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Multi-objective optimization of water supply network rehabilitation

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***Abstract*— Water network rehabilitation is a complex problem, and many facets should be concerned in the solving process. It is a discrete variables, non-linear, multi-objective optimal problem. An optimization approach is discussed in this paper by transforming the hydraulic constraints into objective functions of optimization model of water supply network rehabilitation problem. The non-dominated sorting Genetic Algorithm-II (NSGA-II) was adopted to solve the altered multi-objective optimal problem. The introduction of NSGA-II for water supply network optimal rehabilitation problem results in solving the conflict between one fitness value of standard genetic algorithm (SGA) and multi-objectives of rehabilitation problem. Moreover, it benefits to control the uncertainties brought by using weighting coefficients or punish functions in conventional methods. In order to accelerate the convergence speed of population, this paper introduces the artificial inducement mutation (AIM). It not only improves the convergence speed, but also improves the rationality and feasibility of solutions.**

***Keywords*— water supply system; water supply network; optimal rehabilitation; multi-objective; non-dominated sorting genetic algorithm (NSGA)**

I. INTRODUCTION

Optimal rehabilitation of water supply network has been a main research subject in water supply area for a long time. Maintenance and rehabilitation are long-term tasks after water utilities have been built. The most important and hardest part of maintenance and rehabilitation jobs in water supply system is water supply network rehabilitation. Therefore how to maintenance and rehabilitate water supply network feasibly, economically is essential issue to managers of water supply companies.

Water network rehabilitation is a complex problem, and many facets should be concerned in the solving process. It is discrete variables, non-linear, multi-objective optimal problem [1]. The concepts of building decision models for this problem are varied, the main methods are: general rehabilitation guides, prioritization models and optimization models [2].

The methods of rehabilitation guides and prioritization models have been applied in the early period. Limited by theoretical basis and algorithm level, they cannot evaluate rehabilitation schemes in a scale of whole network. Several objectives were used as the criteria for mains rehabilitation, such as net present value, critical break rate [3] and cost savings in pumping and energy [4]. These methods analyze the pipes in isolation from water supply network. It is difficult to measure the improvement of rehabilitation.

With the development of optimization theory and modeling technology in water supply system, using a more comprehensive and detail optimal model to solve rehabilitation problem becomes doable.. Optimization techniques consider the interaction of each main with the system as a whole, which enable both the performance and the cost of the rehabilitated system to play a role in the formulation of the rehabilitation program [5][6]. It allows for the trade-off between system performance and cost of rehabilitation. However, such techniques require large numbers of trial evaluations to obtain near-global optimal solutions. By using models of water network system and new optimization methods (SGA, particle swarm optimization (PSO), NSGA, NSGA-II), solving these optimization model of rehabilitation becomes possible.

II. REHABILITATION FORMATION

A. Optomalzation model for rehabilitation

Currently the popular optimization model for rehabilitation of water supply network is using minimization of rehabilitation cost and energy cost per year in the investment period as objective and hydraulic performance of network as constraints. With this concept, the general form of rehabilitation optimization model can be expressed by equations below [7].

Objective

$$Min.\ W = (P + \frac{m}{100})\sum_{i\in N}(a + bD_i^{\alpha})L_i$$

$$+3.58\sum_{i=1}^{3}(r_i E_i T_i)\sum_{j\in N_s}\frac{H_{i,j}Q_{i,j}}{\eta_{i,j}} \tag{1}$$

$$P = \frac{I_c(1+I_c)^T}{(1+I_c)^T - 1} \tag{2}$$

Where where *a*, *b*, *c* are coefficients in formula of pipe construction cost; *Dk* is diameter (mm) of pipe *k*; *Ei* is electricity price in period *i* (Yuan/(kW·h)); *Hij* is pressure of pump *j* in period *i* (m); *I* is norm yield rate (%); *Lk* is length (m) of pipe *k*; *m* is lift to save a rate of capital repairs fund (%); *N* is set of pipes which need be rehabilitated; *n*s is set of pump stations; *P* is equivalent coefficient; *Qij* is flow of pump *j* in

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period I (L/s); ri is coefficient of pumping energy in period i; T is repayment period of investment (year); ti is time in period i (h); ηij is efficiency of pump j in period i (%)

$$i = \begin{cases} 1 & \text{Peak period of electricity consumption} \\ 2 & \text{Low period of electricity consumption} \\ 3 & \text{Normal period of electricity consumption} \end{cases}$$

