Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

A Sensor Placement Measure for Impact Detection in Structural Health Monitoring

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***Abstract*— This paper studies the validity of horizontal dilution of precision (HDOP) measure to evaluate sensor network geometries when localizing impacts in structural health monitoring (SHM). First HDOP is defined similarly to navigation applications. Even though several low complexity closed-form solutions have been proposed recently, HDOP measure has been theoretically justified only for iterative least squares approach. The localization errors of popular impact localization methods are experimentally collected and compared with HDOP data for validation. The experimental setup is also described. It is shown that HDOP in general correlates with the positioning error and can be used to characterize geometry.**

***Keywords*— Acoustic position measurement, Acoustic radiators, Position measurement.**

I. INTRODUCTION

Impact location detection has been a topic of significant interest in structural health monitoring (SHM) [1-2], especially in the fields of aircraft and spacecraft structures [3-7]. Knowledge of impact location and its magnitude can be very useful in damage evaluation. A sensor network can be installed on the structure and may report signal arrival times, received signal strengths, impact duration, etc. to a central processing unit which will determine location and estimate damage extent [1,3]. Sensors are used as beacons: impacts generate acoustic waves in the structure which are received by sensors and arrival times (time-of-arrival methods, TOA) or their differences (time-difference-of-arrival, TDOA) are used for location estimation through trilateration. The positioning performance is affected by the geometry of sensor networks. While this effect has been widely studied by navigation community [8-12] geometry influence was not addressed for structural health monitoring. This paper studies the validity of applying a geometry measure called horizontal dilution of precision (HDOP) for SHM. Previously it has been shown that HDOP is the same for TOA and TDOA methods when location is computed using a class of least squares iterative techniques [10-11]. It is not known if HDOP correlates with Ease of Use the accuracy of other closed-form and iterative localization methods. This paper describes an experimental system and validation results for HDOP measure in a comparative study of popular state-of-the-art impact localization techniques.

The paper is organized as follows. Section II defines HDOP. Section III describes the experimental setup. Section IV derives a typical iterative localization method which is used as a reference. Then Section V tabulates experimental data for various algorithms and compares with HDOP measure.

II. HORIZONTAL DILUTION OF PRECISION (HDOP)

HDOP is introduced next following its definition for TOA techniques [12]. Let vectors $\mathbf{r}_i$ and $\mathbf{r}_s$ denote the locations of sensors and impact source, $\mathbf{1}_i$ is a unit vector along with the line-of-sight from source to sensor i, $(i = 1,...,n)$, and matrices $\mathbf{1}_i$, $\mathbf{G}$ and $\mathbf{A}$ are defined as

$$\mathbf{1} = \frac{\mathbf{r}_i - \mathbf{r}_s}{|\mathbf{r}_i - \mathbf{r}_s|};\ \mathbf{G} = \begin{bmatrix} -\mathbf{1}_1^T & 1 \\ -\mathbf{1}_2^T & 1 \\ \vdots & 1 \\ -\mathbf{1}_1^T & 1 \end{bmatrix};\ \mathbf{A} = \left(\mathbf{G}^T\mathbf{G}\right)^{-1} \qquad (1)$$

Source-to-sensor distance measurements d_{is} are obtained by multiplying propagation times and velocities. As time of the impact is unknown it is predicted and all the distance measurements are biased by the same prediction error d_t. The source location is obtained using the set of equations

$$d_{is} = |\mathbf{r}_i - \mathbf{r}_s| + d_t,\quad i = 1,...,n \qquad (2)$$

where $\mathbf{r}_s$ and d_t are unknowns. These equations can be linearized to relate errors Δd_{is}, $\Delta\mathbf{r}_s$ and Δd_t in d_{is}, $\mathbf{r}_s$ and d_t respectively using Taylor expansion. Denoting $\Delta\mathbf{x}_s = \left[\Delta\mathbf{r}_s\ \Delta d_t\right]^T$ the linearization results in

$$\Delta\mathbf{d}_s = \mathbf{G}\Delta\mathbf{x}_s \qquad (3)$$

The least squares solution of (3) is given by the following:

$$\Delta\mathbf{x}_s = \left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T\Delta\mathbf{d}_s \qquad (4)$$

