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A New Improved Model of the degree of Grey slope Incidences Based on the Change Rate of Slope

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***Abstract*—based on studying all kinds of main degree of grey incidences models, it founded that there are some flaws in those models. Such as, not satisfied with the property of normality, can not reflect the relation of positive-negative between sequences, and so on. In order to solve those problems, a new model for grey incidence based on the change rate of slope is proposed. This model calculates the ratio of absolute value of incremental between a raw data and its next in one sequence, and then, applies the ratio to calculate the degree of grey incidences between sequences. The new model is satisfied with not only the property of normative, but the property of order preservation, at the same time, the result also can reflect the relation of positive-negative .The method is proved to be practiced in both theories and applications.**

I. INTRODUCTION

IN 1982, Professor of Deng JuLong of china set up a theory, so-called "grey system theory", and this theory turned out to be an important fruitful area of research with strong and successful practical applications. After nearly 30 years, this theory has been developed and matured rapidly; it has been widely applied to the significant consequences, such as social, economic, scientific and technological, agricultural, financial and so on.

Grey incidence analysis is an important component of grey system theory; meanwhile, it is the foundation of grey analyses, modeling, predictions and decision-makings. The fundamental idea of grey incidence analysis is that the closeness of relation is judged based on the similarity level of the geometrical patterns of sequence curves. The more similar the curves are the higher degree of incidence between sequences; and vice versa.

Grey incidence analysis is so important that many researchers in the area of grey system theory start to study this theory. Up to now, a lot of models or their improvements on grey incidence analysis were proposed. For instance, Deng JuLong proposed the degree of Deng's incidence based on four axioms of grey incidence [1]; Mei ZhenGuo put forward the degree of absolute incidence according to the degree of closeness of the variation for time sequence curves of factors [2]; Liu SiFeng advanced a degree of grey incidence on the basis of the area of curves of time sequence [3]; Dang YaoGuo improved degree of grey slope incidence [4], and so on[5-9].

But, there exists more or less flaws in those models. For example, some models are not satisfied with the property of normality, some have not the property of order preservation, etc. To take the degree of grey slope incidences for an example, we will discuss those flaws at length in subsequent section II.

II. MAIN DEFECTS

A. Not satisfied with the property of normality

The meaning of normality of grey incidences is as follows, $0<\gamma_{0i}\le 1$, When the relation of X_0 and X_i is linear, for example: $X_0(k)=X_i(k)+c$ or $X_0(k)=aX_i(k)$ (a,c : constant), then $\gamma_{0i}=1$.

Now, let us prove that the degree of grey slope incidences is not satisfied with normality.

Definition 2.1 Assume that $X(t)$ is a sequences representing a system's characteristics; $Y_i(t)$ $(i=1,2,\cdots,m)$ are sequences of relevant factors, and, both $X(t)$ and $Y_i(t)$ are all 1-time-interval sequences,

$$\bar{x}=\frac{1}{n}\sum_{t=1}^{n}x(t)\,,\ \Delta x(t)=x(t+\Delta t)-x(t)$$

$$\bar{y}_i=\frac{1}{n}\sum_{t=1}^{n}y_i(t)\,,\ \Delta y_i(t)=y_i(t+\Delta t)-y_i(t)$$

Then

$$\xi_i(t)=\frac{1+|\frac{\Delta x(t)}{\bar{x}}|}{1+|\frac{\Delta x(t)}{\bar{x}}|+|\frac{\Delta x(t)}{\bar{x}}-\frac{\Delta y_i(t)}{\bar{y}_i}|} \tag{1}$$

$$\varepsilon_i=\frac{1}{n-1}\sum_{t=1}^{n-1}\xi_i(t) \tag{2}$$

ε_i is called the degree of grey slope incidence of $Y_i(t)$ with respect to $X(t)$,and $\xi_i(t)$ the slope incidence coefficient of $Y_i(t)$ with respect to $X(t)$ at the point t .

Theorem1 the degree of grey slope incidence is not satisfied with the property of normality.

We will prove this theorem according to the signification of the property of normality.