Constraints

Continuous equation：

$$Q_i - \sum_{j \in V_i} q_{ij} = 0 \quad (i = 1,2,......,n) \tag{3}$$

Energy balance equation：

$$\sum (h_{ij})_l = 0 \tag{4}$$

where Qi is nodal demand (L/s) of node i; qij is pipe flow (L/s) from node i to node j; Vi is adjacent nodes set of node I; hij is head loss (m) of pipe which from node i to node j; l is loop number

Node pressure constraint：

$$H_{\min} \le H_j \le H_{\max} \qquad j \in J \tag{5}$$

Pipe velocity constraint：

$$v_{i\min} \le v_i \le v_{i\max} \qquad i \in P_s \tag{6}$$

Pipe diameter standard constraint：

$$d_i \in D_s = \{D_1, D_2, \ldots, \mathrm{D}_z\} \tag{7}$$

where Hj is pressure (m) of node j; Hmin is the minimum service pressure (m); Hmax is the maximum service pressure (m); J is node set of network. vi is velocity of pipe i (m/s); $v_{i\max}$ is upper boundary of velocity of pipe i (m/s); $v_{i\min}$ is lower boundary of velocity of pipe i (m/s); PS is pipe set of network. Di is diameter (mm) of pipe i; DS is available standard diameter set

This optimization model is a single-objective model with minimization of construction cost and energy cost as objective, and network performances and available diameters as constraints. In conventional solving methods, constraints will transform as parts of objective function by weighting method or ε-constraint method [8]. These transformations enable the traditional algorithms to be used in solving optimization models of water supply network rehabilitation. Unfortunately, the solution obtained by this process largely depends on the values assigned to the weighting factors used or the design of ε-constraint function. This approach does not provide a dense spread of the Pareto points. So the best way to solve rehabilitation problem is to abstract the problem as a multi-objective functions and solve these functions with a true multi-objective oriented algorithm.

B. Alternative optomalzation model for rehabilitation

The concept of alteration is regarding the constraints that are transformed as parts of objective function with weighting method or ε-constraint function as isolate objective functions. So the constraints which represent performance of water supply network should be expressed as objective function.

Objective function of pipe load is calculated as the sum of the velocity shortfalls or excesses Δv_i at pipes (i), weighted by the pipe lengths (L_i) and diameters (D_i, here use unit of meter):

$$Min.W_2 = \alpha \sum_{i \in pm} (\Delta v_i \cdot L_i \cdot D_i) + \beta \sum_{i \in px} (\Delta v_i \cdot L_i \cdot D_i) \tag{8}$$

Where pm is set of pipes with velocity below the lower boundary of velocity; px is set of pipes with velocity above the upper boundary of velocity; and α , β are weights to allow different emphasis on velocity shortfalls or excess.

Objective function of node pressure is calculated as the sum of the pressure shortfalls or excesses Δh_j at consumer nodes(j), weighted by the nodal demands(Q_j) :

$$Min.W_3 = \rho \sum_{j \in jm} (\Delta h_j \cdot Q_j) + \sigma \sum_{j \in jx} (\Delta h_j \cdot Q_j) \tag{9}$$

Where jm is set of nodes with pressure below the lower boundary pressure; jx is set of nodes with pressure above the upper boundary pressure; and ρ , σ are weights to allow different emphasis on pressure shortfalls or excess.

So the new multi-objective optimization model can be expressed as functions below:

Objective functions：

$$\mathrm{Min}.W_1 = (P + \frac{m}{100}) \sum_{i \in N} (a + bD_i^c) L_i + 3.58 \sum_{i=1}^{3} (r_i E_i T_i) \sum_{j \in N_s} \frac{H_{i,j} Q_{i,j}}{\eta_{i,j}} \tag{10}$$

$$Min.W_2 = \alpha \sum_{i \in pm} (\Delta v_i \cdot L_i \cdot D_i) + \beta \sum_{i \in px} (\Delta v_i \cdot L_i \cdot D_i) \tag{11}$$

$$Min.W_3 = \rho \sum_{j \in jm} (\Delta h_j \cdot Q_j) + \sigma \sum_{j \in jx} (\Delta h_j \cdot Q_j) \tag{12}$$

where ρ , σ are weight coefficients; Δh is node pressure shortfalls or excesses with minimum service pressure (m); Q is nodal demand (L/s); Jm is set of nodes with pressure below the minimum service pressure; Jx is set of nodes with pressure above minimum service pressure.

