Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Concepts of Broad Class Ratio Dispersion and Broad Smooth Degree for Nonhomogeneous Exponential Sequence and Their Application

China West Normal University (Yong Wei, Dahong Hu)
College of Mathematics and Information
Nanchong, China
E-mail:3306866@163.com

***Abstract*—Because the optimized grey models fits the approximate nonhomogeneous index sequences，this paper puts forward the concepts of the broad class ratio approach degree and the broad smooth degree which can judge the approach degree of the raw data and the nonhomogeneous index sequences, and deduces some remarks, then introduces the prior check of modeling the nonhomogeneous exponential sequences by these concepts. Finally, an example verifies the validity and the rationality of this method.**

Keywords—grey model, function Transformation, the smooth degree of data, the class ratio dispersion, comparison criterion

I. INTRODUCTION

The concepts of the traditional class ratio dispersion, the traditional smooth degree sequence and the comparison criterion of the traditional smooth degree can only reflect the approach degree of the raw data and the especial geometric progression—constant sequences, and can't reflect the approach degree of the raw data and the normal geometric progression. However the new models of the reconstruction and the optimization have been fit for both the low-growth and the high-growth sequences which approximate the normal geometric progression [1]–[8] , therefore the concepts of the traditional class ratio dispersion and the traditional smooth degree sequence and the comparison criterion of the traditional smooth degree have not been regarded as the weighing index of the grey modeling conditions. To the applicable scope of the optimum grey models, reference [9] introduced the concepts of the new class ratio dispersion and the new smooth degree sequence and proposed the comparison criterion of the new smooth degree so as to meet the modeling demand of the new models. References [10-11] obtained the high precision optimum grey models which are fit for the nonhomogeneous exponential sequences, in order to meet the prior check and judgment of these grey models, this paper introduces the concepts of the broad class ratio approach degree and the broad smooth degree sequences which can reflect the approach degree of the raw data and the nonhomogeneous exponential sequences, and puts forward the prior check of modeling the nonhomogeneous exponential sequences by these concepts.

II. MAIN RESULTS

Definition 1 If $\{a_k\}_{k=1}^n$ is the monotonic sequence, let

$d_a = (\frac{a_n - a_{n-1}}{a_2 - a_1})^{\frac{1}{n-2}}$, then $\left| d_a - \frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}} \right|$ ($k = 3, 4, \ldots, n$) is said to be the broad class ratio dispersion sequence, $\max\limits_{3 \le k \le n} \left| d_a - \frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}} \right|$ is said to be the maximum broad class ratio dispersion of $\{a_k\}_{k=1}^n$.

Definition 2 Let $\{a_k\}_{k=1}^n$ and $\{b_k\}_{k=1}^n$ be the monotonic sequences, $d_a = (\frac{a_n - a_{n-1}}{a_2 - a_1})^{\frac{1}{n-2}}$, $d_b = (\frac{b_n - b_{n-1}}{b_2 - b_1})^{\frac{1}{n-2}}$, if $\left| d_a - \frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}} \right| \le \left| d_b - \frac{b_k - b_{k-1}}{b_{k-1} - b_{k-2}} \right|$ where $k = 3, 4, \ldots, n$, then the broad class ratio dispersion of $\{a_k\}_{k=1}^n$ is said to be much smaller than the broad class ratio dispersion of $\{b_k\}_{k=1}^n$.

Theorem 1 Let $\{a_k\}_{k=1}^n$ be the monotonic sequence, $d_a = (\frac{a_n - a_{n-1}}{a_2 - a_1})^{\frac{1}{n-2}}$, then $\{a_k\}_{k=1}^n$ is the homogeneous or nonhomogeneous exponential sequence if and only if $\left| d_a - \frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}} \right| = 0$ where $k = 3, 4, \ldots, n$.

