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# Fairness Constrained Optimization of Channel Allocation for Open Spectrum Networks

Tao Zhang
Department of Computer Science and Engineering
Wright State University
Dayton, USA
zhang.66@wright.edu

Bin Wang
Department of Computer Science and Engineering
Wright State University
Dayton, USA
bin.wang@wright.edu

Zhiqiang Wu
Department of Electrical Engineering
Wright State University
Dayton, USA
zhiqiang.wu@wright.edu

***Abstract*—Channel allocation is an important area of research in open spectrum networks which asserts a significant impact on the spectrum utilization and the fairness among users. This paper studies the optimization of channel allocation, considering multiple objectives. For each objective, a binary programming model is described. Then a new optimization objective called fairness constrained maximum throughput is proposed. To achieve this optimization objective, a unified binary linear programming (UBLP) model is constructed which is then solved by the simplex method and branch-and-bound search. The solution to this model satisfies a bandwidth requirement for each user, e.g., the bandwidth for each user is equal to or larger than a per-user bandwidth minimum, and the solution also maximizes the network throughput. We prove that given different per-user bandwidth minimum, the optimal solution to the UBLP model achieves specific optimization objectives, such as the maximum network throughput and the max-min fairness. For the proportional fairness objective, the solution to the UBLP model proves to be within a bound of the optimal solution.**

***Index Terms*—open spectrum networks, cognitive radio, dynamic spectrum access, binary linear programming**

## I. Introduction

The right of use to radio spectrum is coordinated by national agencies. In the U.S. the main authority for radio spectrum regulation is the Federal Communications Commission (FCC) [1] for commercial applications. Spectrum segments are licensed to particular users for exclusive use in specific geographic areas. This rigid control of spectrum has resulted in severe spectrum under-utilization, and as a result, spectrum congestion. A few small unlicensed bands are left open for anyone to use as long as certain power regulations are followed. With the recent advances in wireless technologies, these unlicensed bands have become crowded with everything from wireless networks to digital cordless phones.

Open spectrum access is construed as a powerful concept to increase the efficiency of the radio spectrum. In open spectrum networks, dynamic spectrum access (DSA) techniques [2, 3] have been proposed to combat the inefficient spectrum utilization problem. DSA is enabled by the cognitive radio technology [4, 5]. Advances in this technology provide the capability for a radio device to sense and operate on a wide range of frequencies using appropriate communication mechanisms, and thus enable dynamic and more intense spectrum reuse in space, time, and frequency dimensions.

Intelligent channel allocation is, therefore, essential in open spectrum networks. A weak channel allocation scheme decreases the performance of the network and results in low spectrum utilization. Among others, graph coloring based approaches for channel allocation optimization have been proposed, such as the one presented in [6]. Three optimization functions for channel allocation which correspond to different quality demands in different network scenarios were introduced in [6, 7]. These demands reflect the specific requirements of different network scenarios, such as maximizing the total throughput of the network, maximizing the channel utilization at the bottleneck user, or maximizing the throughput of the network and fairness.

In this work, different binary programming models are constructed for channel allocation optimization while considering different optimization objectives. Binary programming is a special case of integer programming in which all variables are required to take on 0 or 1 only. In addition, a new optimization objective called fairness constrained maximum throughput is proposed. To achieve this optimization objective, a binary linear programming (UBLP) model is constructed which is then solved by the simplex method and branch-and-bound search. This paper makes three main contributions:

1) The mapping between the channel allocation problems and binary programming models, in terms of different optimization objectives, is described;
2) A new optimization objective called fairness constrained maximum throughput is presented. Based on a unified binary linear programming (UBLP) model and a proposed iterative algorithm, different optimal solutions for specific optimization objectives can be obtained;
3) It is proved that with different per-user bandwidth minimums in the UBLP model, the optimal solutions to the channel allocation problem with the maximum network throughput and max-min fairness objectives can be determined. It is also proved that for the objective of proportional fairness, the difference between the optimal solution in the channel allocation problem and the solution to the UBLP model is bounded.

