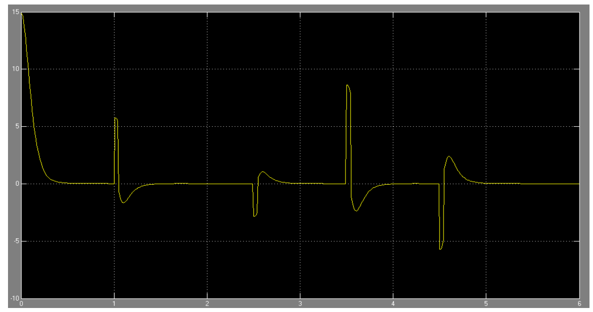

Figure 2. Response with impulse disturbance

The second disturbance imposed on the controllers is a sinusoidal signal. The signal has a magnitude of half a degree and is applied at the angle, analogous to a continuous, perfectly sinusoidal wind affecting the pendulum. The plot below of Figure 3 shows the sinusoidal signal for a duration of about 5.5 seconds.

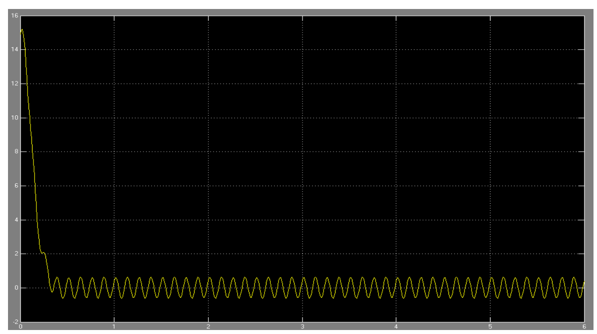

Figure 3. Response with sinusoidal disturbance

The majority of the controllers considered are generated using heuristic information about controlling the system. One of the reasons the inverted pendulum is a prime choice for the demonstration of fuzzy logic techniques is that it is a dynamic system whose control can be easily described with words. One of the advantages of fuzzy control is the freedom available to the designer regarding the inputs of the controllers. Any state of parameter that an operator uses to control a system can be used to design the corresponding fuzzy controller. As already mentioned, the most essential step of the design process is knowing with certainty what the controller is supposed to do given a set of states. The creation of heuristic fuzzy controller is essentially the creation of an arbitrary control surface. The freedom to choose the shape of the control surface accounts for the excellent performance with non-linear systems. PD controllers, for example, have planes as their control surface. A PD controller, with its control surface, can be emulated using Fuzzy Logic [3]. However, the main feature of the fuzzy logic controller is the ability to implement a non-linear mapping of any kind. This is because the fuzzy logic controller has been proved to be an ideal approximator. Of course, this is assuming increased resolution. This means that any conceivable function, linear or not, can be implemented using a fuzzy mapping.

Two different controllers have been designed using heuristic information. One of the controllers uses the states of the system (chosen as the angle and rate of change of the angle) and the other uses the error and the rate of change of the error as inputs. The selection of inputs is arbitrary and the choice is usually based on which states are more convenient to "observe" and write rules about. All fuzzy controllers are designed using a center of gravity defuzzification method and the Min-Max implication.

The first heuristic controller is designed to demonstrate the usefulness of fuzzy logic whenever a model of the system is not available but rules are in place for its control, as it is the case with the inverted pendulum. The controller has two inputs: angle and the rate of change of the angle.

A. Fuzzy Controller #1

Inputs: Angle, Angle Rate
Rules : 11
Input Resolution: 5 Fuzzy Terms
Output Resolution: 5 Fuzzy Terms

This controller is an economical controller as it uses only eleven rules. This controller, as others developed in this effort, has been derived with heuristic rules based on the physical behavior of the pendulum. The reduction of the number of rules (most of the controllers available from the literature span 25 or 49 rules) can be effectively done without any mathematical analysis. For the sake of space, each of the rules of each controller will not be explained but presented in their final form. The main point to remember is that the heuristic controller, based on physical states, is based on rules similar to those that may be given to an imaginary human operator. This is the nature of the rules shown for this controller. For the PI-like controllers presented below in Figure 4, Figure 5, and Figure 6, the focus would be on heuristic rules as in Table 1 based on the control error.

