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On Positioned Solution of Linear Programming with Grey Parameters

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***Abstract*—To surmount the difficulty in solving the problem of linear programming with grey parameters (LPGP), a series of new conceptions such as the positioned linear programming, the ideal model, the critical model, and some other new ideas are put forward. The problem of how the optimum solution is affected by the variation of positioned coefficients and the range of the variation of the optimum positioned values are studied. The satisfying solution for the positioned programming and the satisfying degree of the solution were defined. Thus, the LPGP problem will be converted into several problems of ordinary linear programming. The difficulty in solving the LPGP problem and how to appraise the solution have been surmounted to a certain degree.**

***Keywords*—the LPGP, positioned programming, satisfying solution**

I. INTRODUCTION

The so-called programming mainly studies under certain constraints how to guarantee the objective of achieving the possible optimum. In these problems, if the constraint condition and objective function is linear, the problems are called linear programming problems. Linear programming is one of the most important branches in operations research, which developed early and matured fast with a wide range of practical applications. When grey numbers appear in either the programming model or the constraint of a linear programming model, it is called a linear programming with grey parameters (LPGP).

The LPGP problem is put forward first by Professor Julong Deng who with Huazhong University of Science and Technology, China [1]. He studied the grey drafting linear programming, the grey linear programming of prediction type[2,3]. The confidence degree solution of grey linear programming[4,5] and the cover solution of grey linear programming[6] were studied in recent years. In this paper, we put forward some new thinking to solving the LPGP problem. The work presented in [7,8,9] have been developed.

II. THE LPGP PROBLEM AND ITS POSITIONED PROGRAMMING

Definition 2.1 Assume that

$$X = [x_1, x_2, \cdots, x_n]^T$$

$$C(\otimes) = [c_1(\otimes), c_2(\otimes), \cdots, c_n(\otimes)]$$

$$b(\otimes) = [b_1(\otimes), b_2(\otimes), \cdots, b_m(\otimes)]^T$$

$$A(\otimes) = \begin{bmatrix} a_{11}(\otimes) & a_{12}(\otimes) & \cdots & a_{1n}(\otimes) \\ a_{21}(\otimes) & a_{22}(\otimes) & \cdots & a_{2n}(\otimes) \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1}(\otimes) & a_{m2}(\otimes) & \cdots & a_{mn}(\otimes) \end{bmatrix}$$

where, $c_j(\otimes) \in [\underline{c}_j, \overline{c}_j], \underline{c}_j \geq 0, j = 1,2,\cdots,n$;

$$b_i(\otimes) \in [\underline{b}_i, \overline{b}_i], \underline{b}_i \geq 0, i = 1,2,\cdots,m \text{ ;}$$

$$a_{ij}(\otimes) \in [\underline{a}_{ij}, \overline{a}_{ij}], \underline{a}_{ij} \geq 0; \quad i = 1,2,\cdots,m; \quad j = 1,2,\cdots,n.$$

Then

$$\max S = C(\otimes)X$$

$$\text{s.t} \begin{cases} A(\otimes)X \leq b(\otimes) \\ X \geq 0 \end{cases} \qquad (1)$$

is called a problem of linear programming with grey parameters (LPGP), and $C(\otimes)$ a grey price vector, $A(\otimes)$ a grey consumption matrix, $b(\otimes)$ a grey constraints vector for resource, and X the decision vector of the LPGP.

As a matter of fact, X is a grey vector as well.

Definition 2.2 Suppose that

$$\rho_j, \beta_i, \delta_{ij} \in [0,1], \; i = 1,2,\cdots,m; \; j = 1,2,\cdots,n.$$

and let the white values of grey parameters be, respectively, as follows

$$\tilde{c}_j(\otimes) = \rho_j \overline{c}_j + (1-\rho_j)\underline{c}_j; \; j = 1,2,\cdots,n$$

$$\tilde{b}_i(\otimes) = \beta_i \overline{b}_i + (1-\beta_i)\underline{b}_i; \; i = 1,2,\cdots,m$$

$$\tilde{a}_{ij}(\otimes) = \delta_{ij}\overline{a}_{ij} + (1-\delta_{ij})\underline{a}_{ij}; \; i = 1,2,\cdots,m; \; j = 1,2,\cdots,n$$

where $\tilde{C}(\otimes)$, $\tilde{b}(\otimes)$, $\tilde{A}(\otimes)$ are, respectively, the whitenization vector of price, constraints for resources, and the whitenization matrix of consumption. Then

