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A New Grey-rough Set Model Based on Interval-Valued Grey Sets

Wu Shunxiang
Department of automation, Xiamen University,
Xiamen, P. R. China
E-mail: sxwu@xmu. edu. cn

Lin Lihua,Gao Qinquan
Department of automation, Xiamen University,
Xiamen, P. R. China
E-mail: 270176420@qq. com

***Abstract*—In this paper, a novel grey rough set model for interval-valued grey sets information systems named interval-valued grey-rough set model is proposed and the basic theory and propositions of the interval-valued grey-rough sets is studied. Based on the proposed interval-valued rough grey model, rough similarity degree is defined, and the clustering of the interval-valued grey information systems is also examined, in the meanwhile, some examples are presented respectively.**

***Keywords*—interval-valued grey sets, grey-rough set, grey cluster**

I. INTRODUCTION

Rough set theory primarily proposed by Pawlak [1] is a useful mathematical framework to deal with imprecision, vagueness and uncertainty in information systems. This theory has been amply demonstrated to have usefulness and versatility by successful application in a variety of problems, such as in pattern recognition, machine learning, data mining, classification, clustering, concurrent systems, decision analysis [2,3], image processing, information retrieval, kansei engineering, signal processing, system modeling and voice recognition. Pawlak proposed a minimal decision rule induction from an indiscernibility relation and also proposed two rough approximations an upper approximation that deals with possibility and a lower approximation that deals with certainty of information tables. In particular, rough approximation is a basic mathematical model for handling information tables. However, Classical rough sets mainly deal with nominal data or multi-valued discrete data, such as small, large, short, long, true or false. And the classical rough set theory has difficulty in handling interval-valued of attributes which exist in the real world. Information tables in the real world, however, are quite complicated. Many measured values given as grey data which have a range, for example, ID, mass, No. temperature, and so on. Such value ranges can be described by interval sets in this paper.

Grey system theory, proposed by Deng [4,5], covers grey classification, grey control, grey decision-making, grey prediction, grey structural modeling, grey relational analysis [6], etc. It deals with the uncertainty over how systems with incomplete or lack of information should be controlled. One of the important concepts is a grey number. It is a number whose exact value is unknown but range is known. Thus grey system theory deals with uncertainty unlike those of fuzzy set theory or rough set theory. The grey operation is one of the operations for grey numbers that modifies a range of given intervals of grey sets. It is more suitable to handling information tables containing interval set.

Theories of rough sets and grey sets are generalizations of classical set theories for modeling vagueness and fuzziness respectively [7,8]. It is generally accepted that these two theories are related, but distinct and complementary, to each other [9,10]. As a generalization of grey sets, the notion of interval-valued grey sets was suggested, by which we describe the incomplete information system in this paper [11,12,13,14].

In this paper, we first proposed a novel grey rough set model for interval-valued grey sets data named interval-valued grey-rough set, and its upper approximation and lower approximation is defined, then we investigate its propositions, based on it, rough similarity degree is defined, which is pointed out reasonable, in the meanwhile, some examples are presented respectively.

II. PRELIMINARIES

A. *Interval-valued grey sets*

Let U denote the universal set, x denote an element of U.

Definition 1: An interval-valued grey sets $\otimes$ is a set whose exact elements is uncertain but range is known.

Let G be an interval-valued grey mapping set of U defined by two crisp sets of the upper bound and the lower bound as follows:

$P \subseteq \otimes \subseteq Q$, where, P, Q are two crisp sets, satisfying $P \subseteq Q$, In other words, $\otimes$ denote the set of value range that $\otimes$ may hold.

Throughout this paper, we mark it as $\otimes = \otimes\Big|_P^Q$.

Especially, when $P = Q$, the interval-valued grey sets G becomes a crisp set, which means that interval-valued grey system theory deals with flexibly sets situation.

B. *Tolerance relation based on interval-valued grey sets*

Definition 2: Let U be a non-empty finite universe, $R_A^{\cap}$ be the tolerance relation on the U with respect to all attributes

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in A, $[\otimes]_R^{\cap}$ denotes the tolerant class which including the $\otimes$, then $(U, R^{\cap})$ is the Pawlak approximation space. For any interval-valued grey set $\otimes$, the tolerance relation $R_A^{\cap}$ can be defined as follows:

$$R_A^{\cap} = \{(\otimes(x), \otimes(y)) \in U \times U \mid \forall a \in A, f_a^{+}(\otimes(x)) \cap f_a^{+}(\otimes(y)) \neq \phi\}$$

The tolerant class induced by the tolerance relation $R_A^{\cap}$ is the set of objects $\otimes(x)$, i. e.

$$[\otimes(x)]_A^{\cap} = \{\otimes(y) \in U \mid (\otimes(x), \otimes(y)) \in R_A^{\cap}\}$$
$$= \{\otimes(y) \in U \mid \forall a \in A, f_a^{+}(\otimes(x)) \cap f_a^{+}(\otimes(y)) \neq \phi\}.$$

Where $[\otimes(x)]_A^{\cap}$ describes the set of objects that may tolerant with $\otimes(x)$ in terms of A in an interval-valued grey information system.

