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Forecast of Next Day Clearing Price in Deregulated Electricity Market

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***Abstract*—the daily clearing price curve in electricity market varied with multi-period and strong fluctuation characteristic. When grey GM (1, 1) model is used in forecast, the forecast error exceeded the permitted precision. This is because GM (1, 1) model is invalidated only if the price series did not follow the rule of exponential growth. In this case, grey model with period residual modification is proposed, which inherits the advantages of grey model and makes the forecasting price curve fluctuated. Meanwhile, a series of technology is used, such as smooth processing to original data, improvement of initial condition and period residual modification. Thus the fitting curve is closer to original data and the forecasting precision is improved. Simulation results verified the feasibility of the proposed approach.**

***Keywords*—deregulated electricity market; forecast of next day clearing price; Grey Model of GM (1, 1); period residual modification; quadric exponential smoothing omponent;**

I. Introduction

With competitive market structure having been re-constructed from previous monopolization management in electricity industry, electricity price is the most important parameter concerned by every market participant and regulation committee of electricity market [1-2]. This is because the clearing price, sometimes called equilibrium price, would influence their economic benefit in the transaction of electric power energy. Even in the worse case, fluctuated price with abnormal variation in frequency and amplitude, which may be resulted from the manipulation by generator-side union, would disturb the market order. Wherein, equilibrium price is formed when supply and demand automatically reached on an equilibrium point under market competition condition. Therefore, clearing price forecasting is becoming one of focus issues for domestic and aboard researchers. Wherein, forecast of next day clearing price is considered as one of challenging problems.

In the past, deep research on load forecast has been done for power system. Many mature algorithms and forecast practices have been reported. In some sense, price forecast is able to refer to those algorithms of load forecast partly. Approach used in clearing price forecasting includes time sequence analysis [3-4], artificial neural net [5-7], and combination predication [8]. Wherein, combination approach is a weighted model, which is generated by a group of prediction result of conventional forecasting technology, so it could abstract useful information from every single model. The essence of the approach mentioned above is to find the variation rule of clearing price based on a mass of data, then to construct the forecast model.

More or less, there are some shortcomings. For example, some models require long calculation time or enough samples; some models require data to be conformed to those classical probability distribution. For grey system model, unique advantage has been demonstrated [9], such as no requirement on probability requirement to original data, poor samples, simple calculation processing etc. Actually, in electricity market, equilibrium price could be considered as a grey variable. This is because form mechanism of equilibrium price is very complicated [10]. Meanwhile, equilibrium price is usually influenced by many factors, which are difficult to identify respectively. So clearing price is a synthetic variable and implies the synthetic effect of many factors. As the detailed information of every influencing factor is incomplete and their effect is mostly uncertain, which is accordance with characteristic of grey variable. Consequently, forecast of clearing price is adapted to be solved with uncertain theory, such as grey system theory [11-12].

As we known, model like GM (1, 1) is constructed by one-order or second-order generated sequences, and is especially used in the sequence with approximate exponential variation. In this case, higher fitting precision is guaranteed. If this condition is not be satisfied, the forecasting error will be increased greatly. With exploring the datum of clearing price from California electricity market, America in 2000, which is an open database and is freely shared by all academic researchers, we found that clearing price is a fluctuation sequence with certain cycle, i.e. daily clearing price curve has similar characteristic within a period of time, such as the curve shape, the occurrence interval of maximal clearing price, etc. Then we attempted to use GM (1, 1) with period residual modification [13-14] to handle the discussed problem. With numerical calculation, we found that the approach is effective and its prediction is more accurate than that of GM (1, 1), and prediction precision is improved.