constraints：

$$Q_i - \sum_{j \in V_i} q_{ij} = 0 \quad (i = 1,2,......,n) \tag{13}$$

$$\sum (h_{ij})_l = 0 \tag{14}$$

$$d_i \in D = \{D_1, D_2, \ldots, \mathrm{D}_z\} \tag{15}$$

III. SOLUTION PROCEDUARE

In case of multi-objective optimization, instead of obtaining a unique optimal solution, a set of equally good (non-dominating) optimal solutions is usually obtained (Pareto sets). Within a Pareto set, one objective function improves while the other deteriorates. In absence of any other high level additional information, a decision maker normally cannot choose any one of these non-dominant optimal solutions since all of them are equally competitive and none of them can dominate each other.

Several methods: goal attainment method, ε-constraint method, versions of NSGA and NSGA-II [9] available to solve multi-objective optimization problems, NSGA-II[10][11] is used here to obtain the Pareto set. Use of penalty function is a very popular way of handling constraints. But tuning of the penalty parameter appearing in the penalty function is very time consuming and normally performed on the basis of trial and error. Unless tuned properly, one may get misdirected totally in the search space. NSGA-II based constraint-handling technique, allows one to get rid of the above stated problem of penalty function.

A. Coding

The code string was made with all rehabilitated pipes' diameter code shown in Table1 and Table2. Suppose that the IDs of rehabilitated pipe are 1,2,3,4,5.

Table 1 Code of standard pipe diameters

Table 1 Code of standard pipe diameters

Diameters	code	Diameters	code
100	0	900	8
200	1	1000	9
300	2	1200	10
400	3	1400	11
500	4	1500	12
600	5	1600	13
700	6	1800	14
800	7	2000	15

Table2 code of chromosome of individuals of initial generation

index	Pipes' diameter scheme(order is pipe1-pipe5)	Chromosome Code
1	800,1800,1400,800,100	7,14,11,7,0
2	2000,2000,500,300,400	15,15,4,2,3
3	400,1600,1000,700,1400	3,13,9,6,11
4	300,2000,200,200,1800	2,15,1,1,14
5	1400,2200,400,1500,100	11,16,3,12,0
6	2000,1800,900,800,900	15,14,8,7,8
7	700,2000,100,300,1400	6,15,0,2,11
8	2000,200,1200,1000,800	15,1,10,9,7
9	300,1800,2000,1000,100	2,14,15,9,0
10	1200,2200,900,400,100	10,16,8,3,0

B. Selction operation

Disposal of objective values

The three objective functions of rehabilitation model are all minimization functions, in non-dominated sorting, the individuals that have the larger objective values will dominate the one with smaller objective values. So some disposal should be done to objective values and make the better individual in the preferential rank.

The disposal of objective values can be expressed with the function below:

$$W_i' = Const/(W_i + Const'), \tag{16}$$

where W_i is original objective value; *Const*, *Const′* are constants varying with different objectives.

Individual sorting

The NSGA-II non-dominated sorting process is used a fast non-dominated sorting approach. Individuals will be sorted according to two parameters: non-dominated rank and crowding distance. After obtain the non-dominated sorting rank and crowding-distance of individuals, individuals can be sorted by Crowded Comparison Operator (CCO). The CCO guides the selection process at the various stages of the algorithm toward a uniformly spread-out Pareto optimal front.

Selection operator

In this paper roulette wheel selection was selected as the selection method, which it benefits for selecting potentially useful solutions for recombination. In the selection process, the fitness value of each individual was calculated firstly, and then transfers these fitness values to selection probabilities, whose transfer function is as below

$$p_k = f_k \Big/ \sum_{j=1}^{Pop\_size} f_j . \tag{17}$$

The fitness value of individual f is transferred to selection probability p. The cumulative probability (p_k') of kth individual can be obtained by adding the individual probabilities starting from top of the list till the kth member. The kth individual is represented by the cumulative probability value between p_{k-1}' and p_k'.