From the definition of $[\otimes(x)]_A^{\cap}$, the following properties can be easily obtained.

Property 1: Let (U, A, G) be an interval-valued grey information system and. then

1. $R_A^{\cap}$ is reflexive, symmetric.
2. If $B_1 \subseteq B_2 \subseteq A$, then $R_{B_1}^{\cap} \supseteq R_{B_2}^{\cap} \supseteq R_A^{\cap}$,
   $[\otimes(x)]_{B_1}^{\cap} \supseteq [\otimes(x)]_{B_2}^{\cap} \supseteq [\otimes(x)]_A^{\cap}$.
3. For any $B \subseteq A$, then $R_B^{\cap} = \bigcap_{a \in B} R_{\{a\}}^{\cap}$.

From the definition 2, we know $P = \{[\otimes(x)]_B^{\cap} \mid \otimes(x) \in U\}$ constitute a covering of U.

Definition 3 : Given two interval-valued grey sets $\otimes(x_a) = \otimes(x_a)|_{x_a^-}^{x_a^+}$ and $\otimes(y_a) = \otimes(y_a)|_{y_a^-}^{y_a^+}$ where objects $\otimes x, \otimes y \subseteq U$ The interval-valued grey sets operation is defined as follows:

1. $\otimes(x_a)|_{x_a^-}^{x_a^+} \bigcup \otimes(y_a)|_{y_a^-}^{y_a^+} = \otimes(z_a)|_{x_a^- \cup y_a^-}^{x_a^+ \cup y_a^+}$.
2. $\otimes(x_a)|_{x_a^-}^{x_a^+} \bigcap \otimes(y_a)|_{y_a^-}^{y_a^+} = \otimes(z_a)|_{x_a^- \cap y_a^-}^{x_a^+ \cap y_a^+}$.
3. $\otimes(x_a)|_{x_a^-}^{x_a^+} \subseteq \otimes(y_a)|_{y_a^-}^{y_a^+} \Leftrightarrow x_a^+ \subseteq y_a^+ and x_a^- \subseteq y_a^-$.

Example 1: An interval-valued grey information system is presented in Table 1, Compute the classification induced by the tolerance relation $R_A^{\cap}$ in Table I.

TABLE I. INTERVAL-VALUED GREY INFORMATION SYSTEM

U	a_1	a_2	a_3
$\otimes(x_1)$	$\otimes\|_{\{1\}}^{\{1,2\}}$	$\otimes\|_{\{1\}}^{\{1\}}$	$\otimes\|_{\{2\}}^{\{2\}}$
$\otimes(x_2)$	$\otimes\|_{\{3\}}^{\{3,4\}}$	$\otimes\|_{\{3\}}^{\{3\}}$	$\otimes\|_{\{4\}}^{\{4\}}$
$\otimes(x_3)$	$\otimes\|_{\{1\}}^{\{1\}}$	$\otimes\|_{\{1,2\}}^{\{1,2\}}$	$\otimes\|_{\{2\}}^{\{2\}}$
$\otimes(x_4)$	$\otimes\|_{\{2\}}^{\{1,2\}}$	$\otimes\|_{\{2\}}^{\{1,2\}}$	$\otimes\|_{\{2\}}^{\{2\}}$
$\otimes(x_5)$	$\otimes\|_{\{1\}}^{\{1,3\}}$	$\otimes\|_{\{3\}}^{\{1,3,4\}}$	$\otimes\|_{\{2\}}^{\{2,4\}}$
$\otimes(x_6)$	$\otimes\|_{\{3\}}^{\{3,4\}}$	$\otimes\|_{\{3\}}^{\{3\}}$	$\otimes\|_{\{4\}}^{\{4\}}$
$\otimes(x_7)$	$\otimes\|_{\{3\}}^{\{3\}}$	$\otimes\|_{\{4\}}^{\{4\}}$	$\otimes\|_{\{2\}}^{\{2,4\}}$
$\otimes(x_8)$	$\otimes\|_{\{4\}}^{\{3,4\}}$	$\otimes\|_{\{3\}}^{\{3\}}$	$\otimes\|_{\{4\}}^{\{4\}}$