The solution error covariance can be expressed as

$$E\left[\Delta\mathbf{x}_s\Delta\mathbf{x}_s^T\right] = \left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T E\left[\Delta\mathbf{d}_s\Delta\mathbf{d}_s^T\right]\mathbf{G}\left(\mathbf{G}^T\mathbf{G}\right)^{-1} \qquad (5)$$

Assuming equal variances σ^2 for the measurement errors, i.e. $E\left[\Delta\mathbf{d}_s\Delta\mathbf{d}_s^T\right] = \sigma^2 I$ we obtain

$$E\left[\Delta\mathbf{x}_s\Delta\mathbf{x}_s^T\right] = \sigma^2\left(\mathbf{G}^T\mathbf{G}\right)^{-1} = \sigma^2\mathbf{A} \qquad (6)$$

In case the spatial origin is located in the source, and axes are

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(east, north, up), then the horizontal errors are computed as $\sigma \cdot HDOP$ where

$$HDOP=\sqrt{A_{11}+A_{22}} \tag{7}$$

where A_{11}, A_{22} are two (first) diagonal elements of $\mathbf{A}$. Thus HDOP describes how range measurement errors translate into horizontal positioning errors. For three dimensions position dilution of precision (PDOP) is used $PDOP=\sqrt{A_{11}+A_{22}+A_{33}}$. DOP measures are related to Cramer-Rao Lower Bound (CRLB). It has been shown that DOP measures are similar for both TOA and TDOA methods for certain classes of TDOA algorithms which use iterative least squares technique [10,11]. While HDOP indicates geometry impact on the source localization, it is not clear if it can be used when closed-form TDOA localization solutions are used as HDOP measures for these methods cannot be derived similar to (7). This paper experimentally studies the applicability of HDOP measure (7) for representative state-of-the-art closed-form and iterative methods.

III. EXPERIMENTAL SETUP

Similar to other experimental SHM studies the structures under investigation are aluminum plates. In the following experiments 1.6mm thick plate from Al 7075 alloy is used, 610mm x 915mm size. We study scenarios with three to five sensors attached to the plate. Sensors are Digital Wave B225 piezoelectric transducers. Signals are acquired by parallel 24-bit NI PXI 4472 signal acquisition units from National Instruments (NI). The NI's Labview 8.2 software is used for signal processing.

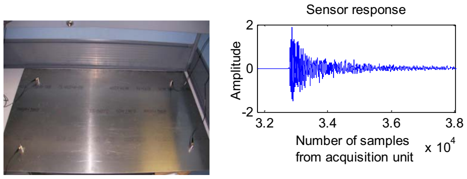

Fig. 1. Experimental aluminum plate with attached sensors (left) and typical sensor response (right).

After the impact the instrumentation detects signal bursts and estimates the time differences of arrival. Fig.1 shows plate with attached sensors and a typical burst. Time differences are obtained as a difference between sample counts from two sensors corresponding to front edges of the bursts which are then multiplied to sampling interval of PXI unit. Front edge is detected by comparing the strength of sensor signal to a threshold. Initially a calibration is performed for estimating the sound propagation speed in the plate. A signal is generated by one of the sensors and time differences are estimated by two others. As all sensors are at known locations the sound speed is estimated assuming isotropic sound propagation as sensor geometry is the main focus, while the reader is referred to [5] on localization in anisotropic propagation materials. In all experiments the time differences translate into source-to-sensor range differences, d_{ij}, given the estimated speed. The processing Labview software is capable to estimate the impact location in real-time, visualize it on the screen and save it in a file for a post-processing.

IV. AN ITERATIVE LOCALIZATION METHOD

Many methods have been proposed for source localization including [3-6, 10-11, 13-14]. For HDOP validation we derive an iterative approach which provides results consistent with the other iterative methods not based on the solution (4). For n sensors the two unknowns in $\mathbf{r}_s$ can be estimated using following system of equations:

$$|\mathbf{r}_i-\mathbf{r}_s|-|\mathbf{r}_1-\mathbf{r}_s|=d_{i1},\quad i=2,\ldots,n \tag{8}$$