The rest of the paper is organized as follows. Section II introduces three common optimization objectives for channel



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allocation. In Section III, the binary programming modeling of the channel allocation problem with different optimization objectives is described. A unified binary linear programming model and a corresponding solution algorithm are proposed in Section IV. We then demonstrate the relationship between the solution to the UBLP model and the optimal solutions to other binary programming models. In Section V, we report and compare the simulation results of the UBLP model and other binary programming models. Finally, conclusions are drawn and future work is summarized in Section VI.

## II. Preliminary

### A. Assumptions

The term, channel, is defined as a basic slice of spectrum allocated to users. It is assumed in open spectrum networks that a user may use any number of available channels simultaneously [8]. Channels are completely orthogonal so that two users will not interfere with each other if they use different channels. Moreover, a channel may be different from other channels in terms of bandwidth and transmission. Users are assumed to be able to sense available channels, evaluate the channel characteristics (e.g., bandwidth), and send the channel information to a central controller. The central controller makes decisions on channel allocation, and releases the allocation scheme to all users. We consider static users or users with low mobility in which spectrum sensing and channel allocation work on a relatively large time scale.

### B. Definitions

In an open spectrum network, let $\mathcal{C} = \{c_1, ..., c_M\}$ be a set comprising $M$ channels. Let $\mathcal{U} = \{1, ..., N\}$ be a set comprising $N$ users. Define an $N \times M$ binary matrix $X$. Set $x_{i,k} = 1$ if user $i$ is assigned the channel $c_k$, and set $x_{i,k} = 0$ otherwise. Let $\mathcal{F}$ be an $N \times N$ binary matrix, and set $f_{i,j} = 1$ if users $i$ and $j$ interfere with each other when they use the same channel, and set $f_{i,j} = 0$ otherwise.

Let the matrix $\mathcal{B} = \{b_{i,k}\}_{N \times M}$ represent the channel bandwidth, i.e., $b_{i,k}$ denotes the bandwidth that can be acquired by user $i$ using channel $c_k$. It is assumed that $b_{i,k} = 1, \forall i \in \mathcal{U}, \forall k \in \mathcal{C}$.

### C. Binary Linear Programming

Linear programs with a few thousand variables and constraints are now viewed as "small". Problems having tens or hundreds of thousands of continuous variables are regularly solved. Tractable integer linear programs are necessarily smaller, but are still commonly in the hundreds or thousands of variables and constraints.

A binary linear programming problem is a special kind of linear integer programming problem in which all of the variables are constrained to be 0 or 1 [9]. A binary linear programming problem can be first solved using the simplex method by relaxing the integer requirements. If binary solutions are found, the process does not need to proceed any further. If not, based on the current non-binary variables, branch-and-bound search [9] is used to partition the problem into mutually exclusive sub-problems. The branching of subproblems is continued until all subproblems have been explored. The efficient warm-start capability of the simplex method helps to solve the subsequent sub-problems very quickly.

Small-to-middle scale channel allocation problems can be modeled as binary linear programming problems if the objective function and the constraints are formulated linearly. In the next section the modeling of channel allocation problems is described in detail.

## III. Problem Modeling

The optimization objective of the channel allocation problem depends on the particular circumstances of networks. In this section three commonly used optimization objectives, maximum network throughput (MNT), max-min fairness (MMF), and proportional fairness (PF) are introduced.