From table I, we can get:

$[\otimes(x_1)]_{a_1}^{\cap} = \{\otimes(x_1), \otimes(x_3), \otimes(x_4), \otimes(x_5)\}$,

$[\otimes(x_2)]_{a_1}^{\cap} = \{\otimes(x_2), \otimes(x_5), \otimes(x_6), \otimes(x_7), \otimes(x_8)\}$,

$[\otimes(x_1)]_{a_2}^{\cap} = \{\otimes(x_1), \otimes(x_3), \otimes(x_4), \otimes(x_5)\}$,

$[\otimes(x_2)]_{a_2}^{\cap} = \{\otimes(x_2), \otimes(x_5), \otimes(x_6), \otimes(x_8)\}$,

$[\otimes(x_1)]_{a_3}^{\cap} = \{\otimes(x_1), \otimes(x_3), \otimes(x_4), \otimes(x_5), \otimes(x_7)\}$,

$[\otimes(x_2)]_{a_3}^{\cap} = \{\otimes(x_2), \otimes(x_5), \otimes(x_6), \otimes(x_7), \otimes(x_8)\}$,

So,

$[\otimes(x_1)]_A^{\cap} = \{\otimes(x_1), \otimes(x_3), \otimes(x_4), \otimes(x_5)\}$,

$[\otimes(x_2)]_A^{\cap} = \{\otimes(x_2), \otimes(x_5), \otimes(x_6), \otimes(x_8)\}$.

Analogously, we can get tolerant class of all the objects in the universe.

III. THE INTERVAL-VALUED GREY-ROUGH SET MODELS

Definition 4: For any interval-valued grey set $\otimes(x)$, we define:

$$\underline{R}(\otimes(x_a)) = \bigcup\{\otimes(s) \in U \mid a \in A, f^+(\otimes(s_a)) \subseteq f^+(\otimes(x_a))$$

$$and f^-(\otimes(s_a)) \supseteq f^-(\otimes(x_a))\}$$

$$\bar{R}(\otimes(x_a)) = \bigcup\{\otimes(s) \in U \mid f_a^+(\otimes(s)) \cap f_a^+(\otimes(x)) \neq \phi\},$$

$$\otimes(s) \to \otimes(x) \Leftrightarrow \forall a \in A, f^+(\otimes(s_a)) \subseteq f^+(\otimes(x_a)),$$

$$and f^-(\otimes(s_a)) \supseteq f^-(\otimes(x_a)).$$

Where $\otimes(s)$, $\otimes(x)$ denote grey objects, $\otimes f(\otimes(s_a))$, $\otimes f(\otimes(x_a))$ denote the value of grey objects $\otimes(s)$, $\otimes(x)$ in terms of attribute a , $\underline{R}(\otimes(x_a))$, $\bar{R}(\otimes(x_a))$ is called the lower approximation and the upper approximation of the interval-valued grey set $\otimes(x)$ about the approximation space $(U, R^{\cap})$ with respect to the attribute a , respectively.

Property 2: Let (U, A, F) be an interval-valued grey information system, from the definition above, we have that:

$$\underline{R}(\otimes(x)) = \bigcap_{a\in A} \underline{R}(\otimes(x_a)),\ \bar{R}(\otimes(x)) = \bigcap_{a\in A} \bar{R}(\otimes(x_a)),$$

and if $\exists a \in A, \underline{R}(\otimes(x_a)) = \phi$, then $\underline{R}(\otimes(x)) = \phi$, especially.

Lemma: Given two interval-valued grey sets $\underline{R}(\otimes(x_a)|_{x_a^-}^{x_a^+})$ and $\underline{R}(\otimes(y_a)|_{y_a^-}^{y_a^+})$, then:

1. If $x_a^+ = y_a^+$ and $x_a^- \supseteq y_a^-$,

   then $\underline{R}(\otimes(x_a)|_{x_a^-}^{x_a^+}) \subseteq \underline{R}(\otimes(y_a)|_{y_a^-}^{y_a^+})$,

   $\bar{R}(\otimes(x_a)|_{x_a^-}^{x_a^+}) = \bar{R}(\otimes(y_a)|_{y_a^-}^{y_a^+})$.