Proof: " $\Rightarrow$ " If $\{a_k\}_{k=1}^n$ is the homogeneous or nonhomogeneous exponential sequence, let $a_k = bd^{k-1} + c$, then $d_a = (\frac{a_n - a_{n-1}}{a_2 - a_1})^{\frac{1}{n-2}} \equiv [\frac{b d^{n-2}(d-1)}{b(d-1)}]^{\frac{1}{n-2}} = d$, so

$\left|d_a - \frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}}\right| = 0$, where $k = 3,4,...,n$

"$\Leftarrow$" If $\left|d_a - \frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}}\right| = 0$ where $k = 3,4,...,n$, then $d_a \equiv (\frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}}) = d$ where $k = 3,4,...,n$, that is, $\{a_k - a_{k-1}\}_{k=2}^{n}$ is the geometric proportion sequence which its common ratio is d, so

$$a_k - a_{k-1} \equiv (a_{k-1} - a_{k-2})d \equiv (a_{k-2} - a_{k-3})d^2 \equiv ... \equiv (a_2 - a_1)d^{k-2}$$

where $k = 3,4,...,n$,

We have

$$\begin{aligned}a_k &\equiv a_{k-1} + (a_2 - a_1)d^{k-2} \equiv a_{k-2} + (a_2 - a_1)d^{k-3} + (a_2 - a_1)d^{k-2}\\ &\equiv a_{k-3} + (a_2 - a_1)d^{k-4} + (a_2 - a_1)d^{k-3} + (a_2 - a_1)d^{k-2}\\ &\dots,\dots,\\ &\equiv a_3 + (a_2 - a_1)d^2 + ... + (a_2 - a_1)d^{k-3} + (a_2 - a_1)d^{k-2}\\ &\equiv a_2 + (a_2 - a_1)d + (a_2 - a_1)d^2 + ... + (a_2 - a_1)d^{k-2}\\ &\equiv a_2 + (a_2 - a_1)d\frac{1 - d^{k-2}}{1 - d}\\ &\equiv a_2 + (a_2 - a_1)\frac{d}{1 - d} - (a_2 - a_1)\frac{d^{k-1}}{1 - d}\end{aligned}$$

Let $b = \frac{-(a_2 - a_1)}{1 - d}$, $c = a_2 + (a_2 - a_1)\frac{d}{1 - d}$, namely, the raw data is the homogeneous or nonhomogeneous exponential sequence.

The proof is completed.

That is, the homogeneous or nonhomogeneous exponential sequence is equivalence to the sequence which the broad class ratio dispersion of its item is 0 constantly. Naturally, the broad class ratio dispersion of the sequence is smaller, the sequence is nearer to the homogeneous or nonhomogeneous exponential sequence. So we introduce the approach degree of the broad class ratio dispersion between the given sequence and the homogeneous or nonhomogeneous exponential sequence below.

Definition 3 Let $\{a_k\}_{k=1}^{n}$ be the sequence, $1 - \max_{3 \le k \le n}\left|d_a - \frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}}\right|$ is said to be the approach degree of the broad class ratio dispersion between $\{a_k\}_{k=1}^{n}$ and the homogeneous or nonhomogeneous exponential sequence.

Theorem 2 The monotonic sequence $\{a_k\}_{k=1}^{n}$ is the homogeneous or nonhomogeneous exponential sequence if and only if the broad class ratio dispersion of $\{a_k\}_{k=1}^{n}$ is 0.

Namely, if the maximum broad class ratio dispersion between $\{a_k\}_{k=1}^{n}$ and the homogeneous or nonhomogeneous exponential sequence is 0, then the approach degree of the broad class ratio dispersion between $\{a_k\}_{k=1}^{n}$ and the homogeneous or nonhomogeneous exponential sequence is 1(100%).

Definition 4 If $\{a_k\}_{k=1}^{n}$ is the monotonic sequence, let $d_a = (\frac{a_n - a_{n-1}}{a_2 - a_1})^{\frac{1}{n-2}}$, then $\frac{\frac{a_k - a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i - a_{i-1}}{d_a^{i-1}}}$ ($k = 3,4,...,n$) is said to be the broad smooth degree sequence of $\{a_k\}_{k=1}^{n}$.

Theorem 3 Let $\{a_k\}_{k=1}^{n}$ be the monotonic sequence, $d_a = (\frac{a_n - a_{n-1}}{a_2 - a_1})^{\frac{1}{n-2}}$, if $\{\frac{a_k - a_{k-1}}{d_a^{k-1}}\}_{k=2}^{n}$ is monotonic, then $\{a_k\}_{k=1}^{n}$ is the homogeneous or nonhomogeneous exponential sequence if and only if its broad smooth degree sequence $\frac{\frac{a_k - a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i - a_{i-1}}{d_a^{i-1}}} \equiv \frac{1}{k-2}$ where $k = 3,4,...,n$.