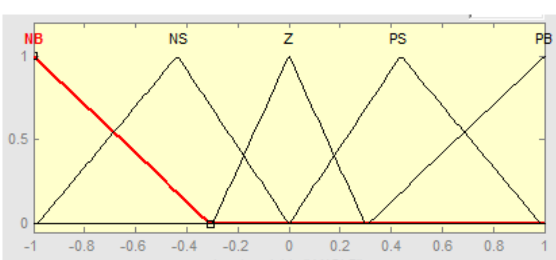

Figure 4. Fuzzy variable angle

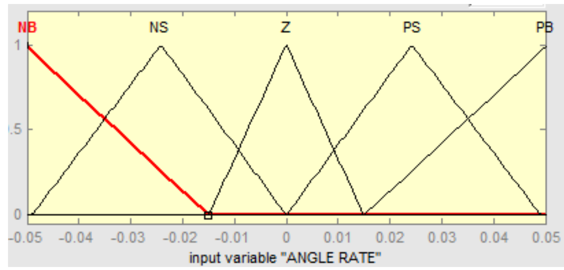

Figure 5. Fuzzy variable angle rate

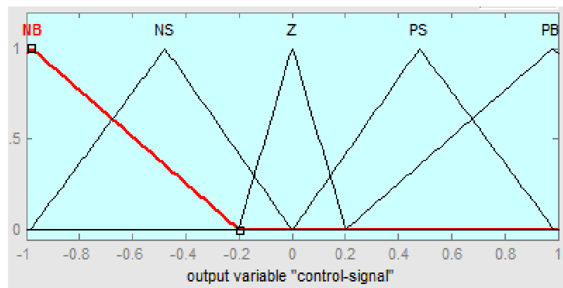

Figure 6. Fuzzy variable control

TABLE I. RULE BASE FOR FUZZY CONTROLLER

NB	**Z**	**NB**
NS	**NS**	**NS**
NS	**PS**	**Z**
Z	**NB**	**NB**
Z	**NS**	**NS**
Z	**Z**	**Z**
Z	**PS**	**PS**
Z	**PB**	**PB**
PS	**NS**	**Z**
PS	**PS**	**PS**
PB	**Z**	**PB**

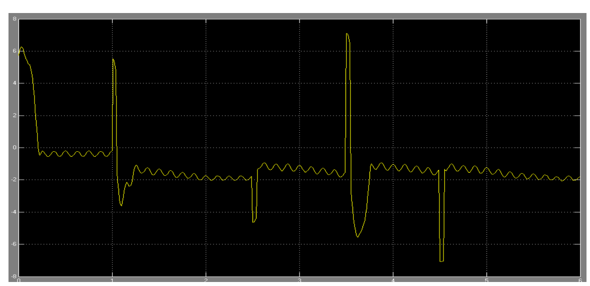

Figure 7. Response with composite disturbance

This fuzzy controller based on physical heuristics is very sensitive. Although the performance shown in Figure 7 is very satisfactory for the given angle, for larger initial conditions, the controller struggles to regulate the angle. The performance of the controller as it is shown is achieved by tuning the controller based on empirical results. This is the tedious and sometimes unattractive part of fuzzy heuristic design. The values for the gains have been found to allow the controller to regulate the angle well, but only for relatively small angles (no larger than 7 degrees). This type of controller, based on rules including the angle and the angular velocity, proved to be the most difficult to work with. They seem to need more tuning than the other types shown later. Several controllers of this type have been tested, but only the one shown was tuned to work properly. Although heuristic controllers are effective in many applications (the pendulum is among the difficult problems), fortunately, many other techniques, some of them covered below, can be used to sidestep or augment the purely heuristic approach.

B. Fuzzy PI-Like Controller

The second kind of heuristic controller presented is the PI-like fuzzy controller. The controller is named PI-like simply because the rule-base is derived from the PI expression for the control effort u. As mentioned above, there is no need to be restricted to these two states; any state definition is allowed as long as it performs well. However, there are many benefits for using the error and its derivative as states. One of the benefits is the easy generation of heuristic rules (as it will be shown) and the other is the compatibility of the controller with the traditional PI controller knowledge available.