Proof: Assume that $X(t) = aY_i(t) + c$ (a, c : constant)

$$\Delta x(t) = x(t + \Delta t) - x(t)$$

$$\Rightarrow x(t + \Delta t) = aY_i(t + \Delta t) + c \,,\, x(t) = aY_i(t) + c$$

So

$$\Delta x(t) = [aY_i(t + \Delta t) + c] - [aY_i(t) + c]$$

$$\Rightarrow \Delta x(t) = a[Y_i(t + \Delta t) - Y_i(t)]$$

$$\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x(t) = \frac{1}{n}\sum_{t=1}^{n}[aY_i(t) + c]$$

$$\Rightarrow \bar{x} = \frac{1}{n}\sum_{t=1}^{n}[aY_i(t)] + c$$

So

$$\left|\frac{\Delta x(t)}{\bar{x}}\right| = \frac{a[Y_i(t + \Delta t) - Y_i(t)]}{\frac{1}{n}\sum_{t=1}^{n}[aY_i(t)] + c}$$

$$\Rightarrow \left|\frac{\Delta x(t)}{\bar{x}}\right| = \frac{Y_i(t + \Delta t) - Y_i(t)}{\frac{1}{an}\sum_{t=1}^{n}[aY_i(t)] + \frac{c}{a}}$$

$$\left|\frac{\Delta y_i(t)}{\bar{y}_i}\right| = \frac{y_i(t + \Delta t) - y_i(t)}{\frac{1}{n}\sum_{t=1}^{n} y_i(t)}$$

So

$$\frac{1}{an}\sum_{t=1}^{n}[aY_i(t)] + \frac{c}{a} \neq \frac{1}{n}\sum_{t=1}^{n} y_i(t),$$

$$\Rightarrow \left|\frac{\Delta x(t)}{\bar{x}}\right| \neq \left|\frac{\Delta y_i(t)}{\bar{y}_i}\right|$$

Therefore

$$\xi_i(t) = \frac{1 + \left|\frac{\Delta x(t)}{\bar{x}}\right|}{1 + \left|\frac{\Delta x(t)}{\bar{x}}\right| + \left|\frac{\Delta x(t)}{\bar{x}} - \frac{\Delta y_i(t)}{\bar{y}_i}\right|} \neq 1$$

So, the degree of grey slope incidence is not satisfied with the property of normality.

B. Can not reflect the relations of positive-negative of sequences

In the case of the degree of grey slope incidence, when the sign of $\Delta x(t)$ is different from $\Delta y_i(t)$, it's shown that the development's form of two sequences at the point t have not any similarity. One is the form of addition, the other is decline, however, the degree of incidences for this two sequences at the point t is positive ($\xi_i(t) > 0$), in other words, they are similarity to some degree at the point of t , it is contradiction in terms, the main cause for that is the degree of grey slope incidence can not reflect the relations of positive-negative of sequences.

III. THE IMPROVED MODEL

In order to solve those defects, a new model for grey incidence analysis based on the change rate of slope is put forward. This model calculates the ratio of absolute value of incremental between a raw data and its next in one sequence, and then, applies the ratio to compute the degree of grey incidences between sequences. We will discuss and analyze the model at length.

Definition2 the time intervals of $[a,b], b > a \geq 0$, take

$$\Delta t_k = t_k - t_{k-1} \,,\, k = 2,3,\cdots n \,,$$

$$[a,b] = \bigcup_{k=2}^{n} \Delta t_k \,,\, \Delta t_k \cap \Delta t_{k-1} = \varnothing \,,\, k = 2,3,\cdots n$$