Proof: "$\Rightarrow$" to a random homogeneous or nonhomogeneous exponential sequence $a_k = bd^{k-1} + c$, where $k = 1,2,...,n$,

then $d_a = (\frac{a_n - a_{n-1}}{a_2 - a_1})^{\frac{1}{n-2}} \equiv (\frac{bd^{n-1} - bd^{n-2}}{bd - b})^{\frac{1}{n-2}} = d$, its broad smooth degree sequence $\frac{\frac{a_k - a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i - a_{i-1}}{d_a^{i-1}}} \equiv \frac{\frac{b(d-1)}{d}}{\frac{(k-2)b(d-1)}{d}}$

$\equiv \frac{1}{k-2}$, where $k = 3,4,...,n$.

"$\Leftarrow$" If $\forall\ k = 3,4,...,n$, $\frac{\frac{a_k - a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i - a_{i-1}}{d_a^{i-1}}} \equiv \frac{1}{k-2}$, then

$$\frac{\frac{a_3 - a_2}{d_a^2}}{\frac{a_2 - a_1}{d_a}} = 1, \quad \frac{\frac{a_4 - a_3}{d_a^3}}{\frac{a_2 - a_1}{d_a^1} + \frac{a_3 - a_2}{d_a^2}} = \frac{1}{2},$$

$$\frac{\frac{a_5 - a_4}{d_a^4}}{\frac{a_2 - a_1}{d_a^1} + \frac{a_3 - a_2}{d_a^2} + \frac{a_4 - a_3}{d_a^3}} = \frac{1}{3}, \dots,$$

$$\frac{\frac{a_n - a_{n-1}}{d_a^{n-1}}}{\frac{a_2-a_1}{d_a}+\frac{a_3-a_2}{d_a^2}+\frac{a_4-a_3}{d_a^3}+\cdots+\frac{a_{n-1}-a_{n-2}}{d_a^{n-2}}} = \frac{1}{n-2},$$

Because $\{\frac{a_k - a_{k-1}}{d_a^{k-1}}\}_{k=2}^n$ is monotonic, we might as well let $\frac{a_k - a_{k-1}}{d_a^{k-1}} \searrow$, then

$$\frac{\frac{a_n - a_{n-1}}{d_a^{n-1}}}{\frac{a_2-a_1}{d_a}+\frac{a_3-a_2}{d_a^2}+\frac{a_4-a_3}{d_a^3}+\cdots+\frac{a_{n-1}-a_{n-2}}{d_a^{n-2}}} \le \frac{1}{n-2},$$

$$\frac{\frac{a_n - a_{n-1}}{d_a^{n-1}}}{\frac{a_2-a_1}{d_a}+\frac{a_3-a_2}{d_a^2}+\frac{a_4-a_3}{d_a^3}+\cdots+\frac{a_{n-1}-a_{n-2}}{d_a^{n-2}}} = \frac{1}{n-2}$$

$$\Leftrightarrow \frac{a_2-a_1}{d_a} = \frac{a_3-a_2}{d_a^2} = \frac{a_4-a_3}{d_a^3} = \cdots = \frac{a_{n-1}-a_{n-2}}{d_a^{n-2}},$$

We have

$a_3 - a_2 = d_a(a_2 - a_1)$,

$a_4 - a_3 = d_a(a_3 - a_2) = d_a^2(a_2 - a_1)$,

$a_5 - a_4 = d_a(a_4 - a_3) = d_a^2(a_3 - a_2) = d_a^3(a_2 - a_1)$

…,

$a_n - a_{n-1} = d_a(a_{n-1} - a_{n-2}) = d_a^2(a_{n-2} - a_{n-3}) = \ldots$
$= d_a^{n-2}(a_2 - a_1)$,

So $a_n = \frac{a_2 - a_1}{1-d_a} d_a^{n-1} - \frac{a_2 - a_1 d_a}{1-d_a}$.

Therefore $\{a_k\}_{k=1}^n$ is the homogeneous or nonhomogeneous exponential sequence.

If $\frac{a_k - a_{k-1}}{d_a^{k-1}} \nearrow$, we can prove it by analogy.

The proof is completed.