II. QUADRIC EXPONENTIAL SMOOTHING APPLIED

If the original sequence has some characteristic of multi-period, frequent variation and strong fluctuation, greater error occurred when we directly use it to modeling [15]. Therefore, quadric exponential smoothing technology was applied to original price sequence and generated a new sequence. And the smoothing formula is denoted as following:

$$\begin{cases} S'(k) = \alpha X^{(0)}(k) + (1-\alpha)S'(k-1) \\ S''(k) = \alpha S'(k) + (1-\alpha)S''(k-1) \end{cases} \quad k=1,n \qquad (1)$$

The meaning of parameter in the equation (1) is interpreted as below.

The parameter α is the smoothing parameter.

$X^{(0)}$ (k) is original sequence of clearing price.

S' (k) is a new sequence generated by $X^{(0)}$ (k) with linear exponential smoothing and called as linear exponential smoothing sequence.

S'' (k) is another sequence, which is generated by S' (k) and called as quadric exponential smoothing sequence.

The first data or average of anterior datum of $X^{(0)}$ (k) can be worked as S' (0). The magnitude of smoothing parameter α is set before calculation, which is usually determined as following principles:

The greater $X^{(0)}$ (k) fluctuates, the smaller α is; vice versa.

S'' (k), as the original sequence, is inputted into GM (1, 1) model. When finished modeling, the forecasted sequence $\tilde{S}''(k)$ should be reduced twice to obtain $\tilde{X}^{(0)}(k)$ according to the equation (2).

$$\begin{cases} \tilde{S}'(k) = [\tilde{S}''(k) - (1-\alpha)\tilde{S}''(k-1)]/\alpha \\ \tilde{X}^{(0)}(k) = [\tilde{S}'(k) - (1-\alpha)\tilde{S}''(k-1)]/\alpha \end{cases} \quad k=1,n \qquad (2)$$

Compared with $X^{(0)}$ (k), randomness of S'' (k) becomes weaker. Although the mathematical expectations E(S'' (k)) still remained constant, its variance VAR(S'' (k)) decreased, which tells us the variation extent of S'' (k) is weakened. Usually, a steady series is helpful to increasing forecasting precision when it is used in modeling.

III. IMPROVED GM (1, 1) FORECASTING MODEL

A. Original GM (1, 1) Model

Grey system theory founded in 1982, has been used in many fields widely [16]. The reason is because it has remarkable advantages such as poor data, simple calculation, excellent fitting precision, back-test criterion etc. wherein, GM (1, 1) is the model with the most extensive application. GM (1, 1) is a first-order differential equation and is used to deal with single variable problem. The steps of GM (1, 1) modeling are listed as following:

- Step 1: Generating first-order accumulative sequence.

Supposing that historical data of clearing price was written as $x^{(0)}$, which is a group of data varied with the time, and the length of sequence is n. $x^{(0)}$ is denoted as below:

$$x^{(0)} = x^{(0)}(1), x^{(0)}(2), \cdots; x^{(0)}(n) \qquad (3)$$

Then, we generated the first-order accumulative sequence:

$$x^{(1)} = x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n) \qquad (4)$$

Where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$$

- Step2: Establishing differential equation based on accumulative sequence.

As we known, when $x^{(1)}$ (k) varied approximately according to exponential growth rule, its expression is the same as the solution of first-order differential equation. Therefore, the new sequence $x^{(1)}$ (k) is considered as meeting the first-order differential equation:

$$\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t} + ax^{(1)}(t) = u \qquad (5)$$

In the equation (5), the meaning of parameter is intercepted as below.

The parameter α, is called as the development parameter of model, standing for development tendency of $x^{(1)}$ as well as original sequence.

The parameter u, is the coordination parameter, tells us transformation relations of these series.

Written A as A= [a, u] ^{T}, every element of matrix A are determined by least square approach. Detailed calculation of matrix A is shown as equation (6).

$$\boldsymbol{A} = (B^{\mathrm{T}}B)^{-1}B^{T}Y \qquad (6)$$

Where

$$\boldsymbol{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} -\frac{1}{2}[x^{(1)}(1) + x^{(1)}(2)] & 1 \\ -\frac{1}{2}[x^{(1)}(2) + x^{(1)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[x^{(1)}(n-1) + x^{(1)}(n)] & 1 \end{bmatrix}$$

- Step3: Establishing grey forecasting model.