Maximum network throughput is an easily understood optimization objective in open spectrum networks. Without considering the fairness, the problem is formulated as follows: Objective Function (MNT):

$$\max \sum_{i=1}^{|\mathcal{U}|} \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \tag{1}$$

Subject to:

$$x_{i,k} + x_{j,k} \leq 1, \text{ if } f_{i,j} = 1, \quad \forall i, j \in \mathcal{U}, \quad \forall k \in \mathcal{C} \tag{2}$$

$$x_{i,k} \in \{0, 1\}, \ \forall i \in \mathcal{U}, \forall k \in \mathcal{C} \tag{3}$$

Note that the objective function (1) is linear, constraints (2) prevent two users from using the same channel if they interfere with each other, and constraints (3) guarantee that the decision variables be 1 or 0 so that the formulation is a binary linear programming problem.

In many situations, the goal of optimization not only includes the network throughput maximization, but also a decent bandwidth minimum for every user. Max-min fairness [10] is a fairness criterion that the user with the smallest throughput gets the priority for channels. In this case, the definition of max-min fairness can be described as:

*Definition 3.1:* A channel allocation scheme $\bar{a}$ is max-min fair if and only if an increase in bandwidth $t_i$ of user $i$ within the domain of feasible allocations must be at the cost of a decrease of some user whose bandwidth is already smaller.

The bandwidth of user $i$ is given as follows:

$$t_i = \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k}, i \in \mathcal{U}, k \in \mathcal{C}.$$

That is, for any other feasible allocation of channels $\bar{b}$, if there is a user $i$ with $t'_i > t_i$, then there must exist some user $j$ such that $t_j \leq t'_i$ and $t'_j < t_j$.

The max-min fairness optimization problem is defined as follows:

Objective Function (MMF):

$$\max w \tag{4}$$

Subject to:

$$x_{i,k} + x_{j,k} \leq 1, \text{ if } f_{i,j} = 1, \quad \forall i, j \in \mathcal{U}, \quad \forall k \in \mathcal{C} \tag{5}$$

$$\sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \geq w, \ \forall i \in \mathcal{U} \tag{6}$$

$$x_{i,k} \in \{0, 1\}, \ \forall i \in \mathcal{U}, \forall k \in \mathcal{C} \tag{7}$$

Constraints (5) avoid the interference between adjacent users. The objective function and constraints (6) guarantee that the objective of optimization maximize the bandwidth of the bottleneck user.

Another very important fairness criterion in channel allocation is proportional fairness (PF) [11–13]. The definition of proportional fairness is given as follows:

*Definition 3.2:* Suppose $t_i$ is the bandwidth of user $i$ for a feasible channel allocation scheme $\bar{a}$, and $t'_i$ is the bandwidth of the same user for scheme $\bar{b}$. The channel allocation scheme $\bar{a}$ is proportional fair if and only if for any other feasible allocation scheme $\bar{b}$,

$$\sum_{i=1}^{|\mathcal{N}|} \frac{t'_i - t_i}{t_i} \leq 0.$$

Practically, the function used to reach a proportional fairness condition is [11–13]:

$$\sum_{i=1}^{|\mathcal{N}|} \ln \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k}. \tag{8}$$

Based on Eq. (8), the PF optimization model is formulated as:

Objective Function (PF):

$$\max \sum_{i=1}^{|\mathcal{N}|} \ln \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \tag{9}$$

Subject to:

$$x_{i,k} + x_{j,k} \leq 1, \text{ if } f_{i,j} = 1, \quad \forall i, j \in \mathcal{U}, \quad \forall k \in \mathcal{C} \tag{10}$$

$$x_{i,k} \in \{0, 1\}, \ \forall i \in \mathcal{U}, \forall k \in \mathcal{C} \tag{11}$$

As explained above, constraints (Eq. (10)) depict the interference relationships and Eq. (11) shows the binary constraints. Obviously the objective function (Eq. 9) of this model is nonlinear. Therefore, it is not a linear programming problem, rather a binary nonlinear programming problem. The objective function (Eq. (9)) is differentiable and strictly concave. In addition, constraints (Eq. (10)) are linear. Therefore, the optimization can in theory be solved by Lagrangian methods [14]. However, in practice a nonlinear programming problem is harder to solve than a linear programming problem. For large problems, it is unlikely that exact optimal solutions can be determined, or even a solution that is close to the optimal.