For the PI controller, the error is defined as the difference between the set point and the actual output of the system (assuming a single output). With this definition in place, it is possible to make four statements that are essential in the formulation of the rule base.

If the error is negative, the process is below the set point.

If the error is positive, the process is above set point.

If the error-rate is negative, the process has increased.

If the error-rate is positive, the process has diminished.

The entire rule-set should be designed to account for all the expected states. As it will be shown below, the PI rule set is quite generic since the inputs are based on an error term that could have been generated by almost any system. Before presenting how each of the rules is derived, the complete rule-base for a PI-like Fuzzy Controller is shown below. This is a "generic" rule base [3] and is illustrated in Figure 8, Figure 9, Figure 10 and Table 2.

Fuzzy PI-like Controller
Inputs: Error, Error Rate
Rules : 49
Input Resolution: 7 Fuzzy Terms
Output Resolution: 7 Fuzzy Terms

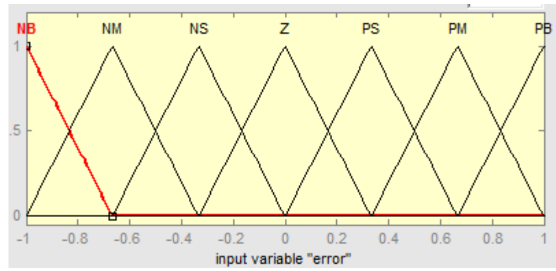

Figure 8. Fuzzy variable error

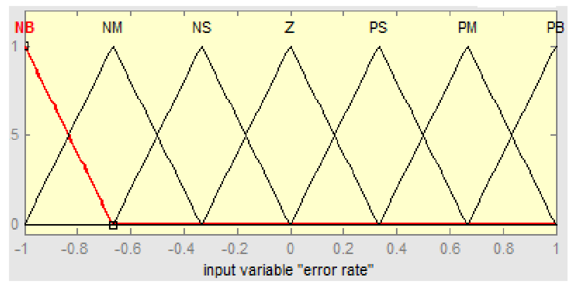

Figure 9. Fuzzy variable error rate

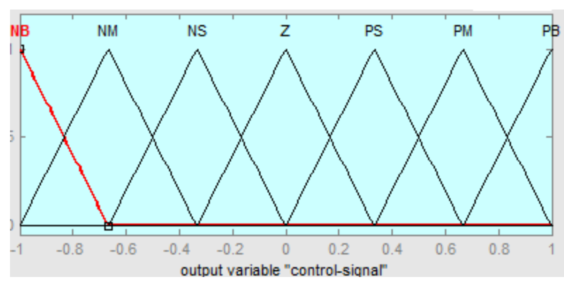

Figure 10. Fuzzy variable control

The PI-like controller presented as a generic fuzzy controller is the most favorable controller presented here. The resolution (49 rules) is sufficient to achieve a smooth surface and the corresponding smooth performance. The controller responds very well to the disturbances and is able to do so well close to 20 degrees in the initial condition setting. This controller provides a good starting point for the design of error-based controllers. Although the controller is heuristic (the rules were explained by groups earlier) the fact that the error and the error rate are used as the state variables encloses the controller more within the P, PI, PD, PID family of controllers rather than with the pure or physical heuristic controller seen earlier. PI-like controllers such as the one presented (as it is shown here) are not optimal and further benefits could be obtained by optimizing the shape of the membership functions and/or the rule-base. The following controllers can be seen as variants of the generic PI-like controller.