We put α and u into equation (5) and got the underlying forecasting expression:

$$\tilde{x}^{(1)}(k+1) = [x^{(0)}(1) - \frac{u}{a}]\mathrm{e}^{-ak} + \frac{u}{a} \quad k=1, n \qquad (7)$$

The equation (7) is called as function of time response for GM (1, 1). With accumulative reduction calculation, the forecasting model of $x^{(0)}$ is described as following:

$$\tilde{x}^{(0)}(k+1)=\tilde{x}^{(1)}(k+1)-\tilde{x}^{(1)}(k)=(1-\mathrm{e}^{a})[x^{(0)}(1)-\frac{u}{a}]\mathrm{e}^{-ak} \quad k=1, n \qquad (8)$$

Wherein, $\tilde{x}^{(1)}(k+1)$ means forecasting sequence of $x^{(1)}$ at the $(k+1)^{th}$ interval.

B. Improvement of Initial Value Condition

According to the equation (7), $\tilde{x}^{(1)}(k+1)$, the solution of differential equation, would be influenced directly by initial value $x^{(0)}(1)$. Actually, $x^{(0)}(1)$ is not the most optimal selection when it is worked as initial value [17]. This is because relation between $x^{(0)}(1)$ and $\tilde{x}^{(1)}(k+1)$ is not closely correlative, in this case, the solution precision of the differential equation mentioned above would be influenced if $x^{(0)}(1)$ with no special proceeding is employed.

Now, some modification into the initial value of sample data, and modified formula is set as underlying expression.

$$x'^{(0)}(1)=x^{(0)}(1)+\sigma \qquad (9)$$

Where, σ is the modification item to $x^{(0)}(1)$, and then the forecasting equation becomes new one:

$$\tilde{x}^{(1)}(k+1)=[x^{(0)}(1)+\sigma-\frac{u}{a}]\mathrm{e}^{-ak}+\frac{u}{a} \qquad (10)$$

$$\tilde{x}^{(0)}(k+1)=(1-\mathrm{e}^{a})[x^{(0)}(1)+\sigma-\frac{u}{a}]\mathrm{e}^{-ak} \qquad (11)$$

Equation (10) and (11) can be transformed into expression (12) and (13):

$$\tilde{x}^{(1)}(k+1)=x^{(1)}(k+1)+\sigma\mathrm{e}^{-ak} \qquad (12)$$

$$\tilde{x}^{(0)}(k+1)=x^{(0)}(k+1)+\sigma(1-\mathrm{e}^{a})\mathrm{e}^{-ak} \qquad (13)$$

Checked (12) (13), we found that the additional modification item, i.e., the item σe^{-ak} or the item $\sigma(1-e^{a})e^{-ak}$, presented modification to the conventional forecast expression. When σ equals to zero, expression (12) and (13) are reduced to (11) and (12).

The parameter σ is solved according to the same approach as introduced above, which means that the sum of error square between original sequence and forecasting sequence arrives to the minimum. The calculation of σ is shown as following:

$$\min\sum_{k=1}^{n}\left[\tilde{x}^{(0)}(k+1)-x^{(0)}(k+1)\right]^2 \qquad (14)$$

$$\sigma=\frac{p}{q}-[x^{(0)}(1)-\frac{u}{a}] \qquad (15)$$

Where

$$p=\sum_{k=1}^{n}x^{(0)}(k)\mathrm{e}^{-a(k-1)} \quad q=(1-\mathrm{e}^{a})\frac{1-\mathrm{e}^{-2na}}{1-\mathrm{e}^{-2a}}$$

IV. GM (1, 1) MODEL WITH PERIOD RESIDUAL MODIFICATION

Considering that clearing price curve varied frequently, extensively, periodically, when we constructed fitting model by GM (1, 1), the positive and negative signs of residual sequence will appear alternately and has irregular variation period. The precision of model is decreased when GM (1, 1) is adopted, and the model is difficult to reflect the fluctuant variation of price sequence.