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$$\max S = \tilde{C}(\otimes)X$$

$$\text{s.t}\begin{cases}\tilde{A}(\otimes)X \le \tilde{b}(\otimes)\\ X \ge 0\end{cases} \qquad (2)$$

is called a positioned programming of the LPGP; and $\rho_j (j = 1,2,\cdots,n)$ the positioned coefficients of price vector, $\beta_i (i = 1,2,\cdots,m)$ the positioned coefficients of constraint vector for resources, and

$\delta_{ij} (i = 1,2,\cdots,m; j = 1,2,\cdots,n)$

the positioned coefficients of consumption.

In Definition 2.2, ρ_j is a reflection of price fluctuation of the *j*th product. It can be determined by using a market analysis. Less ρ_j reflects a lower expected price of the *j*th product, and larger ρ_j reflects a higher expected price of the *j*th product.

The coefficient β_i is a reflection of market supplies of the *i*th resource. Less β_i expresses short supply of the *i*th resource, and larger β_i expresses sufficient supply of the *i*th resource.

Similarly, less δ_{ij} expresses lower consumption of the *i*th resource to produce an unit of the *j*th product, and larger δ_{ij} expresses higher consumption of the *i*th resource to produce the same unit of the *j*th product.

Proposition 2.1. The optimal value maxS of the positioned programming of a LPGP is a function with $m+n+mn$ variables of $\rho_j, \beta_i, \delta_{ij} (i = 1,2,\cdots,m; j = 1,2,\cdots,n)$.

From the Proposition 2.1, the optimal value maxS of the positioned programming can be marked as follows,

$$\max S = f((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

Similarly, the positioned programming can be marked as follows,

$$\text{LP}((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

For the sake of convenience, we first make the following assumptions.

Assumption 1. rank $(\tilde{A}(\otimes)) = m < n$.

Assumption 2. The set composed of the feasible solution of

$$\text{LP}((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

is non-empty.

Assumption 3. The set $\{X \big| \tilde{A}(\otimes)X \le \tilde{b}(\otimes), X \ge 0\}$ composed of real vectors is bounded. At the same time, the positioned programming

$$((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

can be rewritten into the following form,

$$\max S = [\tilde{C}_B(\otimes), \tilde{C}_N(\otimes)]\begin{bmatrix} X_B \\ X_N \end{bmatrix}$$

$$\text{s.t}\begin{cases}[\tilde{B}(\otimes), \tilde{N}(\otimes)]\begin{bmatrix} X_B \\ X_N \end{bmatrix} \le \tilde{b}(\otimes)\\ X_B \ge 0, X_N \ge 0\end{cases} \qquad (3)$$

That is, the first *m* columns of the whitenization matrix of consumption $\tilde{A}(\otimes)$ are the basis matrix $\tilde{B}(\otimes)$; the last *n-m* columns are the non-basis matrix $\tilde{N}(\otimes)$. The basis vectors and non-basis vectors corresponding to $\tilde{B}(\otimes)$ and $\tilde{N}(\otimes)$ can be written, respectively, as X_B, X_N. The whitenization vectors of price corresponding to X_B, X_N can be written, respectively, as $\tilde{C}_B(\otimes)$, $\tilde{C}_N(\otimes)$. From assumption 3, and noticing the fact that $X_N = 0$, it is clear that

$$X = [X_B, X_N]^T = \left[\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes), 0\right]^T$$

$$S = \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes)$$

and the test vector is

$$r = \tilde{C}(\otimes) - \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{A}(\otimes).$$

Proposition 2.2. Suppose that the positioned programming of (3) satisfies the above assumption 1,2,and 3, and

$$X = \left[x_1, x_2, \cdots, x_n\right]^T$$

is the basic solution of the positioned programming of (3). Then,

$$\{x_j \big| j = 1,2,\cdots,n\}$$

is bounded.

Proposition 2.3. There is at least one basic feasible solution of the positioned programming

$$\text{LP}((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m;\; j = 1,2,\cdots,n)$$

which satisfies the assumption 1,2,and 3.

III. THEOREMS ON POSITIONED SOLUTIONS OF LPGP

The LPGP is also called grey drifting linear programming. In reality, a problem of LPGP is a set composed of some ordinary problems of linear programming.