Definition 5 If $\{a_k\}_{k=1}^n$ and $\{b_k\}_{k=1}^n$ are the monotonic sequence, moreover $\left|\frac{1}{k-2} / \frac{\frac{a_k-a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i-a_{i-1}}{d_a^{i-1}}} - \frac{\frac{a_k-a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i-a_{i-1}}{d_a^{i-1}}} / \frac{1}{k-2}\right| \le$ $\left|\frac{1}{k-2} / \frac{\frac{b_k-b_{k-1}}{d_b^{k-1}}}{\sum_{i=2}^{k-1}\frac{b_i-b_{i-1}}{d_b^{i-1}}} - \frac{\frac{b_k-b_{k-1}}{d_b^{k-1}}}{\sum_{i=2}^{k-1}\frac{b_i-b_{i-1}}{d_b^{i-1}}} / \frac{1}{k-2}\right|$, where $k = 3,4,\ldots,n$, then $\{a_k\}_{k=1}^n$ is said to be much smoother than $\{b_k\}_{k=1}^n$.

Theorem 4 In the meaning of the broad smooth degree, the homogeneous or nonhomogeneous exponential sequence is the smoothest sequence.

Proof: by theorem 3, the monotonic sequence $\{a_k\}_{k=1}^n$ is the homogeneous or nonhomogeneous exponential sequence if and only if $\frac{\frac{a_k-a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i-a_{i-1}}{d_a^{i-1}}} \equiv \frac{1}{k-2}$, where $k = 3,4,\ldots,n$.

Thus $\left|\frac{1}{k-2} / \frac{\frac{a_k-a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i-a_{i-1}}{d_a^{i-1}}} - \frac{\frac{a_k-a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i-a_{i-1}}{d_a^{i-1}}} / \frac{1}{k-2}\right| \equiv 0$,

therefore the homogeneous or nonhomogeneous exponential sequence is the smoothest sequence.

The proof is completed.

That is, if the broad smooth degree sequence of $\{a_k\}_{k=1}^n$ is smaller, then $\{a_k\}_{k=1}^n$ is nearer to the homogeneous or nonhomogeneous exponential sequence, like the concept of the broad class ratio approach degree, we can introduce the concept of the broad smooth approach degree between $\{a_k\}_{k=1}^n$ and the homogeneous or nonhomogeneous exponential sequence below.

Definition 6 Let $\{a_k\}_{k=1}^n$ be the sequence, then 1-$\max_{3\le k\le n}\left|\frac{1}{k-2} / \frac{\frac{a_k-a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i-a_{i-1}}{d_a^{i-1}}} - \frac{\frac{a_k-a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i-a_{i-1}}{d_a^{i-1}}} / \frac{1}{k-2}\right|$ is said to be the broad smooth approach degree between $\{a_k\}_{k=1}^n$ and the homogeneous or nonhomogeneous exponential sequence.

Theorem 5 The monotonic sequence $\{a_k\}_{k=1}^n$ is the homogeneous or nonhomogeneous exponential sequence if and only if the broad smooth approach degree of the sequence $\{a_k\}_{k=1}^n$ is 1（100%）.

Definition7 Let $\{a_k\}_{k=1}^n$ be the sequence, if $\exists\, k \in \{3,4,\ldots,n\}$, and $\left|\frac{1}{k-2} / \frac{\frac{a_k-a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i-a_{i-1}}{d_a^{i-1}}} - \frac{\frac{a_k-a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1}\frac{a_i-a_{i-1}}{d_a^{i-1}}} / \frac{1}{k-2}\right| \ge 1$, $\{a_k\}_{k=1}^n$ is said to be not the smooth sequence.

Theorem 6 The monotonic sequence $\{a_k\}_{k=1}^n$ is not the smooth sequence if and only if the broad smooth approach degree between $\{a_k\}_{k=1}^n$ and the homogeneous or nonhomogeneous exponential sequence ≤ 0

When the given sequence is normal geometric progression, by theorem 2 and theorem 3, it is difficult to deduce that the broad class ratio dispersion and the broad smooth degree in this paper are completely consistent with the new class ratio dispersion and the new smooth degree in [9], therefore the class ratio dispersion and the smooth degree in this paper are said to be the broad class ratio dispersion and the broad smooth degree.

III. APPLICATION OF RESULTS

Example let $\{a_k\}_{k=1}^7 = \{6,12,20,43,79,164,318\}$, $\{b_k\}_{k=1}^7 = \{6,15,17,49,73,173,309\}$, then d_a=1.9137, d_b=1.7213, the broad class ratio dispersion of $\{a_k\}_{k=1}^7$ and $\{b_k\}_{k=1}^7$ are compared in table I, the broad smooth degree of them are compared in table II.