C. Crossover operation

Crossover operator is responsible for searching of new individuals, which could possibly have better fitness. There are also several crossover methods: Here we use the one point crossover method as the crossover operator

D. Mutation operation

Since mutation operator has the random attribute, although it can achieve the goal of diversity preservation, it also tampers with the constringency of genetic algorithm. Sometime the bad gene that mutation operator brought in will be gotten rid of by many generations. In order to bringing benefit gene in the mutation operation, a new mutation method: artificial inducement mutation (AIM) was introduced in the NSGA-II in this paper. This operator can steer the population convergence to the field of feasible solutions accelerating, and then use normal mutation operator searching for the best solution in the feasible solutions.

For the optimization model of rehabilitation, the goal of AIM is to make the selected diameters of rehabilitated pipes follow the direction of meeting the constraint of pipe velocity, until the solutions converge to the feasible field.

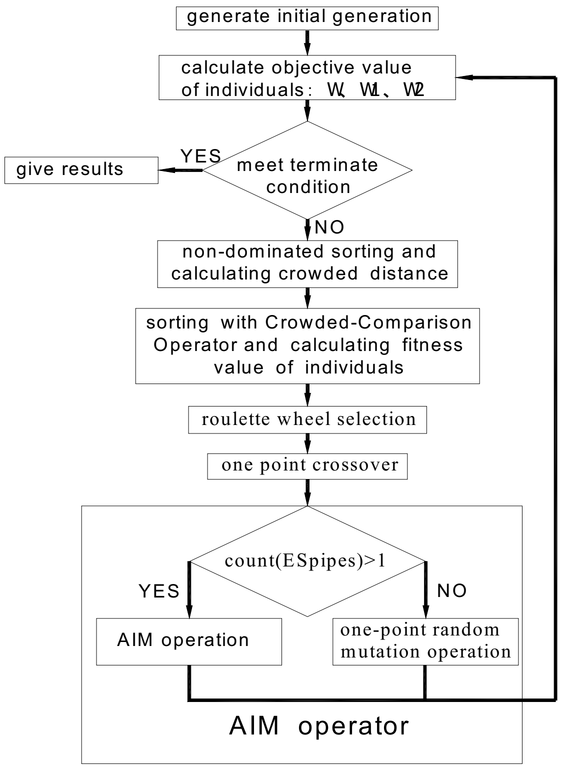

Figure 1 Flow chart of NSGA-II with AIM

IV. CASE STUDY

A. Introduction

The example network [12] with the increase of water consumption, the phenomenon of over fast velocity, high hydraulic slope and lower node pressure has appeared. The chock pipes and lower pressure nodes are shown in table 3 and table 4.

Table 3 Data of chock pipes in case network

Pipe ID	Dia (mm)	length (m)	hydraulic slope (m/Km)	velocity (m/s)
8	300	505.9	5.71	1.32
14	1000	609.5	0.96	1.18
29	750	131.1	3.35	1.93
32	750	179.8	3.05	1.84
35	750	387.1	2.66	1.70
46	600	780.3	2.60	1.34
51	600	350.5	3.02	1.45
52	600	850.3	3.05	1.46
56	400	365.7	6.78	1.74
58	400	368.8	5.51	1.56
90	750	365.7	3.19	1.87
97	600	1300	7.29	2.16
96	900	2100	1.93	1.36
98	500	469	3.07	1.20
99	600	2000	4.87	1.74

Table 4 Data of nodes with low pressure

Node ID	elevation (m)	demand (L/S)	pressure (m)	HGL (m)
60	4.27	9.74	7.92	12.19
61	3.96	6.85	8.05	12.02
62	3.96	14.61	8.05	12.01
63	4.27	6.09	7.72	11.99
64	5.49	10.81	6.45	11.94
65	5.49	19.47	6.45	11.94
66	4	9.74	7.88	11.88
67	3	8.37	8.82	11.82
68	3	6.20	8.88	11.88

The positions of chock pipes and lower pressure nodes are demonstrated in figure 2.

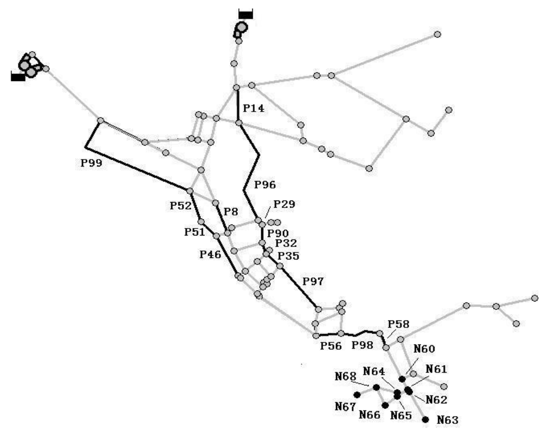

Figure 2 Position of chock pipes and low pressure nodes in case network

The optimization rehabilitation model of case network was solved by NSGA-II without AIM operator and with AIM operator respectively. Since AIM is always benefit to population, so in the algorithm of NSGA-II with AIM operator, its probability is 1, means use AIM unconditional.