In the following proof, we suppose that the whitenization vectors and the whitenization matrix, given in the following

$$\max S = [\tilde{C}_B(\otimes), \tilde{C}_N(\otimes)]\begin{bmatrix} X_B \\ X_N \end{bmatrix}$$

$$\text{s.t}\begin{cases}[\tilde{B}(\otimes), \tilde{N}(\otimes)]\begin{bmatrix} X_B \\ X_N \end{bmatrix} \le \tilde{b}(\otimes)\\ X_B \ge 0, X_N \ge 0\end{cases}$$

still keep the property of non-negativity.

Theorem 3.1. For a positioned programming of a LPGP, when the positioned coefficients of the price vector satisfy

$$\rho_j \le \rho'_j; \quad j = 1,2,\cdots, \quad n$$

we have

$$\max S = f((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

$$\le f((\rho'_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; j = 1,2,\cdots,n) = \max S'$$

Proof: Because $\rho_j \le \rho'_j; \quad j = 1,2,\cdots, \quad n$, we have

$$\tilde{C}(\otimes) \le \tilde{C}'(\otimes)$$

Suppose that $\tilde{C}'(\otimes) = \tilde{C}(\otimes) + \Delta\tilde{C}(\otimes)$ and $\Delta\tilde{C}(\otimes) \ge 0$. There are now the following two cases. Without loss of generality, we assume

that $\tilde{B}(\otimes)$ is the optimal basis of

$$\mathrm{LP}((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; \; j = 1,2,\cdots,n)$$

(1) $\tilde{C}'(\otimes) - \tilde{C}'_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{A}(\otimes) \le 0$

Here, the optimal basis $\tilde{B}(\otimes)$ of the corresponding positioned programming doesn't change, nor does the optimum solution

$$X = [\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes), 0]^T.$$

Obviously,

$$\max S' = \tilde{C}'_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes)$$

$$= \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes) + \Delta\tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes)$$

$$\ge \max S$$

(2) $\tilde{C}'(\otimes) - \tilde{C}'_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{A}(\otimes) don't \le 0$

Suppose that the test number $r'_k(\otimes) > 0$, and that $\tilde{B}(\otimes)$ is not the optimal basis of the positioned programming

$$\mathrm{LP}((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; \; j = 1,2,\cdots,n)$$

Moreover, suppose that we use the simplex method to work out its optimal basis $\tilde{B}_1(\otimes)$ and its optimal solution

$$[\tilde{B}_1^{-1}(\otimes)\tilde{b}(\otimes), 0]^T$$

Notice that $[\tilde{B}_1^{-1}(\otimes)\tilde{b}(\otimes), 0]^T$ is the feasible basic solution of the positioned programming

$$\mathrm{LP}((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; \; j = 1,2,\cdots,n)$$

it is readily to have

$$\max S' = \tilde{C}'_{B_1}(\otimes)\tilde{B}_1^{-1}(\otimes)\tilde{b}(\otimes) \ge \tilde{C}'_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes)$$

$$= [\tilde{C}(\otimes) + \Delta\tilde{C}_B(\otimes)]\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes)$$

$$= \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes) + \Delta\tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes)$$

$$\ge \max S$$

Theorem 3.2. For a positioned programming of a LPGP, when the positioned coefficients of restriction vectors for resource satisfy the following

$$\beta_i \le \beta'_i, \quad i = 1,2,\cdots,m$$

we have

$$\max S = f((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

$$\le f((\rho_j, \beta'_i, \delta_{ij}) \big| i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

$$= \max S'$$

Proof: From $\beta_i \le \beta'_i, \quad i = 1,2,\cdots,m$, we know that

$$\tilde{b}(\otimes) \le \tilde{b}'(\otimes).$$

Suppose that $\tilde{b}'(\otimes) = b(\otimes) + \Delta\tilde{b}(\otimes)$, $\Delta\tilde{b}(\otimes) \ge 0$, then we have

$$\tilde{B}^{-1}(\otimes)\tilde{b}'(\otimes) = \tilde{B}^{-1}(\otimes)\tilde{b}(\otimes) + \tilde{B}^{-1}(\otimes)\Delta\tilde{b}(\otimes)$$

Here, $\tilde{B}(\otimes)$ is the optimal basis of

$$\mathrm{LP}((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; \; j = 1,2,\cdots,n)$$