TABLE II. RULE BASE FOR FUZZY CONTROLLER

	NB	NM	NS	Z	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	Z
NM	NB	NB	NB	NM	NS	Z	PS
NS	NB	NB	NM	NS	Z	PS	PM
Z	NB	NM	NS	Z	PS	PM	PB
PS	NM	NS	Z	PS	PM	PB	PB
PM	NS	Z	PS	PM	PB	PB	PB
PB	Z	PS	PM	PB	PB	PB	PB

V. IMPLEMENTATION

The controllers discussed above have been implemented with the ECP plant hardware using the LabVIEW system [8]. To interface the ECP plant and its encoders, the NI RIO is used. This system does not use the characteristic NI Data Acquisition (DAC) modules that interface real-world signals with the core LabVIEW software. Instead, it employs Field Programmable Gate Array (FPGA) technology to allow for fast, parallel implementation of control loops. The system uses specialized software to compile the LabVIEW "G" code into Hardware Description Language (HDL) structures used to configure that gate arrays. Once the control structure is "flashed" into the gate array, the control loop is implemented by the NI RIO card directly, with minimal computational contribution from the machine running the core LabVIEW software.

The core fuzzy modules have been incorporated into the NI RIO environment using the Fuzzy Logic Control Toolkit. The MATLAB structures for the fuzzy controllers used earlier for analysis and simulation were ported into the LabVIEW environment for implementation as illustrated in Figure 11. The controllers simulated in MATLAB have been slightly adjusted when ported into the LabVIEW environment. The ranges have been expanded to account for the inputs given in encoder counts instead of physical units. The boundaries for active control actions are set to about 16 degrees from the upright position in both directions. The selection of the upright position as the starting point. A "swing-up" controller could have been built to drive the system to the range in which the fuzzy controllers can take over but this is not currently implemented. Currently, at any other angle below 16 degrees from the upright position, the controller has no control efforts sent. Each of the controllers showed performance commensurate to the performance shown in the simulations. The heuristic controller and the PI-like controllers balanced the pendulum, but did so while sending the cart to one of the limits. This was not convenient, especially in comparison to an optimal controller implementation, not described here, in which the optimization of the cart position allowed it to return to the origin. The testing of the fuzzy controllers is short due to the range of motion of the cart with limits but successful in the primal task. The controllers are able to be tested for much longer periods when a disturbance is introduced by simply tapping the pendulum as if to send it away from the limits.

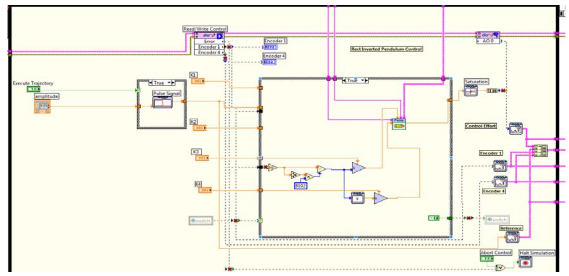

Figure 11. LabVIEW implementation

VI. CONCLUDING REMARKS

An alternative for generating control rules from heuristic linguistic information is to derive them using non-linear control theory. Although this suggests that, as in many control theory schemes, a mathematical model is required, an alternate method can be formulated using a physical model free controller and minimal information about the plant with basic relationships between state variables. The fuzzy rules are then derived in terms of the conditions that validate the behavior for the range of operation. This FLC approach has been successfully developed and implemented in the ECP 210 rectilinear plant with the inverted pendulum feature. This approach with the physical implementation provides new perspectives in the understanding of fuzzy-rule generation and system heuristics. The methodology for the analysis, design, and implementation of such systems into additional plants is the topic of ongoing research.

ACKNOWLEDGMENT

The support gratefully acknowledged are NSF IREE Grant #0738534, and NSF PFI Grant #0438582 from the National Science Foundation.

REFERENCES

[1] G. Chen, Introduction to Fuzzy Sets, Fuzzy Logic, and Fuzzy Control. Boca Raton, FL: CRC, 2001.

[2] C. Chen, Linear System Theory and Design. New York, New York: Oxford University Press, 1998.

[3] D. Driankov, An Introduction to Fuzzy Control. Berlin, Germany: Springer Verlag, 1993.

[4] M. Margaliot, New Approaches to Fuzzy Modeling and Control. River Edge, NJ: World Scientific, 2000.

[5] H. Nguyen, A First Course in Fuzzy and Neural Control. Boca Raton, FL: CRC.

[6] S. Zak, Systems and Control. New York, New York: Oxford University Press, 2003.

[7] ECP Systems, http://www.ecpsystems.com/, 2009.

[8] National Instruments, http://www.ni.com/ , 2009.