B. Results and discussion

3D non-dominated Pareto optimal front in initial, 100 and 200 generations of NSGA-II without AIM are shown in Fig 3.(a)(b)(c). The detail of individuals of non-dominated Pareto optimal front in the 200th generation of NSGA-II without AIM is shown in Table 5. It can be seen from Fig. 3(a) in initial generation only 4 non-dominated Pareto optimal solutions were obtained. With successive generations the dominated solutions were eliminated and replaced by better solutions so that the number of non-dominated Pareto optimal solutions increased to 37 (there are cases that some individuals are superposition) after 200 generations. It is also seen that with the increase in the number of generations, better solutions are obtained, for example each evaluation value of individuals extends follow the positive direction. And in generation 100 node pressure evaluation value achieves its the best, which means all nodes in the network meet the pressure constraint

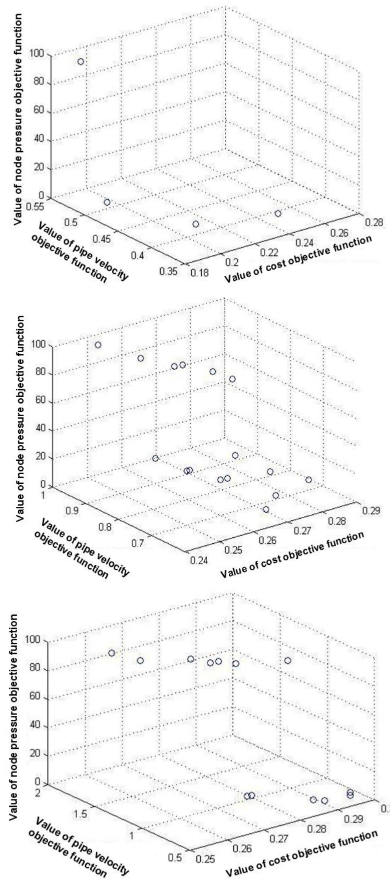

Figure 3(a)(b)(c) 3D Pareto optimal front of initial, 100 and 200 generation of NSGA-II without AIM

Table 5 Individuals of Pareto optimal front in the 200th generation of NAGA-II without AIM

Index	Rehabilitation cost per year (Yuan)	Energy cost per year Yuan	Number of nodes that do not meet pressure constraint	Number of pipes that not meet velocity constraint
0	2433003.50	1451907.50	0	4
2	2247451.25	1375498.75	0	6
4	2096700.50	1382630.75	2	7
6	2329394.50	1398293.37	0	3
10	2232583.00	1420127.37	0	5
11	2078753.75	1384293.25	2	7
14	2265398.00	1373695.25	1	6
15	2297121.25	1400019.50	0	4
22	2104033.25	1411524.37	0	5
24	2400730.50	1454224.00	0	5
38	2062341.37	1292602.50	9	7
40	2031179.00	1334427.50	9	6
42	2279174.5	1401795.50	0	4

From Table 5 it can be seen that the rehabilitation schemes was improved in some extent. But it is found that although the node pressure constraint was met, there are still some pipes, which do not satisfied the velocity constraint. So all these solutions are not the feasible solutions, although the cost was cheap, they can not be adopted. This is mainly because that although new diameters can be brought in by mutation operation, the mutation probability is low and mutation operation has a random attribute, so the searching scope was limited, and the constringency of algorithm was not good. If we want to search in a larger solution scope, a larger population size and generations should be assigned, and also the quality of individuals of initial generation is very important. But all these operations will result in the very time consuming situation. This shortcoming can be solved by the introduction of AIM.

3D non-dominated Pareto optimal front in initial, 10, 50 generations of NSGA-II with AIM are shown in Fig 4 (a)(b)(c) respectively. The detail of individuals of non-dominated Pareto optimal front in the 50th generation with AIM is shown in Table 6.