(1)Assume that $\tilde{B}^{-1}(\otimes)\Delta\tilde{b}(\otimes) \ge 0$, then

$$\tilde{B}^{-1}(\otimes)\tilde{b}'(\otimes) = \tilde{B}^{-1}(\otimes)\tilde{b}(\otimes) + \tilde{B}^{-1}(\otimes)\Delta\tilde{b}(\otimes) \ge 0$$

Hence, $\tilde{B}(\otimes)$ is still the optimal basis of the positioned programming

$$\mathrm{LP}((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m; \; j = 1,2,\cdots,n)$$

So

$$\max S' = \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}'(\otimes)$$

$$= \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes) + \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\Delta\tilde{b}(\otimes)$$

$$= \max S + \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\Delta\tilde{b}(\otimes)$$

$$\ge \max S$$

(2) Assume that $\tilde{B}^{-1}(\otimes)\Delta\tilde{b}(\otimes) don't \ge 0$

Suppose that there is a k, $\Delta x_k < 0$. Now we discuss the situation in two cases as follows

① $x'_k = x_k + \Delta x_k \ge 0$. Then $\tilde{B}(\otimes)$ is still the optimal basis of the positioned programming

$$\mathrm{LP}((\rho_j, \beta'_i, \delta_{ij}) \big| i = 1,2,\cdots,m; \quad j = 1,2,\cdots,n)$$

But the optimal solution of $\mathrm{LP}((\rho_j, \beta_i, \delta_{ij}) \big| i = 1,2,\cdots,m;$

$j = 1,2,\cdots,n)$.

is a feasible basic solution of

$$\mathrm{LP}((\rho_j, \beta'_i, \delta_{ij}) \big| i = 1,2,\cdots,m; \quad j = 1,2,\cdots,n)$$

Therefore, we have

$$\max S = \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes) \le \max S'$$

② $x'_k = x_k + \Delta x_k < 0$. Now $[\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes), 0]^T$ is not the feasible basic solution of

$$\mathrm{LP}((\rho_j, \beta_i', \delta_{ij})|i = 1,2,\cdots,m; \quad j = 1,2,\cdots,n)$$

But $\tilde{B}(\otimes)$ is a regular basis. By using the dual simplex method, we can obtain the optimal basis $\tilde{B}_1(\otimes)$ and the optimal solution $X' = [\tilde{B}_1^{-1}(\otimes)\tilde{b}'(\otimes),0]^T$

And, noticing that the optimal solution of $\mathrm{LP}((\rho_j, \beta_i, \delta_{ij})|i = 1,2,\cdots,m; j = 1,2,\cdots,n)$ is a feasible basic solution of LP $((\rho_j, \beta_i', \delta_{ij})|i = 1,2,\cdots,m$; $j = 1,2,\cdots,n)$

we have

$$\max S = \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes) \le \max S'$$

Theorem 3.3. For a positioned programming $\mathrm{LP}((\rho_j, \beta_i, \delta_{ij})|i = 1,2,\cdots,m; j = 1,2,\cdots,n)$ of an LPGP, when the positioned coefficients of consumption satisfy the following

$$\delta_{ij} \ge \delta_{ij}', \qquad i = 1,2,\cdots,m; \quad j = 1,2,\cdots,n$$

we have

$$\begin{aligned}\max S &= f((\rho_j, \beta_i, \delta_{ij})|i = 1,2,\cdots,m; j = 1,2,\cdots,n) \\ &\le f((\rho_j, \beta_i, \delta_{ij}')|i = 1,2,\cdots,m; j = 1,2,\cdots,n) \\ &= \max S'\end{aligned}$$

Proof: From that $\delta_{ij} \ge \delta_{ij}'$, $i = 1,2,\cdots,m; j = 1,2,\cdots,n$, it follows that

$$\tilde{A}(\otimes) \ge \tilde{A}'(\otimes) \ge 0$$

Assume the kth column satisfies that

$$\tilde{P}_k(\otimes) \ge \tilde{P}_k'(\otimes)$$

(1) $\tilde{P}_k(\otimes)$ is not a basis vector.

When $\tilde{P}_k(\otimes)$ is changed to $\tilde{P}_k'(\otimes)$, the basis $\tilde{B}(\otimes)$ does not changed.