## IV. A Unified Model and Algorithm Based on Binary Linear Programming

In this section, we present a new optimization objective called fairness constrained maximum throughput (FCMT). A unified binary linear programming (UBLP) model is constructed based on this objective. We show that this model captures the optimization objectives of Section III. The UBLP model is illustrated as follows:

Objective Function (FCMT):

$$\max \sum_{i=1}^{|\mathcal{U}|} \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \tag{12}$$

Subject to:

$$x_{i,k} + x_{j,k} \leq 1, \text{ if } f_{i,j} = 1, \quad \forall i, j \in \mathcal{U}, \quad \forall k \in \mathcal{C} \tag{13}$$

$$\sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \geq \xi, \ \forall i \in \mathcal{U}, \forall k \in \mathcal{C} \tag{14}$$

$$x_{i,k} \in \{0, 1\}, \ \forall i \in \mathcal{U}, \forall k \in \mathcal{C} \tag{15}$$

In this model, we add additional constraints (Eq.(14)) and set $\xi$ as the bandwidth minimum for each user. In other words, this optimization objective maximizes the network throughput, given that the bandwidth of each user is larger than or equal to $\xi$ which is the per-user bandwidth minimum. The objective is formally called *fairness constrained maximum throughput.*

Based on the UBLP model, we propose an iterative algorithm (Algorithm1) to obtain solutions with different per-user bandwidth mininums. Given a network topology, a graph $G = (V, E)$, in terms of users (nodes) and interference relationships (edges) between users, can be constructed. Note that all the connected components [15] of $G$ should be determined before running the iterative algorithm. The algorithm is run separately on every connected component of $G$ because the channel allocation for each component is independent of others.

At the beginning of the algorithm, set the per-user bandwidth minimum $\xi = 0$, and then, call the binary linear programming function BLP($G_i$) to obtain an optimal solution. The algorithm then repeats the function calling while increasing the value of $\xi$ by 1 every time until the function BLP($G_i$) cannot obtain an optimal solution.

Suppose the per-user bandwidth minimum in a connected component $G_i$ of $G$ is $\delta$, e.g., $\xi = \delta$. The maximum bandwidth of a user is denoted by $t_{max}$, and assume that the number of total available channels is $M$.

*Lemma 1:* In each connected component $G_i$ of $G$, $t_{max}$ is $M - \delta$.

**Algorithm 1** *UBLP-ALG*($\mathcal{V}, \mathcal{K}$)

$G_i$ is the $i$th *connected component* of graph $G$
**for** every component $G_i$ of $G$ **do**
  $\xi = 0$;
  **while** TRUE **do**
    **if** (BLP($G_i$) cannot obtain a solution ) **then**
      stop;
    **else**
      save the result $r_\xi$ = BLP($G_i$);
      $\xi = \xi + 1$;
    **end if**;
  **end while**;
  **if** (the objective is MNT) **then**
    return $r_0$;
  **else**
    **if** (the objective is MMF) **then**
      return $r_{\xi-1}$;
    **else**
      **if** (the objective is PF) **then**
        return $r_j$ with the maximum value of Eq. (8), $j \in \{0 \cdots \xi - 1\}$;
      **end if**;
    **end if**;
  **end if**;
**end for**;

*Proof:* Because $\xi = \delta$, the degree of each node of $G_i$ is at least 1, and $b_{i,k} = 1, \forall i \in \mathcal{U}, \forall k \in \mathcal{C}$, the maximum number of channels that one user can use is $M - \delta$, thus,

$$\begin{aligned} t_{max} &= \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \\ &= M - \delta. \end{aligned}$$

■

*Lemma 2:* In each connected component $G_i$ of $G$, $M \geq 2\delta$.