Assume that two time sequences defined on $[a,b]$ are

$$X_0 = (x_0(t_1), x_0(t_2), \cdots, x_0(t_n))$$

$$Y_1 = (y_1(t_1), y_1(t_2), \cdots, y_1(t_n))$$

$$\Delta x_0(t_k) = x_0(t_k) - x_0(t_{k-1})$$

$$\Delta x_0(t_{k+1}) = x_0(t_{k+1}) - x_0(t_k)$$

$$\partial_{x_0}(t_{k+1}) = \frac{\Delta x_0(t_{k+1})}{\Delta x_0(t_k)}$$

Similarly

$$\partial_{y_1}(t_{k+1}) = \frac{\Delta y_1(t_{k+1})}{\Delta y_1(t_k)}$$

Then

$$\xi_{01}(k) = \left\{ \begin{array}{l} when \quad \Delta x_0(t_k)\cdot\Delta y_1(t_k) \neq 0 \\ \mathrm{sgn}(\partial_{x_0}(t_{k+1})\cdot\partial_{y_1}(t_{k+1}))\cdot\dfrac{1+|\partial_{x_0}(t_{k+1})|}{1+|\partial_{x_0}(t_{k+1})|+||\partial_{x_0}(t_{k+1})|-|\partial_{y_1}(t_{k+1})||}; \\ when \quad \Delta x_0(t_k)\cdot\Delta y_1(t_k) = 0 \\ 0 \end{array} \right\} \tag{3}$$

$$\varepsilon_{01} = \frac{1}{n-2}\sum_{t=2}^{n-1}\xi_{01}(t) \tag{4}$$

ε_{01} is called the degree of grey incidence of Y_1 with respect to X_0 ,and $\xi_{01}(k)$ the incidence coefficient of Y_1 with respect to X_0 at the point t .

Theorem2 the new model for grey incidence analysis based on the change rate of slope is satisfied with the property of normality.

Proof:

Assume that $X_0(t) = aY_1(t) + c$ (a, c : constant) and $\Delta x_0(t_k)\cdot\Delta y_1(t_k) \neq 0$

$$\Delta x_0(t_k) = x_0(t_k) - x_0(t_{k-1})$$

$$\Rightarrow \Delta x_0(t_k) = [ay_1(t_k) + c] - [ay_1(t_{k-1}) + c]$$

$$\Rightarrow \Delta x_0(t_k) = a(y_1(t_k) - y_1(t_{k-1}))$$

Similarly

$$\Rightarrow \Delta x_0(t_{k+1}) = a(y_1(t_{k+1}) - y_1(t_k))$$

So

$$\partial_{y_1}(t_{k+1}) = \frac{\Delta y_1(t_{k+1})}{\Delta y_1(t_k)} = \frac{y_1(t_k) - y_1(t_{k-1})}{y_1(t_{k+1}) - y_1(t_k)}$$

$$\partial_{x_0}(t_{k+1}) = \frac{\Delta x_0(t_{k+1})}{\Delta x_0(t_k)} = \frac{a(y_1(t_k) - y_1(t_{k-1})}{a(y_1(t_{k+1}) - y_1(t_k))}$$

$$\Rightarrow \partial_{x_0}(t_{k+1}) = \frac{y_1(t_k) - y_1(t_{k-1})}{y_1(t_{k+1}) - y_1(t_k)}$$

So

$$\Rightarrow \xi_{01}(k) = \text{sgn}(\partial_{x_0}(t_{k+1}) \cdot \partial_{y_1}(t_{k+1})) \cdot \frac{1 + |\partial_{x_0}(t_{k+1})|}{1 + |\partial_{x_0}(t_{k+1})| + ||\partial_{x_0}(t_{k+1})| - |\partial_{y_1}(t_{k+1})||}$$

$$\Rightarrow \xi_{01}(k) = \text{sgn}(\partial_{x_0}(t_{k+1}) \cdot \partial_{y_1}(t_{k+1})) \cdot \frac{1 + \left|\frac{\Delta x_0(t_{k+1})}{\Delta x_0(t_k)}\right|}{1 + \left|\frac{\Delta x_0(t_{k+1})}{\Delta x_0(t_k)}\right| + \left|\left|\frac{\Delta x_0(t_{k+1})}{\Delta x_0(t_k)}\right| - \left|\frac{\Delta y_1(t_{k+1})}{\Delta y_1(t_k)}\right|\right|}$$

$$\Rightarrow \xi_{01}(k) = \text{sgn}(\partial_{x_0}(t_{k+1}) \cdot \partial_{y_1}(t_{k+1})) \cdot \frac{1 + \left|\frac{y_1(t_k) - y_1(t_{k-1})}{y_1(t_{k+1}) - y_1(t_k)}\right|}{1 + \left|\frac{y_1(t_k) - y_1(t_{k-1})}{y_1(t_{k+1}) - y_1(t_k)}\right| + \left|\left|\frac{y_1(t_k) - y_1(t_{k-1})}{y_1(t_{k+1}) - y_1(t_k)}\right| - \left|\frac{y_1(t_k) - y_1(t_{k-1})}{y_1(t_{k+1}) - y_1(t_k)}\right|\right|}$$