2. If $x_a^+ \supseteq y_a^+$ and $x_a^- = y_a^-$,

   then $\underline{R}(\otimes(x_a)|_{x_a^-}^{x_a^+}) \supseteq \underline{R}(\otimes(y_a)|_{y_a^-}^{y_a^+})$,

   $\bar{R}(\otimes(x_a)|_{x_a^-}^{x_a^+}) \supseteq \bar{R}(\otimes(y_a)|_{y_a^-}^{y_a^+})$.

3. If $x_a^+ \supseteq y_a^+$ and $x_a^- \subseteq y_a^-$,

   then $\underline{R}(\otimes(x_a)|_{x_a^-}^{x_a^+}) \supseteq \underline{R}(\otimes(y_a)|_{y_a^-}^{y_a^+})$,

   $\bar{R}(\otimes(x_a)|_{x_a^-}^{x_a^+}) \supseteq \bar{R}(\otimes(y_a)|_{y_a^-}^{y_a^+})$.

4. If $x_a^+ \supseteq y_a^+$ and $x_a^- \supseteq y_a^-$,

   then $\bar{R}(\otimes(x_a)|_{x_a^-}^{x_a^+}) \supseteq \bar{R}(\otimes(y_a)|_{y_a^-}^{y_a^+})$.

Proof: It is easy to prove this conclusion by the definition.

Property 3: Let (U, A, G) be an interval-valued grey information system, we have that:

1. $\underline{R}(\otimes(x_a)) \subseteq \bar{R}(\otimes(x_a))$.
2. If $\otimes(x_a) = \Omega$, then $\underline{R}(\otimes(x_a)) = \bar{R}(\otimes(x_a)) = U$,

   if $\otimes(x_a) = \phi$, then $\underline{R}(\otimes(x_a)) = \bar{R}(\otimes(x_a)) = \phi$.
3. If $\otimes(x_a) \subseteq \otimes(y_a)$, then $\underline{R}(\otimes(x_a)) \subseteq \underline{R}(\otimes(y_a))$, $\bar{R}(\otimes(x_a)) \subseteq \bar{R}(\otimes(y_a))$.
4. If $B \subseteq A$, then $\underline{R}(\otimes(x_B)) \supseteq \underline{R}(\otimes(x_A))$.
5. If $\otimes(x) \to \otimes(y) \to \otimes(z)$,

   then $\underline{R}(\otimes(x)) \subseteq \underline{R}(\otimes(y)) \subseteq \underline{R}(\otimes(z))$.
6. If $y \in \underline{R}(\otimes(x))$, then $\underline{R}(\otimes(y)) \subseteq \underline{R}(\otimes(x))$,

   $\bar{R}(\otimes(y)) \subseteq \bar{R}(\otimes(x))$.

Proof: They are straightforward.

Property 4: Let $(U, R^{\cap})$ be a Pawlak approximation space, then the following properties hold for any interval-valued grey sets $\otimes(x)$ and $\otimes(y)$ of the universe U :

1. $\underline{R}((\otimes(x_a)) \cap (\otimes(y_a))) \supseteq \underline{R}((\otimes(x_a))) \cap \underline{R}((\otimes(y_a)))$.
2. $\underline{R}((\otimes(x_a)) \cup (\otimes(y_a))) \supseteq \underline{R}((\otimes(x_a))) \cup \underline{R}((\otimes(y_a)))$.
3. $\bar{R}((\otimes(x_a)) \cap (\otimes(y_a))) = \bar{R}((\otimes(x_a))) \cap \bar{R}((\otimes(y_a)))$.
4. $\bar{R}((\otimes(x_a)) \cup (\otimes(y_a))) = \bar{R}((\otimes(x_a))) \cup \bar{R}((\otimes(y_a)))$.

Where $\underline{R}(\otimes(x_a))$, $\bar{R}(\otimes(x_a))$ denote the lower approximation and the upper approximation of the interval-valued grey set $\otimes(x)$ with respect to the attribute a , and $f^+(\otimes(x_a))$, $f^-(\otimes(x_a))$ denote the upper bound and the lower bound of the interval-valued grey set $\otimes(x)$ in terms of the attribute a , respectively.