TABLE I BROAD CLASS RATIO DISPERSION OF TWO SEQUENCES

k	$\frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}}$	$\left\| d_a - \frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}} \right\|$	$\frac{b_k - b_{k-1}}{b_{k-1} - b_{k-2}}$	$\left\| d_b - \frac{b_k - b_{k-1}}{b_{k-1} - b_{k-2}} \right\|$
3	1.3333	0.5804	0.2222	1.4991
4	2.875	0.9613	16	14.2787
5	1.5652	0.3485	0.75	0.9713
6	2.3611	0.4474	4.1667	2.4454
7	1.8118	0.1019	1.36	0.3613

In table I, we can discover: when $k = 3,4,...,n$, because $\left| d_a - \frac{a_k - a_{k-1}}{a_{k-1} - a_{k-2}} \right| \le \left| d_b - \frac{b_k - b_{k-1}}{b_{k-1} - b_{k-2}} \right|$, that is, the broad class ratio dispersion of $\{a_k\}_{k=1}^n$ is smaller than the broad class ratio dispersion of $\{b_k\}_{k=1}^n$, therefore $\{a_k\}_{k=1}^n$ is nearer to the nonhomogeneous exponential sequence.

Let $\dfrac{\frac{a_k - a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1} \frac{a_i - a_{i-1}}{d_a^{i-1}}} = A_k$, $\dfrac{\frac{b_k - b_{k-1}}{d_b^{k-1}}}{\sum_{i=2}^{k-1} \frac{b_i - b_{i-1}}{d_b^{i-1}}} = B_k$,

$$\left| \frac{1}{k-2} / \frac{\frac{a_k - a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1} \frac{a_i - a_{i-1}}{d_a^{i-1}}} - \frac{\frac{a_k - a_{k-1}}{d_a^{k-1}}}{\sum_{i=2}^{k-1} \frac{a_i - a_{i-1}}{d_a^{i-1}}} / \frac{1}{k-2} \right| = C_k,$$

$$\left| \frac{1}{k-2} / \frac{\frac{b_k - b_{k-1}}{d_b^{k-1}}}{\sum_{i=2}^{k-1} \frac{b_i - b_{i-1}}{d_b^{i-1}}} - \frac{\frac{b_k - b_{k-1}}{d_b^{k-1}}}{\sum_{i=2}^{k-1} \frac{b_i - b_{i-1}}{d_b^{i-1}}} / \frac{1}{k-2} \right| = D_k,$$

then the broad smooth degree of $\{a_k\}_{k=1}^7$ and $\{b_k\}_{k=1}^7$ are compared in table II below.

TABLE II BROAD SMOOTH DEGREE OF TWO SEQUENCES

k	A_k	C_k	B_k	D_k
3	0.6967	0.7385	0.1291	7.6167
4	0.6169	0.6169	1.0628	1.6552
5	0.3121	0.1320	0.2245	0.8113
6	0.2934	0.3218	0.4438	1.2118
7	0.2148	0.1428	0.2429	0.3908

In table II, we can discover: when $k = 3,4,...,n$, because $C_k \le D_k$, that is, $\{a_k\}_{k=1}^n$ is smoother than $\{b_k\}_{k=1}^n$, therefore $\{a_k\}_{k=1}^n$ is nearer to the nonhomogeneous exponential sequence.

We can propose NDGM model in [11] to $\{a_k\}_{k=1}^7$ and $\{b_k\}_{k=1}^7$, then

$\hat{a}^{(1)}(k+1) = 1.972756\hat{a}^{(1)}(k) - 0.14016k + 4.837508$,

$\hat{a}^{(1)}(1) = a^{(1)}(1) + 0.275433$

$\hat{b}^{(1)}(k+1) = 1.882007\hat{b}^{(1)}(k) + 3.111496k + 0.293963$,

$\hat{b}^{(1)}(1) = b^{(1)}(1) + 1.236256$

The simulative results are assembled as follows:

TABLE III ERROR ANALYSIS OF $\{a_k\}_{k=1}^7$

k	$a^{(0)}(k)$	$\hat{a}^{(0)}(k)$	Absolute error	Relative error
1	6	6.28	0.28	4.67%
2	12	10.80	−1.2	−10%
3	20	21.17	1.17	5.85%
4	43	41.62	−1.38	−3.21%
5	79	81.97	2.97	3.76%
6	164	161.56	−2.44	1.49%
7	318	318.59	0.59	0.19%

TABLE IV ERROR ANALYSIS OF $\{b_k\}_{k=1}^7$

k	$b^{(0)}(k)$	$\hat{b}^{(0)}(k)$	Absolute error	Relative error
1	6	7.24	1.24	20.66%
2	15	9.79	−5.21	−34.73%
3	17	21.53	4.53	26.65%
4	49	43.64	−5.36	−10.94%
5	73	85.23	12.23	16.75%
6	173	163.52	−9.48	−5.48%
7	309	310.86	1.86	0.6%

From table III and table IV, we can discover: whether by the absolute error or by the relative error, the simulative effect of $\{a_k\}_{k=1}^7$ is better than that of $\{b_k\}_{k=1}^7$, which is consistent with the foregoing analysis results.

IV. CONCLUSION

The concepts of the traditional class ratio dispersion, the traditional smooth degree sequence and their comparison criterions can only reflect the approach degree of the raw data and the constant sequences, the new class ratio dispersion and the new smooth degree and their comparison criterions in [9] can reflect the approach degree of the raw data and the geometric progression, and meet the modeling demand of the new models which have the white exponential law of coincidence. In order to meet the modeling demand of the optimum models with the widest scope of application, this paper puts forward the concepts of the broad class ratio dispersion and the broad smooth degree, and then introduces the concepts of the approach degree of broad class ratio dispersion and the broad smooth degree which can reflect the approach degree of the raw data and the nonhomogeneous exponential sequences respectively. Combined the precision demand of the practical problems, these concepts can be regarded as the modeling conditions of nonhomogeneous exponential sequences which have the white exponential, coefficient and constant shift law of coincidence.

The broad class ratio and the broad smooth degree is smaller, the modeling effect is better; by contrast with the broad class ratio and the broad smooth degree, the broad class ratio approach degree and the broad smooth approach degree between the raw data and homogeneous or nonhomogeneous exponential sequence is bigger, the modeling effect is better.

The concepts of the traditional class ratio dispersion and the traditional smooth degree sequence are fitted for the original model proposed by professor Deng julong, which has the white exponential law of coincidence only if $a=0$ and has the higher precision when the development coefficient a is smaller. In essence, the smoothest sequence which this kind of grey model demands is the constant sequence, quasi–smooth sequence is the sequence which approximates the constant sequence.

The concepts of the new class ratio dispersion and the new smooth degree sequence in [9] are fitted for the optimum models in [1-8], which have the white exponential law of coincidence when the raw data have the formula be^{ak}. In essence, the smoothest sequence which these kinds of grey models demand is the exponential sequence, quasi–smooth sequence is the sequence which approximates the exponential sequence.

The concepts of the broad class ratio dispersion and the broad smooth degree sequence in this paper are fitted for the optimum models in [10-11], which have the white exponent a, coefficient b and constant c shift law of coincidence when the raw data have the formula $be^{ak}+c$. In essence, the smoothest sequence which these kinds of grey models demand is the exponential sequence such as $be^{ak}+c$, quasi–smooth sequence is the sequence which approximates the nonhomogeneous exponential sequence.

Three prior checks obtained according to three foregoing kinds of class ratio dispersion and the smooth degree must be applied to the corresponding conditions, and can't be applied at random. If the concepts of the traditional class ratio dispersion and the traditional smooth degree sequence are applied to the latter two kinds of optimum grey models, then the high-growth homogeneous and nonhomogeneous exponential sequence which has the white exponential law of coincidence will be mistaken not to be modeled. If the concepts of the new class ratio dispersion and the new smooth degree sequence in [9] are applied to the traditional grey models, then the high-growth exponential sequences which are not fitted for grey modeling will be misunderstood to be the smoothest sequences, however if they are applied to the optimum grey models in [10-11], then a lot of nonhomogeneous exponential sequences will be regarded as not being modeling, there will be confusion.

Which transformations can transform the raw data to the nonhomogeneous exponential sequences so that we can propose the high precision grey models to them? This is the project which is very worthy of study.

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