Because of

$$\frac{1 + \left|\frac{y_1(t_k) - y_1(t_{k-1})}{y_1(t_{k+1}) - y_1(t_k)}\right|}{1 + \left|\frac{y_1(t_k) - y_1(t_{k-1})}{y_1(t_{k+1}) - y_1(t_k)}\right| + \left|\left|\frac{y_1(t_k) - y_1(t_{k-1})}{y_1(t_{k+1}) - y_1(t_k)}\right| - \left|\frac{y_1(t_k) - y_1(t_{k-1})}{y_1(t_{k+1}) - y_1(t_k)}\right|\right|} = 1$$

Therefore

$$\Rightarrow \xi_{01}(k) = \text{sgn}(\partial_{x_0}(t_{k+1}) \cdot \partial_{y_1}(t_{k+1}))$$

So, the new model for grey incidence analysis based on the change rate of slope is satisfied with the property of normality.

Theorem3 the new model for grey incidence analysis based on the change rate of slope has the property of order preservation.

The meaning of order preservation' property on grey incidence is that original sequences which were disposed by all kinds of methods of nondimensionalization have the same degree of grey incidence. In other words, we apply different methods of nondimensionalization to dispose original sequences, however, we will get the same result on degree of grey incidence. The property of order preservation, the other name is "do not generate the effect of ordinal-numeral".

Proof: assume that

D = {initialing operator, averaging operator, interval operator, reversing operator, reciprocating operator…}

Then

$$X_0 D = (x_0(t_1)d, x_0(t_2)d, \cdots, x_0(t_n)d)$$

$$\Delta x_0(t_k) = (x_0(t_k) - x_0(t_{k-1}))d$$

$$\Delta x_0(t_{k+1}) = (x_0(t_{k+1}) - x_0(t_k))d$$

$$\partial_{x_0}(t_{k+1}) = \frac{\Delta x_0(t_{k+1})}{\Delta x_0(t_k)} = \frac{(x_0(t_{k+1}) - x_0(t_k))d}{(x_0(t_k) - x_0(t_{k-1}))d}$$

$$\Rightarrow \partial_{x_0}(t_{k+1}) = \frac{x_0(t_{k+1}) - x_0(t_k)}{x_0(t_k) - x_0(t_{k-1})}$$

Both molecule and denominator in the expression of $\partial_{x_0}(t_{k+1})$ have the same grey incidence operator d, the operator d counteracts the effects acted on the original sequences. Therefore, the value of $\partial_{x_0}(t_{k+1})$ has nothing to do with the grey incidence operator d.

So, no matter which methods of nondimensionalization are applied to the original sequences, we will get the same degree of incidence. The property of order preservation of the new model has been proved to be correct.

At the same time, in the expression of (3), the new model considers the positive-negative of data according to the sign function $\text{sgn}(\partial_{x_0}(t_{k+1}) \cdot \partial_{y_1}(t_{k+1}))$; therefore, the degree of improved grey slope incidence can reflect the relations of positive-negative of sequences.

Proposition1: The degree of improved grey slope incidence satisfies the following properties:

1° $-1 \le \varepsilon_{0i} \le 1$

2° Any two sequences are not absolutely unrelated, that is, $\varepsilon_{0i} \ne 0$.

3° ε_{0i} is only related to the rate of change of slope of X_0 and Y_i, and has nothing to do with the other factors.

4° ε_{0i} has nothing to do with all kinds of methods of nondimensionalization, in other words, the improved model satisfies with the property of order preservation.

5° Symmetry, that is, $\varepsilon_{0i} = \varepsilon_{i0}$

6° Adjacency, the smaller $\| \partial_{x_0}(t_{k+1}) | - | \partial_{y_1}(t_{k+1}) \|$ is, the larger ε_{0i}.