*Proof:* Apparently $t_{max} \geq \xi$, since $\xi = \delta$ and $t_{max} = M - \delta$, we have $M \geq 2\delta$. ■

In connected component $G_i$ of $G$, suppose the optimization objective is PF, let $v_{UBLP}$ denote the value of Eq. (8) obtained from the optimal solution to the UBLP model and let $v_{PF}$ denote the corresponding value of Eq. (8) obtained from the PF model. Denote the difference between the two values by

$$\text{diff} = v_{PF} - v_{UBLP}. \tag{16}$$

*Lemma 3:* In connected component $G_i$ of $G$, if the optimization objective is PF, the lower bound of diff (Eq. (16)) is 0.

*Proof:* Because the solution of the PF model has the optimal value, the value of Eq. (8) obtained from the UBLP model must be equal to or less than that of the PF model. Thus the lower bound of the difference between the above two values is 0. ■

In connected component $G_i$ of $G$, suppose the number of users is $n_i$ and the per-user bandwidth minimum for the optimal solution to the PF model is $\delta$, the following lemma holds.

*Lemma 4:* In connected component $G_i$ of $G$, the upper bound of the difference (diff (Eq. 16)) between the value (Eq. (8)) obtained by the UBLP model with $\xi = \delta$ and that (Eq. (8)) of the PF model is $\ln\left(\frac{M-\delta}{\delta}\right)^{n_i - 1}$.

*Proof:* Because the minimum bandwidth of a user is $\delta$ for both solutions, and given Lemmas 1 and 2, the maximum value of the two solutions is:

$$\ln(\delta \times (t_{max})^{n_i - 1}) \tag{17}$$

and the minimum value of the two solutions is:

$$\ln \delta^{n_i}. \tag{18}$$

On the one hand, due to the PF model, the optimal value (Eq. (8)) obtained from the PF model is definitely larger than or equal to the corresponding value obtained by the UBLP model. On the other hand, again due to the PF model, the value (Eq. (8)) obtained by the UBLP model cannot be Eq. (18) if the value (Eq. (8)) by the PF model is Eq. (17). Therefore, the upper bound of the difference, diff, is:

$$\begin{aligned} \text{diff} &< \ln(\delta(t_{max})^{n_i - 1}) - \ln \delta^{n_i} \\ &< \ln\left(\frac{\delta(t_{max})^{n_i-1}}{\delta^{n_i}}\right) \\ &< \ln\left(\frac{(M-\delta)}{\delta}\right)^{n_i - 1} \\ &= \text{diff}_{sup}. \end{aligned}$$

■

In summary, we have the following propositions:

*Proposition 1: An optimal solution $\bar{a}$ to the UBLP model with $\xi = 0$ is also an optimal solution to the MNT model.*

*Proof:* If $\xi = 0$, constraints of Eq. (14) of the UBLP model become

$$\sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \geq 0, \ \forall i \in \mathcal{U}, \forall k \in \mathcal{C}.$$

Because $\forall i \in \mathcal{U}, \forall k \in \mathcal{C}, x_{i,k} \geq 0$ and $b_{i,k} \geq 0$, constraints of Eq. (14) become unnecessary. Removing Eq. (14), the UBLP model is the same as the MNT model. Therefore, the solution $\bar{a}$ is also an optimal solution to the MNT model. ■

*Proposition 2: An optimal solution $\bar{a}$ to the UBLP model with the maximum value of $\xi$ is also an optimal solution to the MMF model.*

*Proof:* Set the maximum value of $\xi$ as $\delta_{max}$ which is less than or equal to $t_{max}$. Suppose the solution $\bar{a}$ of the UBLP model with $\xi = \delta_{max}$ is not one of the solutions to the MMF model, which means that there must have a solution $\bar{b}$ to the MMF model in which the per-user bandwidth minimum is at least $\delta_{max}+1$. However, according to Algorithm 1, function BLP($G_i$) cannot obtain a solution given the constraints

$$\sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \geq \delta_{max}+1, \ \forall i \in \mathcal{U}, \forall k \in \mathcal{C}.$$

Therefore, $\bar{b}$ does not exist and $\bar{a}$ is also an optimal solution to the MMF model. ■

*Proposition 3: For connected component $G_i$ of $G$,* diff *(Eq. (16)) is bounded by:*

$$0 \leq \text{diff} < \ln(M-1)^{n_i-1},$$

where $n_i$ is the number of users in $G_i$.