Proof:

1. $\underline{R}((\otimes(x_a)) \cap (\otimes(y_a)))$

$$= \bigcup\{\otimes(s) \mid f^+(\otimes(s_a)) \subseteq f^+(\otimes(x_a)) \cap f^+(\otimes(y_a))$$

$$\text{and } f^-(\otimes(s_a)) \supseteq f^-(\otimes(x_a)) \cap f^-(\otimes(y_a))\}.$$

Note that

$$\bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \subseteq f^{+}(\otimes(y_a)) \cap f^{+}(\otimes(x_a))$$

$$\text{and } f^{-}(\otimes(s_a)) \supseteq f^{-}(\otimes(y_a)) \cap f^{-}(\otimes(x_a))\}$$

$$\supseteq \{\bigcup \otimes(s) \mid f^{+}(\otimes(s_a)) \subseteq f^{+}(\otimes(x_a)) \text{ and}$$

$$f^{-}(\otimes(s_a)) \supseteq f^{-}(\otimes(x_a))\}$$

$$\cap\{\bigcup \otimes(s) \mid f^{+}(\otimes(s_a)) \subseteq f^{+}(\otimes(y_a)) \text{ and}$$

$$f^{-}(\otimes(s_a)) \supseteq f^{-}(\otimes(y_a))\},$$

$$= \underline{R}((\otimes(x_a))) \cap \underline{R}((\otimes(y_a))).$$

That is,

$$\underline{R}((\otimes(x_a)) \cap (\otimes(y_a))) \supseteq \underline{R}((\otimes(x_a))) \cap \underline{R}((\otimes(y_a))).$$

2. $\underline{R}(f(\otimes(x_a)) \cup f(\otimes(y_a)))$

$$= \bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \subseteq f^{+}(\otimes(y_a)) \cup f^{+}(\otimes(x_a))$$

$$\text{and } f^{-}(\otimes(s_a)) \supseteq f^{-}(\otimes(y_a) \cup f^{-}(\otimes(x_a))\}$$

Note that

$$\bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \subseteq f^{+}(\otimes(y_a)) \cup f^{+}(\otimes(x_a))$$

$$\text{and } f^{-}(\otimes(s_a)) \supseteq f^{-}(\otimes(y_a)) \cup f^{-}(\otimes(x_a))\}$$

$$\supseteq \{\bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \subseteq f^{+}(\otimes(y_a)) \text{ and}$$

$$f^{-}(\otimes(s_a)) \supseteq f^{-}(\otimes(y_a))\}\} \cup$$

$$\{\bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \subseteq f^{+}(\otimes(x_a)) \text{ and}$$

$$f^{-}(\otimes(s_a)) \supseteq f^{-}(\otimes(x_a))\}\}$$

$$= \underline{R}(f(\otimes(x_a))) \cup \underline{R}(f(\otimes(y_a)))$$

That is,

$$\underline{R}((\otimes(x_a)) \cup (\otimes(y_a))) \supseteq \underline{R}((\otimes(x_a))) \cup \underline{R}((\otimes(y_a))).$$

3. $\bar{R}((\otimes(x_a)) \cap (\otimes(y_a)))$

$$= \bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \cap [f(\otimes(x_a)) \cap f(\otimes(y_a))]^{+} \neq \phi\}$$

$$= \bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \cap f^{+}(\otimes(x_a)) \cap f^{+}(\otimes(y_a)) \neq \phi\}$$

$$= \bigcup\{\otimes(s) \mid f^{+}(\otimes s_a) \cap f^{+}(\otimes(x_a)) \neq \phi \text{ and}$$

$$f^{+}(\otimes(s_a)) \cap f^{+}(\otimes(y_a)) \neq \phi\}$$

$$= \{\bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \cap f^{+}(\otimes(x_a)) \neq \phi\}\} \cap$$

$$\{\bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \cap f^{+}(\otimes(y_a)) \neq \phi\}\}$$

$$= \bar{R}((\otimes(x_a))) \cap \bar{R}((\otimes(y_a)))$$

4. $\bar{R}((\otimes(x_a)) \cup (\otimes(y_a)))$

$$= \bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \cap [f(\otimes(x_a)) \cup f(\otimes(y_a))]^{+} \neq \phi\}$$
$$= \bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \cap [f^{+}(\otimes(x_a)) \cup f^{+}(\otimes(y_a))] \neq \phi\}$$
$$= \bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \cap f^{+}(\otimes(x_a)) \neq \phi \cup$$

$$f^{+}(\otimes(s_a)) \cap f^{+}(\otimes(y_a)) \neq \phi\}$$

$$= \{\bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \cap f^{+}(\otimes(x_a)) \neq \phi\}\} \cup$$

$$\{\bigcup\{\otimes(s) \mid f^{+}(\otimes(s_a)) \cap f^{+}(\otimes(y_a)) \neq \phi\}\}$$

$$= \bar{R}((\otimes(x_a))) \cup \bar{R}((\otimes(y_a)))$$

The proof is complete.