In this case, we analyzed the period of residual sequence in the grey model and found that if we divided the residual sequence into a few sections, of which period and amplitude is different, then used sine (or cosine) curve to fit the residual sequence, the improved model are able to approximate the special variation of curve.

The residual modified item in every section could be calculated according to (16), referred to reference [13].

In this case, we analyzed the period of residual sequence in the grey model and found that if we divided the residual sequence into a few sections, of which period and amplitude is different, then used sine (or cosine) curve to fit the residual

$$\hat{E}(t_i)=A_i\sin\frac{2\pi t_i}{T_i} \qquad (16)$$

The parameter $\hat{E}(t_i)$ is the modified item at the t^{th} interval in the i^{th} period.

A_i and T_i stand for the maximum amplitude and the length at the t^{th} interval in the i^{th} period respectively.

In order to simplify calculation, amplitude of every section can be taken as average of residual absolute value, listed as follows:

$$\overline{A}=\frac{\sum_{j=1}^{M}|\varepsilon(j)|}{M} \qquad j=1, M$$

M is the number of concerned residual sequence.

The length of divided section is determined by the interval of sign alteration of residual sequence. Generally, the residual sequence contains a few of sections, some are referred to the segment with positive sign, and others are referred to the segment negative sign. Of course, the division is able to do some adjustment according to requirement from actual situation. Then, every residual modified item is added to the corresponding reduced expression at the same interval, shown as equation (17).

$$\hat{x}^{(0)}(t+1)=x^{(0)}(t+1)+\hat{E}(t+1) \qquad (17)$$

After the disposal as discussed above, the residual is decreased. This means fitting curve is closer to the original curve; therefore, precision of forecasting model is increased.

For clearing price in next day, the characteristic of periodical variation is suggested to be similar to the known sequence, the amplitude in every divided section is assumed to be within the permitted range. Once these parameters are determined, the residual modification item at every interval in future is calculated and is added to the forecasting expression.

V. ANALYSIS OF EXAMPLES

In our research, the data of clearing price in California market during the 5th-11th, March, 2000 are taken as original data. Conventional grey model and modified grey model are established respectively, and these models are used to forecast the clearing price on the 12th, 13th, and 14th, March, 2000. Forecasting result is evaluated according to the following indexes: percentage of relative error δ_{APE} and average of relative error δ_{MAPE}, which are written as equation (18).

$$\delta_{\text{APE}} = \frac{\left|x_i - \tilde{x}_i\right|}{x_i} \times 100\%$$

$$\delta_{\text{MAPE}} = \frac{1}{T}\sum_{i=1}^{T}\frac{\left|x_i - \tilde{x}_i\right|}{x_i} \qquad i=1,T \qquad (18)$$

T is the number of forecasting value.

x_i represents the history clearing price;

$\tilde{x}_i$ represents the forecasted clearing price.

In table 1, we listed out the actual clearing price, forecasted result of two models as well as their error on the 14th, March, 2000.

After completing the forecasting calculation on the 14th, March, 2000, we drew out the curves of the actual clearing price curve and the forecasted curve of two models, which is displayed in figure 1.

As figure 1 shown, the forecasting precision has increased by the GM (1, 1) model with period residual modification. Table 2 showed the forecasting error with improved GM (1, 1) model and GM (1, 1) model on the12th, 13th, and 14th March, 2000. We noticed that the error of model with period residual modification has decreased, when compared with general grey model. The average percentage of relative error in three days is 7.38%, which reached the requirement of engineering.

In addition, we use the forecast result on the same day to compare the modified GM (1, 1) with ARIMA, which is one of commonly algorithms used in time series forecast. Both of the models have represented relatively approximate forecast ability.