However, the test number

$$r_k' = \tilde{C}_k(\otimes) + \tilde{C}_B(\otimes)\tilde{B}^{-1}\tilde{P}_k'(\otimes)$$

may have been changed.

① If $r_k' \le 0$, then the optimal solution of

$$LP((\rho_j, \beta_i, \delta_{ij})|i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

is still the optimal solution of

$\mathrm{LP}((\rho_j, \beta_i, \delta_{ij}')|i = 1,2,\cdots,m; j = 1,2,\cdots,n)$.

And, the optimal value doesn't changed. So,

$$\max S = \max S'$$

② If $r_k' > 0$, then x_k' which corresponds to $\tilde{P}_k'(\otimes)$ will become a basis variable. We can obtain the optimal solution

$$X' = [\tilde{B}_1^{-1}(\otimes)\tilde{b}(\otimes),0]^T \text{ of}$$

$$\mathrm{LP}((\rho_j, \beta_i, \delta_{ij}')|i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

by using the simplex algorithm.

Noticing that $[\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes),0]^T$ is the feasible basic solution of

$$\mathrm{LP}((\rho_j, \beta_i, \delta_{ij}')|i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

we have

$$\begin{aligned}\max S &= \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes) \\ &\le \tilde{C}_{B_1}(\otimes)\tilde{B}_1^{-1}(\otimes)\tilde{b}(\otimes) = \max S'\end{aligned}$$

(2) $\tilde{P}_k(\otimes)$ is a basis vector.

When $\tilde{P}_k(\otimes)$ is changed to $\tilde{P}_k'(\otimes)$, whether the basis of

$$LP((\rho_j, \beta_i, \delta_{ij})|i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

is a basis of $\mathrm{LP}((\rho_j, \beta_i, \delta_{ij}')|i = 1,2,\cdots,m; j = 1,2,\cdots,n)$ or not, the optimal solution of the former is a feasible basic solution of the latter. But $X' = [\tilde{B}_1^{-1}(\otimes)\tilde{b}(\otimes),0]^T$ is the optimal solution of the latter, therefore

$$\begin{aligned}\max S &= \tilde{C}_B(\otimes)\tilde{B}^{-1}(\otimes)\tilde{b}(\otimes) \\ &\le \tilde{C}_{B_1}(\otimes)\tilde{B}_1^{-1}(\otimes)\tilde{b}(\otimes) = \max S'\end{aligned}$$

According to Theorems 3.1, 3.2 and 3.3, we know that the optimal value of a positioned programming is an increasing function about the positioned coefficients $\rho_j(j = 1,2,\cdots,n)$ of the price vector and the positioned coefficients of the constraint vector, and a decreasing function about the positioned coefficients $\delta_{ij}(i = 1,2,\cdots,m$; $j = 1,2,\cdots,n)$ of consumption.

Definition 3.1. Assume that $\forall i = 1,2,\cdots,m$ and $j = 1,2,\cdots,n$, we have

$$\rho_j = \rho, \quad \beta_j = \beta, \quad \delta_{ij} = \delta$$

Then, the corresponding positioned programming is called a (ρ, β, δ) -positioned programming. It is written as $\mathrm{LP}(\rho, \beta, \delta)$. Its optimal value is denoted $\max S(\rho, \beta, \delta)$, called the (ρ, β, δ) -positioned optimal value.

Theorem 3.4. For a positioned programming $\mathrm{LP}(\rho, \beta, \delta)$, we have

1^0 When $\rho = \rho_0, \beta = \beta_0, \delta_1 \le \delta_2$,

$$\max S(\rho_0, \beta_0, \delta_1) \ge \max S(\rho_0, \beta_0, \delta_2)$$

2^0 When $\rho_1 \le \rho_2, \beta = \beta_0, \delta = \delta_0$,

$$\max S(\rho_1, \beta_0, \delta_0) \le \max S(\rho_2, \beta_0, \delta_0)$$

3^0 When $\rho = \rho_0, \beta_1 \le \beta_2, \delta = \delta_0$,

$$\max S(\rho_0, \beta_1, \delta_0) \le \max S(\rho_0, \beta_2, \delta_0)$$

Here, ρ reflects the general price level of *n* kinds of products; β reflects the general supplying state of *m* kinds of resources; and δ is a collective reflection of the level of manufacturing technique, the quantity of the labor force, and managerial level applied in production.