Example 2 :(continue example 1) Compute the lower approximation and the upper approximation of the interval-valued grey objects $\otimes x_1, \otimes x_2$:

$$\underline{R}(\otimes(x_{1_{a_1}})) = \{\otimes(x_1), \otimes(x_3)\},$$

$$\underline{R}(\otimes(x_{1_{a_2}})) = \{\otimes(x_1)\},$$

$$\underline{R}(\otimes(x_{1_{a_3}})) = \{\otimes(x_1), \otimes(x_3), \otimes(x_4)\},$$

$$\bar{R}(\otimes(x_{1_{a_1}})) = \{\otimes(x_1), \otimes(x_3), \otimes(x_4), \otimes(x_5)\},$$

$$\bar{R}(\otimes(x_{1_{a2}})) = \{\otimes(x_1), \otimes(x_3), \otimes(x_4), \otimes(x_5)\},$$

$$\bar{R}(\otimes(x_{1_{a_3}})) = \{\otimes(x_1), \otimes(x_3), \otimes(x_4)\},$$

$$\underline{R}(\otimes(x_{2_{a_1}})) = \{\otimes(x_2), \otimes(x_6), \otimes(x_7)\},$$

$$\underline{R}(\otimes(x_{2_{a_2}})) = \{\otimes(x_2), \otimes(x_6), \otimes(x_8)\},$$

$$\underline{R}(\otimes(x_{2_{a_3}})) = \{\otimes(x_2), \otimes(x_6), \otimes(x_8)\},$$

$$\bar{R}(\otimes(x_{2_{a_1}})) = \{\otimes(x_2), \otimes(x_5), \otimes(x_6), \otimes(x_7), \otimes(x_8)\},$$

$$\bar{R}(\otimes(x_{2_{a_2}})) = \{\otimes(x_2), \otimes(x_5), \otimes(x_6), \otimes(x_8)\},$$

$$\bar{R}(\otimes(x_{2_{a_3}})) = \{\otimes(x_2), \otimes(x_5), \otimes(x_6), \otimes(x_7), \otimes(x_8)\},$$

Then, $\underline{R}(\otimes(x_{1_A})) = \{\otimes(x_1)\}$,

$\bar{R}(\otimes(x_{1_A})) = \{\otimes(x_1), \otimes(x_3), \otimes(x_4)\}$,

$\underline{R}(\otimes(x_{2_A})) = \{\otimes(x_2), \otimes(x_6)\}$,

$\bar{R}(\otimes(x_{2_A})) = \{\otimes(x_2), \otimes(x_5), \otimes(x_6), \otimes(x_8)\}$.

Similarly, we can get all the objects as follows:

$\underline{R}(\otimes(x_{3_A})) = \{\otimes(x_3)\}$,

$\bar{R}(\otimes(x_{3_A})) = \{\otimes(x_1), \otimes(x_3), \otimes(x_4), \otimes(x_5)\}$,

$\underline{R}(\otimes(x_{4_A})) = \{\otimes(x_4)\}$,

$\bar{R}(\otimes(x_{4_A})) = \{\otimes(x_1), \otimes(x_3), \otimes(x_4), \otimes(x_5)\}$,

$\underline{R}(\otimes(x_{5_A})) = \{\otimes(x_5)\}$,

$\bar{R}(\otimes(x_{5_A})) = \{U\}$,

$\underline{R}(\otimes(x_{6_A})) = \{\otimes(x_2), \otimes(x_6)\}$,

$\bar{R}(\otimes(x_{6_A})) = \{\otimes(x_2), \otimes(x_5), \otimes(x_6), \otimes(x_8)\}$,

$\underline{R}(\otimes(x_{7_A})) = \{\otimes(x_7)\}$,

$\bar{R}(\otimes(x_{7_A})) = \{\otimes(x_5), \otimes(x_7)\}$,

$\bar{R}(\otimes(x_{8_A})) = \{\otimes(x_2), \otimes(x_5), \otimes(x_6), \otimes(x_8)\}$.

IV. SIMILARITY DEGREE AND PROPERTIES IN THE INTERVAL-VALUED GREY SET INFORMATION SYSTEM

Inspired by the way of fuzzy clustering in fuzzy set, we introduce a novel similarity degree to measure the similarity of two objects. Further more, we point out that the new similarity degree we propose in this paper is meeting with the axiomatic definition of similarity degree. We first give the axiomatic definition of similarity degree as follows.