Referred to the maximal relative error for single interval in forecasted day, the former is 22.80%, slightly lower than 23.85% of the latter. As for the average of relative error, the former is 7.87%, slightly higher than 7.69% of the latter.

TABLE I. COMPARISON OF CLEARING PRICE BETWEEN TWO MODELS
UNIT: USD/MWH, %

Interval	History	GM(1,1)	Improved GM(1,1)	Error of Improved GM(1,1)
01:00	14.892 4	17.194 8	15.944 9	7.07
02:00	14.001 1	15.692 0	14.662 5	4.72
03:00	13.440 8	13.854 3	13.549 0	0.81
04:00	12.992 5	13.704 7	13.403 0	3.16
05:00	13.994 0	16.271 4	15.407 0	10.1
06:00	14.741 7	16.224 4	14.904 4	1.1
07:00	16.990 8	17.936 4	15.939 6	6.19
08:00	17.995 8	20.681 2	19.598 1	8.9
09:00	19.498 1	21.952 8	21.256 8	9.02
10:00	20.007 4	22.268 8	22.081 2	10.37
11:00	19.997 9	22.159 9	21.748 3	8.75
12:00	19.994 2	21.648 9	21.489 5	7.48
13:00	20.063 5	20.900 5	20.577 1	2.56
14:00	20.110 8	20.613 5	20.154 0	0.21
15:00	19.999 6	20.256 4	19.802 5	0.99
16:00	19.993 4	19.593 8	19.267 8	3.63
17:00	19.992 0	19.859 0	19.626 4	1.83
18:00	21.565 8	21.764 8	21.764 8	0.92
19:00	28.006 8	25.330 3	26.182 2	6.51
20:00	27.999 7	24.758 2	25.318 2	9.58
21:00	23.063 6	21.567 1	21.191 8	8.12
22:00	21.095 3	20.950 0	20.950 0	0.69
23:00	16.999 4	20.356 9	19.851 6	16.78
24:00	14.983 4	17.371 5	16.441 4	9.73

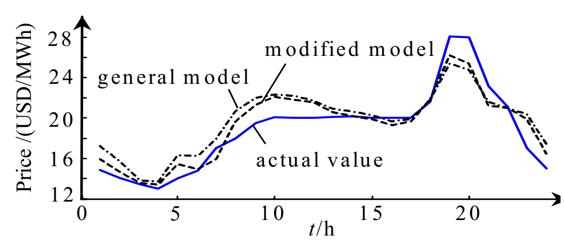

Figure 1. Comparison between two models in forecasting price

TABLE II. ERROR PERCENTAGE OF TWO MODELS
UNIT: %

Data	Error of GM(1,1)	Error of Modified GM(1,1)
12	9.96%	7.53%
13	11.61%	8.81%

Data	Error of GM(1,1)	Error of Modified GM(1,1)
14	8.39%	5.8%

VI. CONCLUSIONS

In this paper, we proposed a new approach of clearing price forecasting, that is grey model with period residual modification.

Firstly, we analyzed the characteristic of the clearing price, i.e. it is with uncompleted and uncertain information. Therefore, clearing price is taken as a grey variable. Meanwhile, we made used of advantage of grey system theory, which manifested in poor samples, simple calculation and forecast result being tested.

Secondary, quadric smoothing technology was applied to original price sequence, and then generated a new sequence, which decrease the fluctuation of sequence. Considering the initial value of grey model would influence the solution of differential equation, new approach of initial value design is adopted. Adding item with period residual modification into GM (1, 1) makes forecast curve fluctuated, which would trace the tendency of original sequence. With the above disposals, the forecast precision has been improved compared with GM (1, 1).

Finally, an example of California electricity market has verified that when applying proposed approach to the actual system, the forecast result is satisfactory. And the forecast precision of modified GM (1, 1) is compared with that of ARIMA, both is relatively close.

Deep research in grey model is helpful to improve the forecast precision of clearing price, which would give grey system model more extensive application.

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