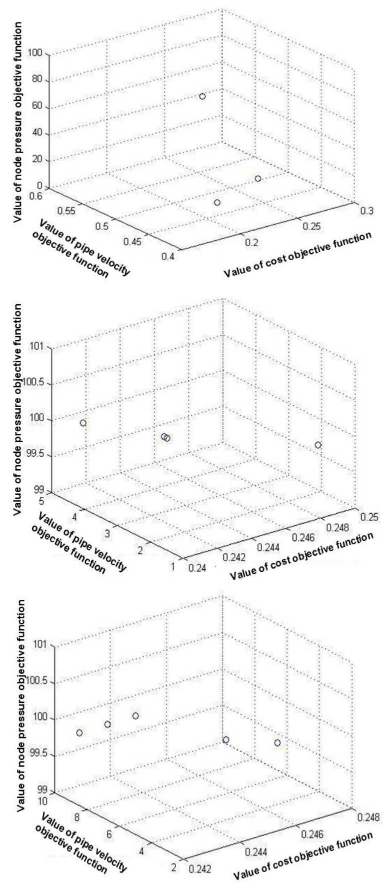

Figure.4 (a)(b)(c) 3D Pareto optimal front of initial,10th and 50th generations of NSGA-II with AIM

It can be seen from Fig.4(c) that individuals which meet both constraints of node pressure and pipe velocity were generated. This indicates that artificial mutation does guide the population convergence to feasible solutions field, accelerating and searching best solutions in feasible field with rest of evolution process

Table 6 Individuals of Pareto optimal front of the 50th generation of NSGA-II with AIM

index	Rehabilitation cost per year (Yuan)	Energy cost per year Yuan)	Number of nodes that do not meet pressure constraint	Number of pipes that not meet velocity constraint
8	2600978.25	1489380.75	0	0
18	2624264.5	1488686.25	0	0
31	2574135.75	1494308.25	0	1
38	2649502.00	1488393.37	0	0
40	2595291.00	1493950.75	0	0

The hydraulic information of rehabilitated pipes in one of best solution of NSGA-II with AIM is shown in table 7, and the pressure of lower pressure nodes in the rehabilitated network are shown in Table 8.

Table 7 Data of chock pipes after rehabilitation

ID	Diameter (mm)	Length (m)	hydraulic slope (m/Km)	velocity (m/s)
8	300	300	4.44	1.16
14	1000	1200	0.41	0.83
29	750	1200	0.38	0.8
32	750	1000	0.88	1.11
35	750	900	1.42	1.36
46	600	800	0.51	0.67
51	600	600	2.73	1.38
52	600	600	2.76	1.39
56	400	600	0.95	0.78
58	400	500	1.87	1.00
90	750	1200	0.35	0.78
97	600	900	1.36	1.13
96	900	1200	0.50	0.8
98	500	700	0.59	0.62
99	600	800	1.60	1.14

Table 8 Data of lower pressure nodes after rehabilitation

Node ID	Pressure (m)	Node ID	Pressure (m)
58	23.98	64	21.48
60	22.95	65	21.48
61	23.08	66	22.32
62	23.08	67	21.62
63	22.75	68	22.32

From Table 7 and 8, it can be seen that the chock pipes have been eliminated and pressure of the lower pressure nodes has been improved to an acceptable level. In this rehabilitation scheme, the diameters of some chock pipes have not been modified, this is because after optimization, the diameters of other pipes has been magnified, these pipes' hydraulic condition also improved, so these pipes do not need to be modified.

V. CONCLUSION

A multi-objective optimal model of water supply network rehabilitation was discussed and solved with NSGA-II and AIM in this paper. By introduction of NSGA-II, the problem of multi-objective of optimal rehabilitation model with one fitness value of conventional GA is solved. The shortcomings that brought in by weighting method or ε-constraint method have been eliminated. By introduction of artificial inducement mutation (AIM), the population is directed to feasible solutions field rapidly, and searching the best solution in the feasible field. So the convergence of algorithm has been improved and can give more feasible and better solutions. By comparing the results of two NSGA-IIs with and without artificial inducement mutation in the case study, the advantage and feasibility of artificial inducement mutation are shown and evaluated. In fact there are several different concepts in building multi-objective optimal model for water supply network rehabilitation. NSGA-II and AIM can still work on solving other multi-objective optimal models.

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