IV EXAMPLES

Example 1 Assume that sequences

$X_0 = (10, 20, 30, 40, 50, 60)$ ($x_0(t) = t$)

and

$Y_1 = (20, 40, 60, 80, 100, 120)$ ($y_1(t) = 2x_0(t)$)

$Y_2 = (110, 120, 130, 140, 150, 160)$ ($y_2(t) = x_0(t) + 10$)

are given, calculate the ε_{01} and ε_{02}.

Solution 1° calculate $\Delta x_0(t_k)$, $\Delta y_1(t_k)$ and $\Delta y_2(t_k)$

$\Delta x_0(t_2) = x_0(t_2) - x_0(t_1)$ =20-10=10

$\Delta x_0(t_3)$ =10, $\Delta x_0(t_4)$ =10, $\Delta x_0(t_5)$ =10

So

ΔX_0 =(10, 10, 10, 10)

Similarly

ΔY_1 = (20, 20, 20, 20) ΔY_2 = (10, 10, 10, 10)

2° Calculate $\partial_{x_0}(t_{k+1})$, $\partial_{y_1}(t_{k+1})$ and $\partial_{y_2}(t_{k+1})$

$$\partial_{x_0}(t_2) = \frac{\Delta x_0(t_3)}{\Delta x_0(t_2)} = \frac{10}{10} = 1, \partial_{x_0}(t_3) = \frac{\Delta x_0(t_4)}{\Delta x_0(t_3)} = \frac{10}{10} = 1$$

$$\partial_{x_0}(t_4) = \frac{\Delta x_0(t_5)}{\Delta x_0(t_4)} = \frac{10}{10} = 1, \partial_{x_0}(t_5) = \frac{\Delta x_0(t_6)}{\Delta x_0(t_5)} = \frac{10}{10} = 1$$

$$\partial_{y_1}(t_2) = \frac{\Delta y_1(t_3)}{\Delta y_1(t_2)} = \frac{10}{10} = 1, \partial_{y_1}(t_3) = \frac{\Delta y_1(t_4)}{\Delta y_1(t_3)} = \frac{10}{10} = 1$$

$$\partial_{y_1}(t_4) = \frac{\Delta y_1(t_5)}{\Delta y_1(t_4)} = \frac{10}{10} = 1, \partial_{y_1}(t_5) = \frac{\Delta y_1(t_6)}{\Delta y_1(t_5)} = \frac{10}{10} = 1$$

$$\partial_{y_2}(t_2) = \frac{\Delta y_2(t_3)}{\Delta y_2(t_2)} = \frac{10}{10} = 1, \partial_{y_2}(t_3) = \frac{\Delta y_2(t_4)}{\Delta y_2(t_3)} = \frac{10}{10} = 1$$

$$\partial_{y_2}(t_4) = \frac{\Delta y_2(t_5)}{\Delta y_2(t_5)} = \frac{10}{10} = 1, \partial_{y_2}(t_5) = \frac{\Delta y_2(t_6)}{\Delta y_2(t_5)} = \frac{10}{10} = 1$$

3° Calculate ε_{01} and ε_{02}

$$\xi_{01}(2) = \mathrm{sgn}(\partial_{x_0}(t_2) \cdot \partial_{y_1}(t_2)) \cdot \frac{1 + |\partial_{x_0}(t_2)|}{1 + |\partial_{x_0}(t_2)| + \|\partial_{x_0}(t_2)| - |\partial_{y_1}(t_2)\|}$$

$$\Rightarrow \xi_{01}(2) = \mathrm{sgn}(1 \cdot 1) \cdot \frac{1 + |1|}{1 + |1| + \|1| - |1\|} = 1$$

Similarly, $\xi_{01}(3) = 1$, $\xi_{01}(4) = 1$, $\xi_{01}(5) = 1$

$$\varepsilon_{01} = \frac{1}{n-2} \sum_{t=2}^{n-1} \xi_{01}(t)$$

$$\Rightarrow \varepsilon_{01} = \frac{1}{6-2}(\xi_{01}(2) + \xi_{01}(3) + \xi_{01}(4) + \xi_{01}(5))$$

$$\Rightarrow \varepsilon_{01} = \frac{1}{4}(1+1+1+1) = 1$$

$$\varepsilon_{02} = \frac{1}{n-2} \sum_{t=2}^{n-1} \xi_{02}(t)$$

$$\Rightarrow \varepsilon_{02} = \frac{1}{6-2}(\xi_{02}(2) + \xi_{02}(3) + \xi_{02}(4) + \xi_{02}(5))$$

$$\Rightarrow \varepsilon_{02} = \frac{1}{4}(1+1+1+1) = 1$$

Because the relations of X_0, Y_1 and Y_2 are linear, this example once again proves that the new model satisfies with the property of normality.