Definition 5: Let $\gamma : (x, y) \to \gamma(x, y)$ be the binary function on the U, and it satisfy that:

1. $\gamma(x, x) = 1$, $\gamma(x, \phi) = 0$.

2. $\gamma(x, y) = \gamma(y, x)$.

3. $x \to y \to z \Rightarrow \gamma(x, z) \le \min\{\gamma(x, y), \gamma(y, z)\}$.

Then we call it similarity degree.

Definition 6: Given interval-valued grey objects $\otimes(x_1)$, $\otimes(x_2)$ of the U, and $(\underline{R}(\otimes(x)), \bar{R}(\otimes(x)))$, $(\underline{R}(\otimes(y)), \bar{R}(\otimes(y)))$, then we define similarity degree as follows:

$$S(\otimes(x), \otimes(y)) = \alpha \times \frac{|\underline{R}(\otimes(x)) \cap \underline{R}(\otimes(y))|}{|\underline{R}(\otimes(x)) \cup \underline{R}(\otimes(y))|} + \beta \times \frac{|\bar{R}(\otimes(x)) \cap \bar{R}(\otimes(y))|}{|\bar{R}(\otimes(x)) \cup \bar{R}(\otimes(y))|}$$

Where, $\alpha + \beta = 1$ and $|*|$ denotes the cardinality of a set.

Especially, $\underline{S} = \frac{|\underline{R}(\otimes(x)) \cap \underline{R}(\otimes(y))|}{|\underline{R}(\otimes(x)) \cup \underline{R}(\otimes(y))|}$ is called the similarity degree of lower approximation, and $\bar{S} = \frac{|\bar{R}(\otimes(x)) \cap \bar{R}(\otimes(y))|}{|\bar{R}(\otimes(x)) \cup \bar{R}(\otimes(y))|}$ is called the similarity degree of upper approximation.

From the definition above, we have that:

1. $S(\otimes(x), \otimes(x)) = 1$, $S(\otimes(x), \phi) = 0$,

2. $S(\otimes(x), \otimes(y)) = S(\otimes(y), \otimes(x))$,

3. If $\otimes(x) \to \otimes(y) \to \otimes(z)$,

then $S(\otimes(x), \otimes(z)) \le \min\{S(\otimes(x), \otimes(y)), S(\otimes(y), \otimes(z))\}$.

Proof: 1, 2 is straightforward.

1. If $\otimes(x) \to \otimes(y) \to \otimes(z)$, then
2. $\underline{R}(\otimes(x)) \subseteq \underline{R}(\otimes(y)) \subseteq \underline{R}(\otimes(z))$, and

$$|\underline{R}(\otimes(x))| \le |\underline{R}(\otimes(y))| \le |\underline{R}(\otimes(z))|$$
$$|\bar{R}(\otimes(x))| \le |\bar{R}(\otimes(y))| \le |\bar{R}(\otimes(z))|,$$

Further more,

$$S(\otimes(x), \otimes(y)) = \alpha \times \frac{|\underline{R}(\otimes(x)) \cap \underline{R}(\otimes(y))|}{|\underline{R}(\otimes(x)) \cup \underline{R}(\otimes(y))|} + \beta \times \frac{|\bar{R}(\otimes(x)) \cap \bar{R}(\otimes(y))|}{|\bar{R}(\otimes(x)) \cup \bar{R}(\otimes(y))|}$$
$$= \alpha \times \frac{|\underline{R}(\otimes(x))|}{|\underline{R}(\otimes(y))|} + \beta \times \frac{|\bar{R}(\otimes(x))|}{|\bar{R}(\otimes(y))|}$$

$$S(\otimes(y), \otimes(z)) = \alpha \times \frac{|\underline{R}(\otimes(y)) \cap \underline{R}(\otimes(z))|}{|\underline{R}(\otimes(y)) \cup \underline{R}(\otimes(z))|}$$

$$+\beta\times\frac{|\bar{R}(\otimes(y))\cap\bar{R}(\otimes(z))|}{|\bar{R}(\otimes(y))\cup\bar{R}(\otimes(z))|}$$

$$=\alpha\times\frac{|\underline{R}(\otimes(y))|}{|\underline{R}(\otimes(z))|}+\beta\times\frac{|\bar{R}(\otimes(y))|}{|\bar{R}(\otimes(z))|}$$

$$S(\otimes(x),\otimes(z))=\alpha\times\frac{|\underline{R}(\otimes(x))\cap\underline{R}(\otimes(z))|}{|\underline{R}(\otimes(x))\cup\underline{R}(\otimes(z))|},$$

$$+\beta\times\frac{|\bar{R}(\otimes(x))\cap\bar{R}(\otimes(z))|}{|\bar{R}(\otimes(x))\cup\bar{R}(\otimes(z))|}$$

$$=\alpha\times\frac{|\underline{R}(\otimes(x))|}{|\underline{R}(\otimes(z))|}+\beta\times\frac{|\bar{R}(\otimes(x))|}{|\bar{R}(\otimes(z))|}$$