The spatial origin is mapped to the first sensor. Then the distance between the source and sensor i is the following:

$$|\mathbf{r}_i-\mathbf{r}_s|=|\mathbf{r}_s|+d_{i1} \tag{9}$$

We square (9) and denote $\mathbf{S}=[\mathbf{r}_2 \ldots \mathbf{r}_n]^T$, $\mathbf{d}=[d_{21}\ldots d_{n1}]^T$, $\mathbf{D}=\left[|\mathbf{r}_2|^2-d_{21}^2,\ldots,|\mathbf{r}_n|^2-d_{n1}^2\right]^T$. After arithmetic manipulations the following equation is obtained:

$$\mathbf{S}\mathbf{r}_s=\frac{1}{2}\mathbf{D}-|\mathbf{r}_s|\,\mathbf{d} \tag{10}$$

For an iterative solution $|\mathbf{r}_s|$ is fixed and the least squares solution of (10) is the following:

$$\mathbf{r}_s=\frac{1}{2}\left[\mathbf{S}^T\mathbf{S}\right]^{-1}\mathbf{S}^T\left(\mathbf{D}-|\mathbf{r}_s|\mathbf{d}\right) \tag{11}$$

Then $\mathbf{r}_s$ is obtained by iterating (11) with fixed $|\mathbf{r}_s|$ and updating $|\mathbf{r}_s|$ from $\mathbf{r}_s$ for next iteration. Typically only few iterations are required for the convergence. [13] provides a similar solution. For HDOP validation and comparative study the location is also estimated using two popular closed-form solutions including a spherical-intersection method (closed-form-1) from [13] and a closed-form least squares approach from [3] (closed-form-2). The approach in [3] is similar to methods in [5] and [14]. Various impact locations are selected for geometry studies (red dotes indicate true impact locations in the figures). The results are summarized in the next section.

TABLE I
CORRELATION OF POSITION ROOT MEAN SQUARE (RMS) ERROR AND HDOP

Method	Left source	Center source	Right source
3 Sensors (RMS in cm), Sensor geometry 1			
Iterative	3.44	4.91	6.49
HDOP	**1.24**	**2.18**	**5.3**
3 Sensors (RMS in cm), Sensor geometry 2			
Iterative	4.87	5.05	6.93
HDOP	**3.16**	**6.34**	**31.02**

TABLE II

Correlation of Position Root Mean Square (RMS) Error and HDOP

Method	Left source	Center source	Right source
4 Sensors (RMS in cm)			
Iterative	2.08	1.07	1.80
HDOP	**1.34**	**1.15**	**1.34**
5 Sensors (RMS in cm), Sensor geometry 1			
Iterative	3.21	3.56	4.44
Closed-Form 1	2.98	3.65	4.42
Closed-Form 2	3.35	3.05	4.01
HDOP	**1.05**	**1.81**	**4.46**
5 Sensors (RMS in cm), Sensor geometry 2			
Iterative	3.21	3.56	4.44
Closed-Form 1	2.98	3.65	4.42
Closed-Form 2	3.35	3.05	4.01
HDOP	**1.05**	**1.81**	**4.46**

TABLE III

Minimum Number of Sensors

Method	Minimum number of required sensors
Iterative	3
Closed-Form 1	4
Closed-Form 2	5

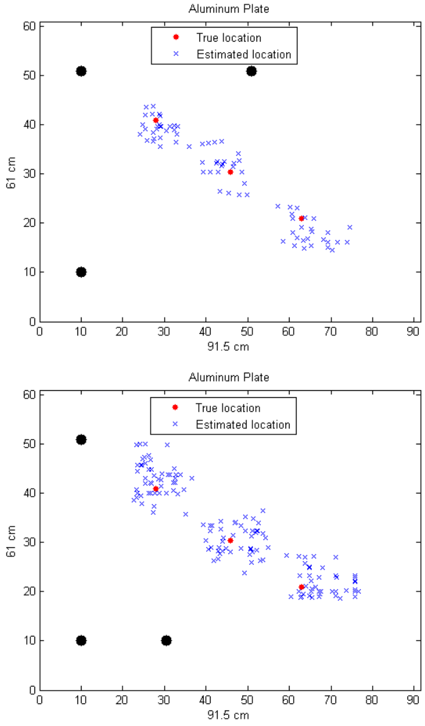

Fig. 2. Impact location estimates for 3 sensors with sensor geometries 1(top) and 2 (bottom).