Example 2 Assume that sequences

$X_0 = (10, 18, 23, 24, 28, 48, 58)$

and

$Y_1 = (10, 22, 25, 21, 30, 42, 54)$

are given,

1° Apply the initial image of each sequence to compute the degree of grey incidences ε'_{01} of X_0 and Y_1

2° Apply the average image of each sequence to calculate the degree of grey incidences ε_{01} of X_0 and Y_1

Solution :

1° The initial image of sequence X_0, named X'_0

$X'_0 = \frac{X_0}{x_i(1)}$ =(1,1.8,2.3,2.4,2.8,4.8,5.8)

$\Delta X'_0$ =(0.8,0.5,0.1,0.4,2.0,1.0)

$\partial_{x'_0}(t_2) = \frac{\Delta x'_0(t_3)}{\Delta x'_0(t_2)} = \frac{0.5}{0.8}$ =0.625, $\partial_{x'_0}(t_3)$ =0.2

$\partial_{x'_0}(t_4)$ =4, $\partial_{x'_0}(t_5)$ =5, $\partial_{x'_0}(t_6)$ =0.5

Similarly

$\partial_{y'_1}(t_2)$ =0.25, $\partial_{y'_1}(t_3)$ =-1.33, $\partial_{y'_1}(t_4)$ =2.25

$\partial_{y'_1}(t_5)$ =1.33, $\partial_{y'_1}(t_6)$ =1.00

$\xi'_{01}(2)$ =0.730, $\xi'_{01}(3)$ =-0.515, $\xi'_{01}(4)$ =0.741,

$\xi'_{01}(5)$ =0.620, $\xi'_{01}(6)$ =0.750

Then

$\varepsilon'_{01} = \frac{1}{5}$ (0.730-0.515+0.741+0.620+0.750)=0.4472

2° The average image of sequence X_0, named $\bar{X}_0$

$\bar{x}_0 = \sum_{i=1}^{7} x_0(i)$ =29.857

$$\bar{X}_0 = \frac{X_0}{\bar{x}_0} = (0.335,0.603,0.77,0.803,0.938,1.608,1.943)$$

$$\Delta \bar{X}_0 = (0.268,0.167,0.033,0.135,0.670,0.335)$$

$$\partial_{\bar{x}_0}(t_2) = 0.625,\ \partial_{\bar{x}_0}(t_3) = 0.199,\ \partial_{\bar{x}_0}(t_4) = 4.09,$$

$$\partial_{\bar{x}_0}(t_5) = 4.99,\ \partial_{\bar{x}_0}(t_6) = 0.5$$

Similarly

$$\partial_{\bar{y}_1}(t_2) = 0.25,\ \partial_{\bar{y}_1}(t_3) = -1.333,\ \partial_{\bar{y}_1}(t_4) = 2.25,$$

$$\partial_{\bar{y}_1}(t_5) = 1.33,\ \partial_{\bar{y}_1}(t_6) = 1.00,$$

Then

$$\varepsilon_{01} = 0.4472$$

So

$$\varepsilon_{01}^{'} = \varepsilon_{01}$$

This example once again shows that the new model has the property of order preservation.

V Conclusion

The property of normality and order preservation is very important to grey incidence analysis; however, up to now, there is scarcely a model which is satisfied with those properties. In order to solve those problems, a new model is proposed. The new model calculates the ratio of absolute value of incremental between a raw data and its next in one sequence, and then, applies the ratio to calculate the degree of grey incidences between sequences.

On both a theoretical and a practical level, it has been proved that this model is satisfied with the property of normative and order preservation, at the same time, the result also can reflect the relation of positive-negative .when the incremental of slope is not zero, this model provides a good solution for grey incidence analysis.

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