IV. SATISFYING SOLUTIONS OF GREY LINEAR PROGRAMMING

In this section, we study grey linear programming problems such that not necessarily the optimal solutions but satisfying solutions can be practically reached.

Definition 4.1. When $\rho = \beta = 1, \delta = 0$, the correspond -ding positioned programming LP(1, 1, 0) is called an ideal model of the LPGP. Its optimal value is written as $\max \overline{S}$. The ideal model stands for an ideal condition such that the highest prices of its products, the most sufficient resource supply, the most developed manufacturing technique, and the quality of labor force and managerial level are all at their optimal states. In fact, only a few firms can potentially come up to the ideal state.

Definition 4.2. When $\rho = \beta = 0, \delta = 1$, the corresponding positioned programming LP(0, 0, 1) is called a critical model of the LPGP. Its optimal value is written as $\max \underline{S}$. The critical model stands for a condition such that the lowest prices, the shortest resource supply, the less-developed manufacturing technique, and the lowest quantity of labor force and managerial level are employed. With such a condition in place, the firm is at the edge of bankruptcy. The only choice for the firm to take is to change its products, to improve its production techniques, to find alternative resources, and to reeducate its managers and workers all around.

Definition 4.3. When $\rho = \beta = \delta = \theta$, the corresponding positioned programming is called a θ-positioned program - ming. It is written as LP(θ).
Similarly, its optimal value is written as $\max S(\theta)$, which is called the θ-positioned optimal value.
Especially when θ = 0.5, the corresponding θ- positioned programming LP(0.5) is called the mean whitenization prog - ramming. Generally, the mean whitenization programming is the most typical one for LPGP.

Theorem 4.1. $\forall\ \rho, \beta, \delta \in [0,1]$, we have

(1) $\max \underline{S} \le \max S(\rho, \beta, \delta) \le \max \overline{S}$;

(2) $\max \underline{S} \le \max S(\theta) \le \max \overline{S}$.

Proof: We prove (1). only. The second statement is left to the reader to prove.
Because $0 \le \rho \le 1$, $0 \le \beta \le 1$, $0 \le \delta \le 1$,
from Theorem 3.4, it follows that

$$\max \underline{S} = \max S(0,0,1) \le \max S(\rho,0,1) \le \max S(\rho, \beta, 1) \le \max S(\rho, \beta, \delta).$$

Similarly, we can prove that $\max \overline{S} \ge \max S(\rho, \beta, \delta)$.

Definition 4.4. For the given $\rho, \beta, \delta \in [0,1]$,

$$\mu(\rho, \beta, \delta) = \frac{1}{2}\left(1 - \frac{\max \underline{S}}{\max S(\rho, \beta, \delta)}\right) + \frac{1}{2}\,\frac{\max S(\rho, \beta, \delta)}{\max \overline{S}} \qquad (4)$$

is called the satisfying degree of the positioned programming LP(ρ, β, δ).

The satisfying degree of LP(ρ, β, δ) reflects the relation -ship among the positioned optimal value $\max S(\rho, \beta, \delta)$, the optimal value $\max \underline{S}$ of its critical model, and the optimal value $\max \overline{S}$ of the ideal model. The nearer $\max S(\rho, \beta, \delta)$ approaches $\max \overline{S}$, the bigger $\mu(\rho, \beta, \delta)$ is; the nearer $\max S(\rho, \beta, \delta)$ approaches $\max \underline{S}$, the smaller $\mu(\rho, \beta, \delta)$ is.

Similarly, we can define the concept of satisfying degree of $\mu(\theta)$ for θ-positioned programming LP(θ).

Proposition 4.1. $\forall\ \rho, \beta, \delta \in [0,1]$, we have that

$$0 \le \mu(\rho, \beta, \delta) \le 1$$

Definition 4.5. Given a grey target $D = [\mu_0, 1]$, if $\mu(\rho, \beta, \delta) \in D$, then the corresponding optimal solution is called the satisfying solution of the LPGP.

V. CONCLUDING REMARKS

Grey parameters often included in linear programming problems. As a result of lack information, the price vector, the consumption matrix, and the constraints vector for resource in linear programming problems all with some uncertainty. In the process of research on solving the LPGP problem, many people have probed from different angles. In this paper, we go further into the positioned solution of LPGP. By positioning an LPGP problem first, we can solve the LPGP problem by solve several ordinary linear programming problems.

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