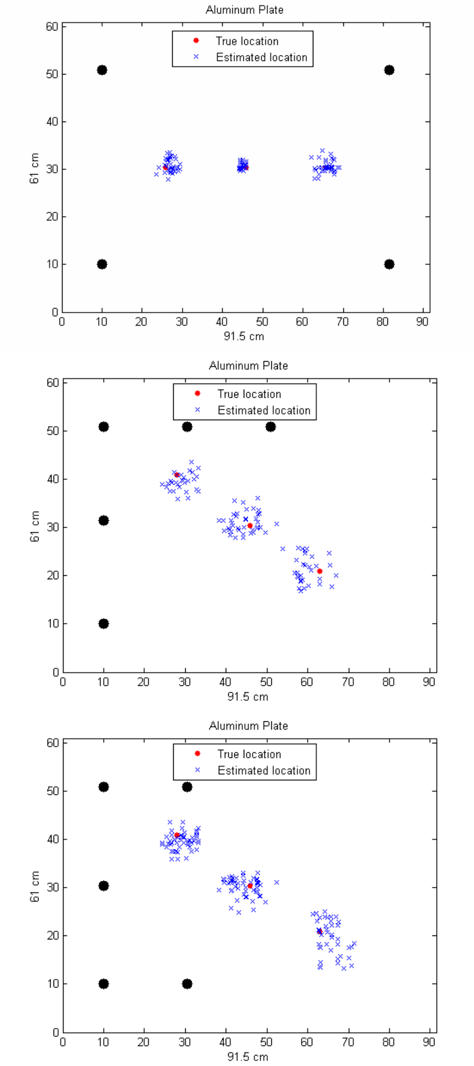

Fig. 3. Estimated impact locations for 4 sensors (top), 5 sensors with sensor geometry 1 (middle) and 2 (bottom).

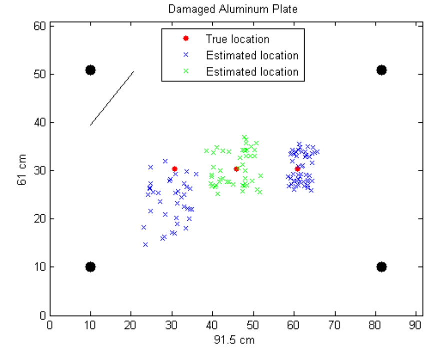

Fig. 4. Estimated impact locations for 4 sensors on damaged aluminum plate

V. EXPERIMENTS

In all experiments corresponding to different sensor placement geometries three impact source locations are considered. Fig. 2 shows experiments with three sensors and two geometries. Only iterative techniques can be used with three sensors. Table I shows the correlation between experimental positioning errors and HDOP. One can observe that HDOP correlates with positioning errors. For the right source HDOP figure is very high indicating bad geometry and potential big errors while experimental error is more moderate.

Fig. 3 shows experimental data for four and five sensors, while Table II tabulates the errors and HDOP values. One can observe that in general HDOP correlates with geometry impact and relative error values but in many cases for the left and middle sources experimental errors are relatively high which indicates on error factors different from the geometry. One should note that all the methods under study are highly nonlinear functions of the measurements and measurement errors can be affected by nonlinearity factors in addition to geometry.

Finally Fig. 4 demonstrates the influence of propagation media on the positioning accuracy. The aluminum plate is cut close to the top left sensor to eliminate line-of-sight propagation. The signal propagation time from the source to this sensor is higher due to the longer paths – mostly due to diffraction on the cut boundaries. While this moderately affects the accuracy of the middle and right source positioning as propagation distance does not increase essentially one can clearly observe significantly biased estimation for the left source. To avoid such errors one can use preliminary recorded list of eligible sensors for different structure areas which are used after a coarse estimation is performed with all responding sensors. Other techniques may use higher sensitivity thresholds or integrity monitoring in which positioning is performed for different sensor subsets and resulting estimates are compared for integrity tracking.

VI. CONCLUSIONS

In this paper we validated the concept of dilution of precision (DOP) for impact localization in SHM. Sensor placement geometry is important and the localization algorithm can provide an accuracy estimate by multiplying DOP to distance measurement error. We described the experimental setup which is used for the validation. While DOP concepts were known in navigation community they were not applied in SHM. There are no theoretically justified DOP concepts for all the state-of-the-art localization methods used for SHM. We experimentally demonstrated that the known DOP measure which is derived for iterative least squares algorithms can be also used for other localization methods including closed-form solutions.

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