*Proof:* According to Lemma 3, the lower bound is, diff $\geq 0$. Now we prove the upper bound. Assume $n_i \geq 2$. The derivative of $\text{diff}_{sup}$ in Lemma 4 is:

$$\begin{aligned} {\text{diff}_{sup}}' &= \left(\ln(\frac{(M-\delta)}{\delta})^{n_i-1}\right)' \\ &= (n_i-1) \times \left(\frac{\delta}{M-\delta}\right) \times \left(-\frac{M}{\delta^2}\right). \end{aligned}$$

Note that

$$-\frac{M}{\delta^2} < 0,$$

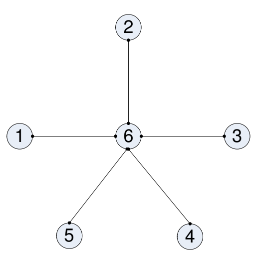
and $M > \delta$, $\text{diff}_{sup}$ is a monotonically decreasing function of $\delta$. Hence when $\delta = 1$, the upper bound is:

$$\text{diff} < \ln(M-1)^{n_i-1}.$$

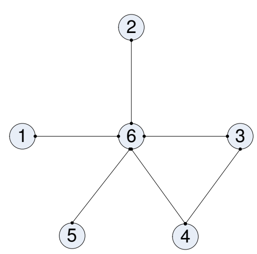
■

## V. Simulation Evaluation and Discussions

To demonstrate the effectiveness of the UBLP model, simulation studies are performed under the assumptions that all users are static. All available channels are also assumed to have the same bandwidth. Two fixed topologies in Figure 1 are used in the studies. In Figure 1, a node represents a user and an edge represents the interference relationship between two users. In the two topologies, there are 6 users and the total number of available channels is set to 5.



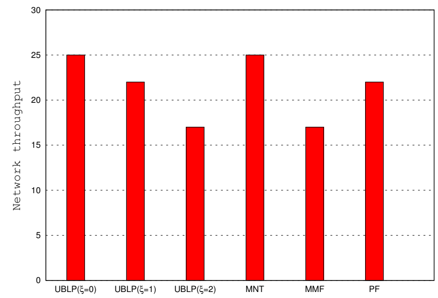

(a) Topology 1



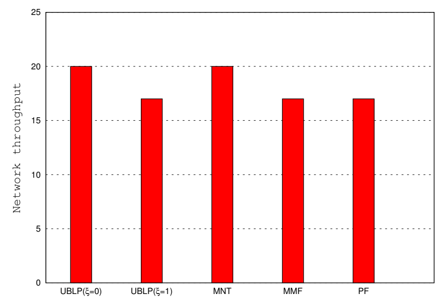

(b) Topology 2

Fig. 1. Two network topologies used in studies.

For each topology, we run the iterative algorithm (Algorithm 1) with different values of $\xi$. In Figure 2, the network throughput decreases with increasing $\xi$ in both cases. The