So we have $S(\otimes(x),\otimes(\mathrm{z}))\le S(\otimes(x),\otimes(y))$, and $S(\otimes(x),\otimes(\mathrm{z}))\le S(\otimes(y),\otimes(z))$,

That is

$S(\otimes(x),\otimes(\mathrm{z}))\le\min\{S(\otimes(x),\otimes(y)),S(\otimes(\mathrm{y}),\otimes(\mathrm{z}))\}$,

From above, we can conclude safely that the definition of similarity degree is reasonable.

Property 5 : If $\underline{R}(\otimes(x))=\underline{R}(\otimes(y))$ and $\bar{R}(\otimes(x))=\bar{R}(\otimes(y))$, then $S(\otimes(x),\otimes(y))=1$.

Especially, when $y\in\underline{R}(\otimes(x))$,

$$S(\otimes(x),\otimes(y))=\alpha\times\frac{|\underline{R}(\otimes(y))|}{|\underline{R}(\otimes(x))|}+\beta\times\frac{|\bar{R}(\otimes(y))|}{|\bar{R}(\otimes(x))|}.$$

Example 3: (continue example 1) Let $\alpha=0.5$, then similarity degree of each object can be calculated according to the definition as follows:

$$S(\otimes(x_1),\otimes(x_2))=0.5\times\frac{|\underline{R}(\otimes(x_1))\cap\underline{R}(\otimes(x_2))|}{|\underline{R}(\otimes(x_1))\cup\underline{R}(\otimes(x_2))|}$$

$$+0.5\times\frac{|\bar{R}(\otimes(x_1))\cap\bar{R}(\otimes(x_2))|}{|\bar{R}(\otimes(x_1))\cup\bar{R}(\otimes(x_2))|}$$

$$=0.5\times\frac{|\{\otimes(x_1),\otimes(x_3),\otimes(x_4),\otimes(x_5)\}\cap\{\otimes(x_2),\otimes(x_5),\otimes(x_6),\otimes(x_8)\}|}{|\{\otimes(x_1),\otimes(x_3),\otimes(x_4),\otimes(x_5)\}\cup\{\otimes(x_2),\otimes(x_5),\otimes(x_6),\otimes(x_8)\}|}$$

$=1/14$

In a similar way, we can also get all the objects, and construct a similarity degree matrix (see table II).

TABLE II. SIMILARITY DEGREE MATRIX

S	$\otimes(x_1)$	$\otimes(x_2)$	$\otimes(x_3)$	$\otimes(x_4)$	$\otimes(x_5)$	$\otimes(x_6)$	$\otimes(x_7)$	$\otimes(x_8)$
$\otimes(x_1)$	1	1/14	1/2	1/2	1/4	1/14	1/10	1/14
$\otimes(x_2)$		1	1/14	1/14	1/4	1	1/10	1/2
$\otimes(x_3)$			1	1/2	1/4	1/14	1/10	1/14
$\otimes(x_4)$				1	1/4	1/14	1/10	1/14
$\otimes(x_5)$					1	1/4	1/8	1/4
$\otimes(x_6)$						1	1/10	1/2
$\otimes(x_7)$							1	1/10
$\otimes(x_8)$								1

From the table above, if let $\lambda=0.5$, then the universe can be clustered into four classes:

$\{\otimes(x_1),\otimes(x_3),\otimes(x_4)\}$, $\{\otimes(x_2),\otimes(x_6),\otimes(x_8)\}$,

$\{\otimes(x_5)\}$, $\{\otimes(x_7)\}$.

V. CONCLUSION

Rough sets and grey sets theory are two mathematical tools to deal with uncertainty. Combining them together is of both theoretical and practical importance. This paper combines the interval-valued grey sets with the rough sets, proposes a novel grey rough set model for interval-valued grey sets data named interval-valued grey-rough set and studies the basic theory of the interval-valued rough grey sets. Based on the generalized interval-valued rough grey sets, the clustering of the interval-valued grey information systems is also studied. Our future work will concentrate on the interval-valued grey rough set model in the interval-valued grey information systems. An application of the model which is presented in this paper will also be researched in the future.

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