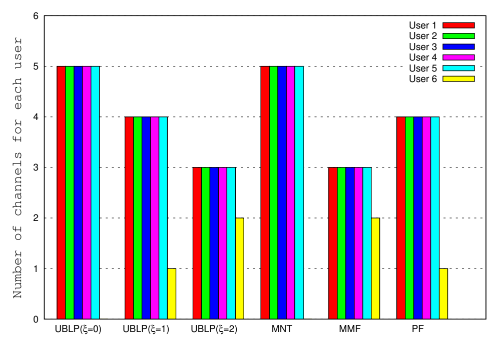



(a) Topology 1

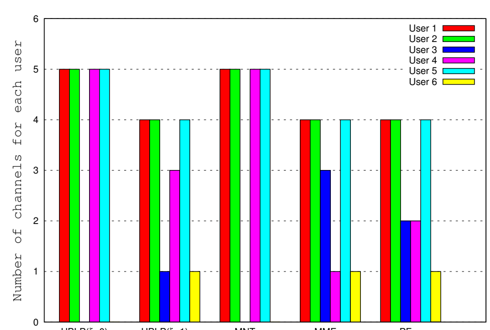



(b) Topology 2

Fig. 2. Network throughput.




(a) Topology 1




(b) Topology 2

Fig. 3. Number of channels for each user.

| Proportional Fairness | | |
|---|---|---|
| Topology | Model | Value |
| Topology 1 | UBLP($\xi = 0$) | $-\infty$ |
| | UBLP($\xi = 1$) | 6.931 |
| | UBLP($\xi = 2$) | 6.186 |
| | PF | 6.931 |
| Topology 2 | UBLP($\xi = 0$) | $-\infty$ |
| | UBLP($\xi = 1$) | 5.257 |
| | PF | 5.545 |

TABLE I
COMPARISON OF PROPORTIONAL FAIRNESS.

total network throughput with $\xi = 0$ is equal to the optimal value of the MNT model. In Figure 3(a), the solution to the UBLP model with the maximum $\xi$ is the same as the solution to the MMF model, and in Figure 3(b), the solution to the UBLP model with the maximum $\xi$ satisfies the optimization constraints of the MMF model although it is not entirely the same as the solution to the MMF model.

Figure 3 also shows an obvious fact that the essence of max-min fairness is to decrease the difference of bandwidth between any two users. In Table I, the values of logarithmic utility function Eq. (8) for the two topologies are presented. The value of the logarithmic utility function of the UBLP model for proportional fairness and the PF model are exactly the same in Figure 1(a). However, in some scenarios, for instance in Figure 1(b), the value of the logarithmic utility function of the UBLP model for proportional fairness is smaller (or less fair) than the one obtained from the PF model. In this case, the value of of the PF model is $5.545$ while the value is $5.257$ under the UBLP model.

Just as it has been proved in the previous section, the solution to the UBLP model with the maximum $\xi$ always satisfies the constraints and is a solution of the MMF model. In addition the UBLP model with $\xi = 0$ is clearly the MNT model. According to the results shown in Table I, solutions for proportional fairness obtained under the UBLP model are a compromise between the MNT model and the MMF model. However the objective of the UBLP model is to maximize the total network throughput while providing bandwidth satisfaction for each user, whereas the optimization objective of the PF model focuses on improving the bandwidth of the bottleneck user. In this respect the UBLP model and the PF model are different.

## VI. CONCLUSION AND FUTURE WORK

Channel allocation in open spectrum networks is essential in ensuring a high network throughput and user fairness. In this paper binary linear programming modeling of the channel allocation problem is presented. An optimization objective called fairness constrained maximum throughput is also proposed. Based on the UBLP model and a proposed iterative algorithm, solutions for different optimization objectives can be obtained. It is proved that with different per-user bandwidth minimums, our approach achieves the clear optimization for the objectives of maximum network throughput and max-min fairness. It is also proved that for the objective of proportional fairness, the solution to the UBLP model is within a bound compared with the optimal solution. The simulation studies indicate that the proposed algorithm achieves excellent performance. However, for large scale channel allocation problems, the corresponding binary linear programming problems are difficult to solve because the number of sub-problems increases much faster than the size of the problem. Moreover, the mobility of users/spectrum must be considered in practical networking scenarios. Our future work will look into designing distributed heuristic algorithms for large-scale channel allocation problems that are adaptive to the network dynamics.

## ACKNOWLEDGMENT

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